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ARITHMETIC.

DEFINITIONS, NOTATION, AND NUMERATION.

ARTICLE 1. By a UNIT is meant a single object or thing, considered as one and undivided.

2. NUMBER is the name by which we signify how many objects or things are considered, whether *one* or *more*. When, for instance, we speak of one horse, two apples, three yards, or four hours, the number of the things referred to will be one, two, three, or four, according to the case; and so one, two, three, four, and the rest, are called numbers.

3. NUMBERS are considered either as ABSTRACT or CONCRETE.

Abstract numbers are those which have no reference to any particular kind of unit; thus, five, as an abstract number, signifies five units only, without any regard to particular objects.

Concrete numbers are those which have reference to some particular kind of unit; thus, when we speak of five hours, six yards, seven horses, the numbers five, six, seven, are said to be concrete numbers, having reference to the particular units one hour, one yard, one horse, respectively.

4. ARITHMETIC is the science of Numbers.

5. All numbers in common Arithmetic are expressed by means of the figure 0, commonly called zero or a cypher, which has no value in itself, and nine significant figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, which denote respectively the numbers one, two, three, four, five, six, seven, eight, nine. These ten figures are sometimes called DIGITS; but this name is often improperly limited to the nine significant figures above mentioned, which are then called the nine digits.

The number one, which is represented by the figure 1, is called UNITY.

6. When any of these figures stands by itself, it expresses its simple or intrinsic value; thus, 9 expresses nine abstract units, or nine particular

things: but when it is followed by another figure, it then expresses ten times its simple value; thus, 94 expresses ten times nine units, together with four units more: when it is followed by two figures, it then expresses one hundred times its simple value; thus, 943 expresses one hundred times nine units, together with ten times four units, and also three units more: and so on by a tenfold increase for each additional figure that follows it.

The value, which thus belongs to a figure in consequence of its position or place, is called its LOCAL VALUE.

Therefore all numbers have a simple or intrinsic value, and also a local value.

7. It appears then, that, in common Arithmetic we proceed towards the left from units to tens of units; from tens of units to tens of tens of units, or hundreds of units; from hundreds of units to tens of hundreds of units, or thousands of units; from thousands of units to tens of thousands of units; from tens of thousands of units to tens of tens of thousands of units, that is, to hundreds of thousands of units; thence to tens of hundreds of thousands of units, or millions of units; thence to tens of millions of units, hundreds of millions of units, &c., till we come to millions of millions of units, which are called billions of units, and so on to trillions, quadrillions, &c.

Thus, 10 represents one ten of units, together with no units; or, as it is briefly read, ten. 11 represents one ten of units, together with one unit; or, as it is briefly read, eleven. Similarly 12, 13, 14, 15, 16, 17, 18, 19, respectively represent one ten of units together with two, three, four, five, six, seven, eight, nine units; they are respectively read twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

The next ten numbers are expressed by 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 which respectively represent two tens of units together with no one, two, three, four, five, six, seven, eight, nine units; they are briefly read twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

The next ten numbers are expressed by 30, 31, 32, 33, 34, 35, 36, 37, 38, 39 which are respectively read thirty, thirty-one, thirty-two, thirty-three, thirty-four, thirty-five, thirty-six, thirty-seven, thirty-eight, thirty-nine: we thus arrive at 40 (forty), 50 (fifty), 60 (sixty), 70 (seventy), 80 (eighty), 90 (ninety).

99 is the largest number which can be expressed by two figures, since it represents nine tens of units together with nine units; the next number

to this is 100, which represents ten tens of units, or one hundred of units, together with no tens of units, together with no units; or, as it is briefly read, one hundred.

By pursuing the same system in higher numbers the figure occupying the fourth place from the right hand will represent so many tens of hundreds of units, or thousands of units; the figure in the fifth place will represent so many tens of thousands of units; and so on.

205 represents two hundreds of units, together with no tens of units, together with five units; or, as it is briefly read, two hundred and five.

5473 represents five thousands of units, together with four hundreds of units, together with seven tens of units, together with three units; or, as it is briefly read, five thousand, four hundred and seventy-three.

7040730 represents seven millions of units, together with no hundreds of thousands of units, together with four tens of thousands of units, together with no thousands of units, together with seven hundreds of units, together with three tens of units, together with no units; or, as it is briefly read, seven millions, forty thousand, seven hundred and thirty.

107834265 represents one hundred of millions of units, together with no tens of millions of units, together with seven millions of units, together with eight hundreds of thousands of units, together with three tens of thousands of units, together with four thousands of units, together with two hundreds of units, together with six tens of units, together with five units; or, as it is briefly read, one hundred and seven millions, eight hundred and thirty-four thousand, two hundred and sixty-five.

8. **Notation** is the art of expressing any number by figures which is already given in words. **Numeration** is the converse of Notation, being the art of expressing any number in words which is already given in figures.

9. The method above explained of denoting numbers by means of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and combinations of them, was brought into Europe by the Arabs, and it is therefore often called the **Arabic Notation**. It was derived by the Arabs from the Hindoos. This method of notation is now in common use, not only in this country, but throughout Europe.

Ex. I.

Exercises in Notation and Numeration.

Express the following numbers in figures:

(1) Sixty-three; eighty-one; ninety-nine; forty; thirteen.

1—2

(2) Two hundred; three hundred and three; seven hundred and sixty-four; eight hundred and eighty-eight.

(3) Four thousand; one thousand, four hundred and seventy-one; six thousand, nine hundred and thirty; nine thousand and nine.

(4) Twenty-seven thousand, five hundred and four; thirty-three thousand; nine thousand and sixteen.

(5) One hundred thousand; six hundred and seventy-six thousand and fifty; two hundred and two thousand, five hundred and ninety-three.

(6) Seven millions, three thousand; eleven millions, one hundred and eight thousand, one hundred and six; fifty-four millions, fifty-four thousand and eighty-eight; six hundred and thirteen millions, twenty thousand, three hundred and three.

(7) Two billions; nine billions, three hundred thousand and twenty-one; ninety-four billions, ninety millions, ninety-four thousand, nine hundred and four.

Write down in words at full length the following numbers:

(1) 43; 60; 88; 97; 59; 12; 21; 19.
(2) 256; 401; 500; 999; 365; 578; 837.
(3) 2000; 1724; 3003; 7584; 1075; 4541.
(4) 37003; 47049; 63090; 80008; 341323.
(5) 6850406; 8080808; 7849630; 418254.
(6) 10000001; 20220022; 92568987; 30180070.
(7) 2560530200; 800309560; 9738413208.
(8) 7070000423; 987654321; 5707068080.
(9) 100198700010090; 48726870634103264.

ADDITION.

10. Addition is the method of finding a number, which is equal to two or more numbers taken together.

The number found by adding two or more numbers together is called the sum or amount of the several numbers so added.

11. There are two kinds of Addition, Simple and Compound.

It is Simple Addition, when the numbers to be taken together are all abstract numbers; or when they are all concrete numbers of the same denomination, as *all* pence, *all* days, *all* pints.

It is Compound Addition, when the numbers to be taken together are concrete numbers of the same kind, but of different denominations of that kind; as pounds, shillings, and pence; or years, months, and days; or gallons, quarts, and pints.

12. The sign +, PLUS, placed between two or more numbers, signifies that the numbers are to be added together: thus 2+5+7 signifies that 2, 5, and 7 are to be added together, and denotes their sum.

The sign =, EQUAL, placed between two numbers, signifies that the numbers are equal to one another.

The sign, $\overline{\quad\quad}$ VINCULUM, placed over numbers, and the sign ( ) or { }, called a BRACKET, enclosing numbers within it, are used to denote that all numbers under the vinculum, or within the bracket, are equally affected by all numbers not under the vinculum or within the bracket: thus $\overline{2+3}$ or (2+3) or {2+3}, each signify, that whatsoever is outside the vinculum or bracket which affects 2 in any way, must also affect 3 in the same way, and conversely.

The sign ∴ signifies 'therefore.'

SIMPLE ADDITION.

13. RULE. Write down the given numbers under each other, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lowest line.

Find the sum of the column of units; if it be under ten, write it down under the column of units, below the line just drawn; if it exceed ten, then write down the last figure of the sum under the column of units, and carry to the next column the remaining figure or figures; treat each succeeding column in the same way, and write down the full sum of the extreme left-hand column. The entire sum so marked down will be the sum or amount of the separate numbers.

14. Add together 5469, 743, and 27.

Proceeding by the Rule given above, we obtain

$$\begin{array}{r}5469\\743\\27\\\hline 6239\end{array}$$

The reason for the Rule will appear from the following considerations.

When we take the sum of 7 units and 3 units and 9 units, we get 10 units and 9 units, or 19 units; we therefore place the 9 units under the column of units and carry on the 1 ten units to the next column, viz. the column of tens.

Now the sum of 1 ten, 2 tens, 4 tens, and 6 tens, is 10 tens and 3 tens, or 13 tens; we therefore place the 3 tens under the column of tens and carry on the 1 hundred units to the next column, viz. the column of hundreds.

Again, the sum of 1 hundred, 7 hundreds, and 4 hundreds, is 10 hundreds and 2 hundreds, or 12 hundreds; we therefore place the 2 hundreds under the column of hundreds, and carry on the 1 thousand units to the next column, viz. the column of thousands.

Again, the sum of 1 thousand and 5 thousands, is 6 thousands; we therefore place the 6 under the column of thousands, and the entire sum is 6239.

15. The above example might have been worked thus, putting down at full length the local value of all the figures.

$$\begin{array}{rl} \text{Thus } 5469 = & 5000+400+60+9 \\ +743 = & +700+40+3 \\ +27 = & +20+7. \end{array}$$

Now adding the columns, we get the sum

$$=5000+1100+120+19$$

$$=5000+\overline{1000+100}+\overline{100+20}+\overline{10+9},$$

(since $1100=1000+100$, $120=100+20$, and $19=10+9$)

$$=6000+200+30+9,$$

(collecting the thousands together, the hundreds together, and so on)

$$=6239.$$

Note. The truth of all results in Addition may be proved by adding the columns first upwards as in the above example, and then adding them downwards; if the results be the same, the operation in each case will in all probability have been performed correctly.

Ex. II.

Examples in Simple Addition.

$$\begin{array}{lr} (1) & 12 \\ & 35 \\ & 56 \\ & \underline{80} \\ & 183 \end{array} \qquad \begin{array}{lr} (2) & 57 \\ & 87 \\ & 65 \\ & \underline{43} \\ & \end{array} \qquad \begin{array}{lr} (3) & 234 \\ & 567 \\ & 753 \\ & \underline{345} \\ & 1899 \end{array} \qquad \begin{array}{lr} (4) & 654 \\ & 321 \\ & 804 \\ & \underline{509} \\ & \end{array}$$

(5)
494
587
656
336

(6)
1721
3333
5046
2754

(7)
750
36
1843
561
3190

(8)
4789
2346
3857
5005

(9)
9102
479
8776
901

(10)
84670
5437
29
21904

(11)
1790621
206803
353
9003766

(12)
250783
21003
5734
40036
21
100001
423578

(13)
627432
543201
678641
548200
868759
345678

(14)
892764
93687
9482
100
152346
11

(15)
1807353
298743
5987
760003
247
50705

(16)
117064
92073
827569
351
777777
65656

(17) Add together 7384, 326, 6780, and 57; also 6740, 9745, 5769, 8031, 6543, 2002, and 9999; also 89, 4500, 423, 2024, 5408, 60546, and 9401.

(18) Add together 83746, 2478, 692577, 456, and 7; also 935478, 262, 13897, 598453, 25, 3734, 724008, and 649768.

(19) Find the sum of 4738685, 237869513, 148794343678, 865, 4647, and 250; also of 685339582, 78602045, 370489000, 7055591234, 276, 9123456789, and 5000: also of 888929944, 73600, 279784662, 333, 5875396006, 4827532, 486684836, 80032148379, 12345, 1112858673, and 53800000835.

(20) Add together one thousand, four hundred and eighty-three; seven hundred and ninety-six; thirty-nine; forty thousand, seven hundred and forty-four; five thousand, eight hundred and sixty; fifty thousand and seven.

(21) Add together the following numbers: fifteen thousand, seven hundred and ninety-six; four hundred and nine; two hundred and thirty-four thousand and fifty; four millions, three thousand and seventy-six; forty thousand and thirty six; ten thousand, nine hundred and one.

(22) Add together the following numbers: twenty-two millions, six hundred thousand, five hundred and three; five hundred and sixty-three

millions, seventy-six thousand and thirty-four; one hundred and eleven millions, six hundred and fifty thousand and fifty; three hundred and twenty-six millions, seven thousand, nine hundred and ninety-one; one thousand seven hundred and ten millions, one thousand seven hundred and ten; one billion, three hundred thousand and five.

SUBTRACTION.

16. Subtraction is the method of finding what number remains when a smaller number is taken from a greater number.

The number found by subtracting the smaller of two numbers from the greater is called the Remainder.

17. There are two kinds of Subtraction, SIMPLE and COMPOUND, which differ from each other in precisely the same way, in which Simple and Compound Addition differ from each other.

18. The sign –, minus, placed between two numbers, signifies that the second number is to be subtracted from the first number.

SIMPLE SUBTRACTION.

19. RULE. Place the less number under the greater number, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lower line.

Take, if possible, the number of units in each figure of the lower line from the number of units in each figure of the upper line which stands immediately over it, and put the remainder below the line just drawn, units under units, tens under tens, and so on: but if the units in any figure in the lower line exceed the number of units in the figure above it, add ten to the upper figure, and then take the number of units in the lower figure from the number in the upper figure thus increased; put the remainder down as before, and then carry one to the next figure of the lower line. The entire difference or remainder, so marked down, will be the difference or remainder of the given numbers.

20. Ex. Subtract 4938 from 5123.

Proceeding by the Rule given above, we obtain

$$\begin{array}{r} 5123 \\ \underline{4938} \\ 185 \end{array}$$

so that the remainder is one hundred and eighty-five (185).

The reason for the Rule will appear from the following considerations.

We cannot take 8 units from 3 units, we therefore add 10 units to the 3 units, which are thus increased to 13 units; and taking 8 units from 13 units we have 5 units left; we therefore place 5 under the column of units: but having added 1 ten units to the upper number, we must add the same number of units (1 ten units) to the lower number, so that the difference between the two numbers may not be altered; and adding 1 ten units to the 3 ten units in the lower number, we obtain 4 tens or 40 instead of 3 tens or 30.

Again, we cannot take 4 tens from 2 tens; we therefore add 10 tens or 1 hundred to the 2 tens, which thus become 12 tens or 120; and then taking 4 tens or 40 from 12 tens or 120, we have 8 tens or 80 remaining; we therefore place 8 under the column of tens: but having added 1 hundred to the upper number, we must add 1 hundred to the lower number for the reason given above; and adding 1 hundred to the 9 hundreds in the lower number, we obtain 10 hundreds or 1000 instead of 900.

Again, we cannot take 10 hundreds from 1 hundred, and we therefore add 10 hundreds or 1 thousand to the 1 hundred, which thus becomes 11 hundreds or 1100: and taking 10 hundreds or 1000 from 11 hundreds or 1100, we have 1 hundred or 100 left; we therefore place 1 under the column of hundreds: but having added 10 hundreds or 1 thousand to the upper number, we must add 1 thousand to the lower number for the reason given above; and adding 1 thousand to the 4 thousands in the lower number, we obtain 5 thousands or 5000;

5000 taken from 5000 leaves 0;

therefore the whole difference or remainder is 185.

21. The above Example might have been worked thus, putting down at full length the local values of the figures:

$$\begin{aligned}5123 &= 5000 + 100 + 20 + 3\\ &= \overline{4000 + 1000} + 100 + 20 + 3\\ &= \overline{4000 + 1000} + 100 + \overline{10 + 10} + 3\\ &= 4000 + 1000 + 110 + 13\end{aligned}$$

(collecting the first 10 with the 100, and the second 10 with the 3)

$$4938 = 4000 + 900 + 30 + 8.$$

Therefore subtracting the columns, thousands from thousands, &c. we get the remainder or difference

$$\begin{aligned}&= 100 + 80 + 5\\ &= 185.\end{aligned}$$

Note. The truth of all results in Subtraction may be proved by adding the less number to the difference or remainder; if this sum equals the larger number, the result obtained by subtraction may be presumed to be correct.

Ex. III.

Examples in Simple Subtraction.

(1)	663	(2)	976	(3)	704	(4)	806
	580		531		483		720
	83						
(5)	4236	(6)	80502	(7)	46095	(8)	555555
	3089		38672		28736		123456

(9)	1000000	(10)	400357261	(11)	89437182
	100101		99988877		15790293

(12) Find the difference between 6543756 and 412848; 7863927 and 826957; 303233334 and 192001222.

(13) How much greater is 164326289 than 48476798?
.... 10000001000 than 7077070077?
.... 7559030640021 than 6990040005679?

(14) Take two thousand and nine, from ten thousand and ninety-six; three thousand and eight, from seven thousand, nine hundred and forty-four.

(15) Required the difference between four and four millions; also between one hundred millions and three hundred thousand.

(16) Subtract five hundred and eighty-four thousand and seventy-six, from fifteen millions, one hundred thousand and three.

---

22. The following method of expressing numbers was used by the Romans, and it is still in occasional, though not in common use, among ourselves. They represented the number one by the character I; five by V; ten by X; fifty by L; one hundred by C; five hundred by D or IↃ; one thousand by M or CIↃ.

All other numbers were formed by a combination of the above characters, subject to the following Rules:

First; When a character was *followed* by one of *equal or less* value, the whole expression denoted the *sum* of the values of the single characters; for instance, II stood for 2; III for 3; VI for 6; VIII for 8; LV for 55; LXXVII for 77; CCXI for 211.

Secondly; When a character was *preceded* by one of *less* value, the whole expression denoted the *difference* of the values of the single characters; for instance, IV stood for $5-1$, or 4; IX for $10-1$, or 9; XIX for $10+\overline{10-1}$ or 19; XL for $50-10$, or 40; XC for $100-10$, or 90.

Thirdly; Every Ↄ annexed to IↃ increased the value of the latter tenfold; for instance, IↃↃ stood for 5000; IↃↃↃ for 50000; and so forth. And every C prefixed and Ↄ annexed to CIↃ increased the value of the latter tenfold; for instance, CCIↃↃ stood for 10000; CCCIↃↃↃ for 100000; and so forth.

Fourthly; A line drawn over a character or characters increased the value of the latter a thousand-fold; for instance, $\overline{\text{V}}$ stood for 5000; $\overline{\text{C}}$ for 100000; $\overline{\text{IX}}$ for 9000; and so forth.

It follows then that either XXXXVI or XLVI will represent 46: and that either M.DCCC.LIV, or CIↃ.IↃCCCLIV, or $\overline{\text{I}}$.DCCCLIIII will represent 1854.

Ex. IV.

(1) Express in Roman characters, thirty; forty-eight; fifty-nine; 222; 6000; 1843.

(2) Express in words, and also in Arabic figures, the values of XXIII; LXIX; CCXVIII; $\overline{\text{V}}$I; $\overline{\text{C}}$LDCIII; $\overline{\text{MM}}$C.

MULTIPLICATION.

23. Multiplication is a short method of finding the sum of any given number repeated as often as there are units in another given number: thus, when 3 is multiplied by 4, the number produced by the multiplication is the sum of 3 repeated 4 times, which sum is equal to $3+3+3+3$ or 12.

The number to be repeated or added to itself, is called the Multiplicand.

The number which shews how often the multiplicand is to be repeated or added to itself, is called the Multiplier.

The number found by multiplication is called the Product.

The multiplicand and multiplier are sometimes called 'Factors,' because they are factors or makers of the product.

24. Multiplication is of two kinds, Simple and Compound. It is termed Simple Multiplication, when the multiplicand is either an abstract number, or a concrete number of one denomination.

It is termed Compound Multiplication, when the multiplicand contains numbers of more than one denomination, but all of the same kind.

25. The sign ×, placed between two numbers, signifies that the numbers are to be multiplied together.

26. The following Table ought to be learned correctly:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

In the above Table, the second line from the top shews the product of each of the numbers, 1, 2, 3, 4, &c. 11, 12, in the first line, when multiplied by 2; the several products being placed under the respective numbers of the line above, from the multiplication of which they arise: the third line shews the several products, when the figures in the first line are respectively multiplied by 3: and so on.

Note. One of the factors, namely the multiplier, must necessarily be an 'abstract number'; since it would be absurd to speak of 6 shillings multiplied by 4 shillings. We can multiply 6 shillings by 4, *i. e.* we can find how many shillings there are in four times six shillings; but there is no meaning in 6 shillings multiplied by 4 shillings.

SIMPLE MULTIPLICATION.

27. RULE. Place the multiplier under the multiplicand, units under units, tens under tens, and so on. Multiply each figure of the multiplicand, beginning with the units, by the figure in the units' place of the multiplier (by means of the table given for Multiplication); set down and carry as in Addition. Then multiply each figure of the multiplicand,

beginning with the units, by the figure in the tens' place of the multiplier, placing the first figure so obtained under the tens of the line above, the next figure under the hundreds, and so on. Proceed in the same way with each succeeding figure of the multiplier. Then add up all the results thus obtained, by the rule of Simple Addition.

Note. If the multiplier does not exceed 12, the multiplication can be effected easily in one line, by means of the Table given above.

28. Ex. Multiply 7654 by 397.

Proceeding by the Rule given above, we obtain

$$\begin{array}{r} 7654 \\ 397 \\ \hline 53578 \\ 68886 \\ 22962 \\ \hline 3038638 \end{array}$$

The reason for the Rule will appear from the following considerations.

When 7654 is to be multiplied by 7, we first take 4 seven times, which by the Table gives 28, *i.e.* 8 units and 2 tens; we therefore place down 8 in the units' place and carry on the 2 tens: again, 5 tens taken 7 times give 35 tens, to which add 2 tens, and we obtain 37 tens, or 7 tens and 3 hundreds; we put down 7 in the tens' place, and carry on 3 hundreds: again, 6 hundreds taken 7 times give 42 hundreds, to which add 3 hundreds, and we obtain 45 hundreds, or 4 thousands and 5 hundreds; we put down 5 in the hundreds' place, and carry on the 4 thousands: again, 7 thousands taken 7 times give 49 thousands, to which we add the 4 thousands, thus obtaining 53 thousands, which we write down.

Next, when we multiply 7654 by the 9, we in fact multiply it by 90; and 4 units taken 90 times give 360 units, or 3 hundreds, 6 tens, and 0 units; therefore, omitting the cypher, we place the 6 under the tens' place, and carry on the 3 to the next figure, and proceed with the operation as in the line above.

When we multiply 7654 by the 3, we in fact multiply by 300; and 4 multiplied by 300 gives 1200, or 1 thousand, 2 hundreds, 0 tens, and 0 units; therefore, omitting the cyphers, we place the first figure 2 under the hundreds' place, and proceed as before. Then adding up the three lines of figures which we have just obtained, we obtain the product of 7654 by 397.

29. The above Example might have been worked thus, putting down at full length the local values of the figures:

$$\begin{array}{rrrrrrr}
7654= & & & 7\times1000+ & 6\times100+ & 5\times10+ & 4 \\
397= & & & & 3\times100+ & 9\times10+ & 7 \\ \cline{4-7}
 & & & 49\times1000+ & 42\times100+ & 35\times10+ & 28 \\
 & & 63\times10000+ & 54\times1000+ & 45\times100+ & 36\times10 & \\
 & 21\times100000+ & 18\times10000+ & 15\times1000+ & 12\times100 & & \\ \cline{2-7}
 & 21\times100000+ & 81\times10000+ & 118\times1000+ & 99\times100+ & 71\times10+ & 28
\end{array}$$

which =

$$\begin{array}{rlllllll}
20\times100000+ & 1\times100000 & & & & & \\
 & +\ 8\times100000+ & 1\times10000 & & & & \\
 & +\ 1\times100000+ & 1\times10000+ & 8\times1000 & & & \\
 & & & +\ 9\times1000+ & 9\times100 & & \\
 & & & & +\ 7\times100+ & 1\times10 & \\
 & & & & & +2\times10+ & 8 \\ \hline
\end{array}$$

$$\begin{aligned}
&2000000+10\times100000+2\times10000+17\times1000+16\times100+3\times10+8\\
&=2000000+1000000+2\times10000+\overline{10\times1000+7\times1000}+\overline{10\times100+6\times100}+3\times10+8\\
&=3000000+\overline{2\times10000+1\times10000}+\overline{7\times1000+1\times1000}+6\times100+3\times10+8\\
&=3000000+3\times10000+8\times1000+600+30+8\\
&=3000000+30000+8000+600+30+8\\
&=3038638
\end{aligned}$$

30. If the multiplier or multiplicand, or both, end with cyphers, we may omit them in the working; taking care to affix to the product as many cyphers as we have omitted from the end of the multiplier or multiplicand, or both. Thus, if 263 be multiplied by 6200, and 570 be multiplied by 3200, we have

$$\begin{array}{r}
263 \\
6200 \\ \hline
526 \\
1578 \\ \hline
1630600
\end{array}
\qquad\qquad
\begin{array}{r}
570 \\
3200 \\ \hline
114 \\
171 \\ \hline
1824000
\end{array}$$

The reason is clear: for, in the first case, when we multiply by the 2, in fact we multiply by 200; and 3 multiplied by 200 gives 600: in the second case, the 7 multiplied by the 2 is the same as 70 multiplied by 200; and 70 multiplied by 200 gives 14000.

31. If the **Multiplier** contain any cypher in any other place, then, in multiplying by the different figures of the multiplier, we may pass over the cypher; taking care, however, when we multiply by the next figure, to place the first figure arising from that multiplication, under the

third figure of the line above instead of the second figure. The reason of this is clear: for, if we were multiplying by 206, when we multiply by the 6 we take the multiplicand 6 times, when we multiply by the 2 we really take the multiplicand, not 20 times, but 200 times.

32. When two numbers are to be multiplied together, it is a matter of indifference, so far as the product is concerned, which of them be taken as the multiplicand or multiplier; in other words, the product of the first multiplied by the second, will be the same as the product of the second multiplied by the first.

$$\begin{aligned} \text{Thus, } 2\times4&=2+2+2+2=8,\\ 4\times2&=4+4\qquad\;\;\,=8; \end{aligned}$$

therefore the results are the same, that is, $2\times4=4\times2$.

That the product of one number multiplied by another, will be equal to the product of the latter multiplied by the former, may perhaps appear more clearly from the following mode of shewing this equality in the case of the numbers 3 and 5.

$$3 \text{ is } =1+1+1;$$

$$\therefore 3\times5=(1+1+1)+(1+1+1)+(1+1+1)+(1+1+1)+(1+1+1)$$

$$\left.\begin{array}{l} =1+1+1\\ +1+1+1\\ +1+1+1\\ +1+1+1\\ +1+1+1 \end{array}\right\}=15.$$

Now, if we regard the *ones* from left to right, there are 3 *ones* taken 5 times; if we regard them taken from top to bottom, we have 5 *ones* repeated 3 times; and the number of ones in each case is the same; *i.e.* $3\times5=5\times3$: and so in the case of any two other numbers multiplied together.

33. The truth of all results in Multiplication may be proved by using the multiplicand as multiplier, and the multiplier as multiplicand: if the product thus obtained be the same as the product found at first, the results are in all probability true.

34. We have hitherto confined our attention to products formed by the multiplication of two factors only. Products may however arise from the multiplication of three or more factors; this is termed CONTINUED MULTIPLICATION: thus $2\times3\times4$ denotes the continued multiplication of the factors 2, 3, and 4; and means that 2 is to be first multiplied by 3,

and the product thus obtained to be then multiplied by 4. The result of such a process would be 24, which is, therefore, the continued product of 2, 3, and 4: we may express it thus, $2 \times 3 \times 4 = 24$.

Ex. V.

Examples in Simple Multiplication.

(1) 534 4 2136	(2) 673 3	(3) 2867 5	(4) 7492 6
(5) 2057 7	(6) 57409 8	(7) 2745638 9	(8) 5763 11 63393
(9) 35976 11	(10) 91525 12	(11) 257 53 771 1285 13621	(12) 96843 17
(13) 87298 46	(14) 16097 59	(15) 296897 83	(16) 69284 90 6235560
(17) 840607 80	(18) 175 189	(19) 6298 769	(20) 5423 603 16269 32538 3270069
	(21) 25607 5004	(22) 78847 8803	

(23) Find the product of 234578 by 18, by 29, and also by 53; of 924846 by 67, by 95, and also by 430; 2846067 by 206, by 1008, and also by 907; 8409631 by 21711, by 7009, by 8435, and also by 7980.

(24) Find the product of 1754 and 9306; of 47506 and 4500; of 149570 and 15790; of 554768 and 39314; of 815085 and 20048; of 123456789 and 987654321; and of 57298492692 and 7008090050321.

(25) Multiply 9487352 by 4731246; 4342760 by 599999; 17376872 by 7399078; 38015732 by 400700065; 574585614865 by 2837154309.

(26) Multiply six hundred and fifty thousand and ninety, by three thousand and eight; also seventy-six millions, eight thousand, seven hundred and sixty-five, by nine millions, nine thousand and nine.

(27) Find the continued product of 12, 17, and 19; of 3781, 3782, and 3783; and of 6565, 6786, and 9898.

(28) Multiply 20470 by 1030, and 2958 by 476, explaining the reason of each step in the process.

DIVISION.

35. Division is the method of finding how often one number, called the Divisor, is contained in another number, called the Dividend. The result is called the Quotient.

36. Division is of two kinds, Simple and Compound. It is called Simple Division, when the dividend and divisor are, both of them, either abstract numbers, or concrete numbers of one and the same denomination.

It is called Compound Division, when the dividend, or when both divisor and dividend contain numbers of different denominations, but of one and the same kind.

37. The sign ÷, placed between two numbers, signifies that the first is to be divided by the second.

38. In Division, if the dividend be a concrete number, the divisor may be either a concrete number or an abstract number, and the quotient will be an abstract number or a concrete number, according as the divisor is concrete or abstract. For instance, 5 shillings taken 6 times give 30 shillings, therefore 30 shillings divided by 5 shillings give the abstract number 6 as quotient; and 30 shillings divided by 6 give the concrete number 5 shillings as quotient.

SIMPLE DIVISION.

39. Rule. Place the divisor and dividend thus:

divisor) dividend (quotient.

Take off from the left-hand of the dividend the least number of figures which make a number not less than the divisor; then find by the Multiplication Table, how often the first figure on the left-hand side of the divisor is contained in the first figure, or the first two figures, on the left-hand side of the dividend, and place the figure which denotes this number of times in the quotient: multiply the divisor by this figure, and bring down the product, and subtract it from the number which was taken off at the left of the dividend: then bring down the next figure of the dividend, and place it to the right of the remainder, and proceed as before; if the divisor be greater than this remainder, affix a cypher to the quotient, and bring down the next figure from the dividend to the right of the remainder, and proceed as before. Carry on this operation till all the figures of the dividend have been thus brought down, and the quotient, if there be no remainder, will be thus determined, or if there be a remainder, the quotient and the remainder will be thus determined.

Note 1. If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value: but if any remainder be greater than the divisor, or equal to it, the last figure of the quotient must be changed for a greater.

Note 2. If the divisor does not exceed 12, the division can easily be effected in one line, by means of the Multiplication Table.

40. Ex. Divide 2338268 by 6758.

Proceeding by the Rule given above, we obtain

```
6758) 2338268 (346
      20274
      -----
       31086
       27032
       -----
        40548
        40548
        -----
```

Therefore the quotient is 346.

The reason for the Rule will appear from the following considerations.

The divisor represents six thousand, seven hundred and fifty-eight: the first five figures on the left-hand side of the dividend represent two millions, three hundred and thirty-eight thousand, and two hundred.

Now the divisor is contained in this 300 times; and 6758 × 300 = 2027400, or omitting the two cyphers at the end for convenience in working, we properly place the 4 under the 2 in the line above; we subtract the product thus found, and we obtain a remainder of 3108, which represents three hundred and ten thousand, and eight hundred. Bring down the 6 by the Rule; this 6 denotes 6 tens or 60, but the cypher is omitted for the reason above stated: the number now represents three hundred and ten thousand, eight hundred and sixty: 6758 is contained 40 times in this, and 6758 × 40 = 270320; we omit the cypher at the end as before, and subtract the 27032 from the 31086; and after subtraction the remainder is 4054, which represents forty thousand, five hundred and forty. Bring down the 8 by the Rule, and the number now represents forty thousand, five hundred and forty-eight: 6758 is contained 6 times exactly in this number.

Therefore 346 is the quotient of 2338268 by 6758.

41. The above example worked without omitting the cyphers would have stood thus:

$$\begin{array}{rrl}
6758) & 2338268 & (300+40+6 \\
 & \underline{2027400} & \\
 & 310868 & \\
 & \underline{270320} & \\
 & 40548 & \\
 & \underline{40548} &
\end{array}$$

hence it appears that the divisor is subtracted from the dividend 300 times, and then 40 times from what remains, and then 6 times from what then remains, and there being now no remainder, 6758 is contained exactly 346 times in 2338268.

The truth of the above method might have been shewn as follows:

$$2338268 = 2027400 + 270320 + 40548$$

$$\begin{array}{rrrrl}
6758) & 2027400 & +270320 & +40548 & (300+40+6 \\
 & \underline{2027400} & & & \\
 & & +270320 & & \\
 & & \underline{+270320} & & \\
 & & & +40548 & \\
 & & & \underline{+40548} &
\end{array}$$

42. Ex. Divide 56438971 by 4064.

$$\begin{array}{rrl}
4064) & 56438971 & (13887 \\
 & \underline{4064} & \\
 & 15798 & \\
 & \underline{12192} & \\
 & 36069 & \\
 & \underline{32512} & \\
 & 35577 & \\
 & \underline{32512} & \\
 & 30651 & \\
 & \underline{28448} & \\
 & 2203 &
\end{array}$$

therefore 4064 is contained in 56438971, 13887 times, with the remainder 2203.

43. *If the divisor terminate with cyphers, the process can be abridged by the following Rule.*

Rule. Cut off the cyphers from the divisor, and as many figures from the right-hand of the dividend, as there are cyphers so cut off at the right-hand end of the divisor; then proceed with the remaining figures according to the Rule, Art. (39); and to the last remainder annex the figures cut off from the dividend for the total remainder.

Ex. Divide 537523 by 3400.

Proceeding by the Rule,

$$\begin{array}{rl}
34{,}00)\ & 5375{,}23\ (158 \\
& \underline{34}\phantom{75{,}23\ (158} \\
& 197\phantom{5{,}23\ (158} \\
& \underline{170}\phantom{5{,}23\ (158} \\
& 275\phantom{{,}23\ (158} \\
& \underline{272}\phantom{{,}23\ (158} \\
& 3\phantom{{,}23\ (158}
\end{array}$$

therefore 3400 is contained in 537523, 158 times with remainder 323.

The reason for the Rule will appear from the following considerations.

537523 is 5375 hundreds and 23, of which 537500 contains 3400, 158 times with a remainder 300 over; and as 23 does not contain 3400 at all, the quotient will evidently be 158, with remainder 300 + 23, or 323.

Note. The same rule applies when the divisor and dividend both terminate with cyphers.

44. Definitions. A number which cannot be separated into factors, which are respectively greater than unity, is called a PRIME number. Thus 3, 5, 7, 11, 13 are prime numbers.

A number which can be separated into factors respectively greater than unity, or which, in other words, is produced by multiplying together two or more numbers respectively greater than unity, is called a COMPOSITE number. Thus 4 which is $= 2 \times 2$, 6 which is $= 2 \times 3$, 8 which is $= 2 \times 2 \times 2$, are composite numbers; because they are composed or consist of the product of two or more numbers, each of which is greater than unity.

Numbers which have no common factor greater than unity, are said to be PRIME to one another. Thus the numbers 3, 5, 8, 11, are prime to each other.

45. When the divisor is a composite number, and made up of two factors, neither of which exceeds 12, the dividend may be divided by one of the factors in the way of Short Division, and then the result by the other factor: if there be a remainder after each of these divisions, the true remainder will be found by multiplying the second remainder by the first divisor, and adding to the product the first remainder.

Ex. Divide 56732 by 45.

$$45\begin{cases}9\\5\end{cases}\left|\begin{array}{l}56732\\ \hline 6303-5\\ \hline 1260-3\end{array}\right.$$

the total remainder is $9\times3+5$, or $27+5=32$.

Therefore the quotient arising from the division of 56732 by 45 is 1260, with a remainder 32 over.

The reason for the above Rule is manifest from the following considerations.

6303 is 5 times 1260 together with 3,
and 56732 is 9 times 6303 together with 5,
or is 9 times (5 times $1260+3$), together with 5,
or is 45 times $1260+27+5$,
or is 45 times $1260+32$.

46. The accuracy of results in Multiplication is often tested by the following method, which is termed "CASTING OUT THE NINES": add together all the figures in the multiplicand, divide their sum by 9, and set down the remainder; then divide the sum of the figures in the multiplier by 9, and set down the remainder: multiply these remainders together, and divide their product by 9, and set down the remainder: if this remainder be the same as the remainder which results after dividing the product, or the sum of the digits in the product, of the multiplicand and multiplier by 9, the sum is very probably right; but if different, it is sure to be wrong.

This test depends upon the fact that "if any number and the sum of its digits be each divided by 9, the remainders will be the same." The proof of which may be shewn thus:

$$100 = 99 + 1,$$

where the remainder must be one, whether 100, or the sum of the digits in 100, viz. 1, be divided by 9, since 99 is divisible by 9 without a remainder.

Similarly,

$$\begin{aligned} 200 &= 2 \times 99 + 2, \\ 300 &= 3 \times 99 + 3, \\ 400 &= 4 \times 99 + 4, \\ 500 &= 5 \times 99 + 5, \\ \&c. &= \&c. \end{aligned}$$

Hence it appears that if 100, 200, 300, 400, 500, &c. be each divided by 9, and the sum of the digits making up the respective numbers be also divided by 9, the two remainders in each case will be the same.

Also the number

$$\begin{aligned} 532 &= 500 + 30 + 2 \\ &= 5 \times 100 + 3 \times 10 + 2 \\ &= \overline{5 \times 99 + 5} + \overline{3 \times 9 + 3} + 2; \end{aligned}$$

whence it appears that if the parts 5 × 100, 3 × 10, and 2, which make up the entire number, be each divided by 9, the remainders will be 5, 3, 2 respectively; and therefore the remainder, when 532 is divided by 9, will clearly be the same, as when $5+3+2$ is divided by 9.

To explain why the test holds, let us take as an example 533 multiplied by 57.

$$\begin{array}{r} 533 \\ \underline{57} \\ 3731 \\ \underline{2665} \\ 30381 \end{array}$$

Now

$$\begin{aligned} 533 &= 9 \times 59 + 2 = 531 + 2 \\ 57 &= 9 \times 6 + 3 = 54 + 3. \end{aligned}$$

It is clear, since 531 contains 9 without a remainder, that 531 × 57 contains 9 without a remainder; therefore the remainder which is left after dividing the product of 533 and 57 by 9, must be the same as the remainder which is left after dividing the product of 2 and 57 by 9.

Again, since the product of 57 and $2 = (54 + 3) \times 2$, and the product of 54 and 2 when divided by 9 leaves no remainder, therefore the remainder which is left after dividing the product of 533 and 57 by 9, must be the same as the remainder left after dividing the product of 3 and 2 by 9, *i.e.* after dividing the product of the remainders which are left after the division of the multiplicand and multiplier respectively by 9.

Now on dividing either 30381, or the sum of its digits, which is 15, by 9, the remainder left is 6, and 3 × 2 divided by 9 also leaves 6 as remainder. Therefore we conclude that 30381 is the correct product of 533 and 57.

Note. If an error of 9, or any of its multiples, be committed, the results will nevertheless agree, and so the error in that case remains undetected.

Ex. VI.

Examples in Simple Division.

(1) 456 ÷ 2. (2) 90680 ÷ 2. (3) 261070308 ÷ 2.
(4) 6378 ÷ 3. (5) 470850 ÷ 3. (6) 385734 ÷ 3.
(7) 372096 ÷ 4. (8) 47392488 ÷ 4. (9) 337625 ÷ 5.
(10) 9876540 ÷ 5. (11) 890106 ÷ 6. (12) 3782046 ÷ 6.
(13) 623399 ÷ 7. (14) 78432407 ÷ 7.
(15) 164864 ÷ 8. (16) 3812312 ÷ 8.
(17) 7869231 ÷ 9. (18) 39237840 ÷ 9.
(19) 407792 ÷ 11. (20) 91875342 ÷ 11.
(21) 211632 ÷ 12. (22) 43600391 ÷ 12.
(23) 404586 ÷ 13. (24) 786543318 ÷ 17.
(25) 1234560 ÷ 20. (26) 8224776 ÷ 18.
(27) 14683059 ÷ 27. (28) 817286228 ÷ 44.
(29) 54906734 ÷ 59. (30) 6848734752 ÷ 96.
(31) 70865432 ÷ 87. (32) 649305745 ÷ 55.
(33) 28894545 ÷ 123. (34) 433418175 ÷ 615.
(35) 1674918 ÷ 189. (36) 31884740 ÷ 779.
(37) 536819741 ÷ 907. (38) 1111111111111 ÷ 50160.
(39) 8235460800 ÷ 1440. (40) 57380625 ÷ 7575.
(41) 353008972662 ÷ 5406. (42) 599961567212 ÷ 2468.
(43) 26799534687 ÷ 7890000. (44) 57111104051 ÷ 3851.
(45) 1000000000000000 ÷ 1111, and also by 11111.
(46) 634394567 ÷ 164600. (47) 67157148372 ÷ 90009.
(48) 1220225292 ÷ 200563. (49) 7428927415293 ÷ 8496427.
(50) 60435674536845 ÷ 79094451. (51) 65358547823 ÷ 5578.

(52) $3968901531620 \div 687637943$.

(53) Divide 152181255 by 3854, and explain the process.

(54) Divide 143255 by 4093. Explain the operation, and shew that it is correct.

(55) Divide 203534191 by 72, first by Long Division, and then by its factors 8 and 9; and shew that the results in both cases coincide

GREATEST COMMON MEASURE.

47. A MEASURE of any given number is a number which will divide the given number exactly, *i.e.* without a remainder.

Thus, 2 is a measure of 6, because 2 is contained 3 times exactly in 6.

When one number is a measure of another, the former is said to measure the latter.

48. A MULTIPLE of any given number is a number which contains it an exact number of times. Thus 6 is a multiple of 2.

49. A COMMON MEASURE of two or more given numbers is a number which will divide each of the given numbers exactly: thus, 3 is a common measure of 18, 27, and 36.

The GREATEST COMMON MEASURE of two or more given numbers, is the greatest number which will divide each of the given numbers exactly: thus, 9 is the greatest common measure of 18, 27, and 36.

50. *If a number measure each of two others, it will also measure their sum, or difference; and also, any multiple of either of them.*

Thus, 3 being a common measure of 9 and 15, will measure their sum, their difference, and also any multiple of either 9 or 15.

The sum of 9 and $15 = 9 + 15 = 24 = 3 \times 8$;

therefore 3 measures their sum 24.

The difference of 15 and $9 = 15 - 9 = 6 = 2 \times 3$;

therefore 3 measures their difference 6.

Again, 36 is a multiple of 9, and $36 = 3 \times 12$;
therefore 3 measures this multiple of 9; and similarly any other multiple of 9.

Again, 75 is a multiple of 15; and $75 = 3 \times 25$;
therefore 3 measures this multiple of 15; and similarly any other multiple of 15.

51. *To find the greatest common measure of two numbers.*

Rule. Divide the greater number by the less; if there be a remainder, divide the first divisor by it; if there be still a remainder, divide the second divisor by this remainder, and so on; always dividing the last preceding divisor by the last remainder, till nothing remains. The last divisor will be the greatest common measure required.

Ex. Required the greatest common measure of 475 and 589.

Proceeding by the Rule given above,

$$\begin{array}{rrrrl} 475) & 589 & (1 & & \\ & \underline{475} & & & \\ & 114) & 475 & (4 & \\ & & \underline{456} & & \\ & & 19) & 114 & (6 \\ & & & \underline{114} & \\ & & & 0 & \end{array}$$

therefore 19 is the greatest common measure of 475 and 589.

Reason for the above process.

Any number which measures 589 and 475,
also measures their difference, or 589 − 475, or 114, Art. (50),
also measures any multiple of 114, and therefore 4 × 114, or 456, Art. (50);
and any number which measures 456 and 475,
also measures their difference, or 475 − 456, or 19;

and no number greater than 19 can measure the original numbers 589 and 475; for it has just been shewn that any number which measures them must also measure 19.

Again, 19 itself will measure 589 and 475.
For 19 measures 114 (since 114 = 6 × 19);
therefore 19 measures 4 × 114, or 456, Art. (50);
therefore 19 measures 456 + 19, or 475, Art. (50);
therefore 19 measures 475 + 114, or 589;

therefore since 19 measures them both, and no number greater than 19 can measure them both,

19 is their greatest common measure.

52. *To find the greatest common measure of three or more numbers.*

Rule. Find the greatest common measure of the first two numbers; then the greatest common measure of the common measure so found and the third number; then that of the common measure last found and the fourth number, and so on. The last common measure so found will be the greatest common measure required.

Ex. Find the greatest common measure of 16, 24, and 18

Proceeding by the Rule given above,

```
16) 24 (1
    16
    --
     8) 16 (2
        16
        --
```

therefore 8 is the greatest common measure of 16 and 24.

Now to find the greatest common measure of 8 and 18,

```
8) 18 (2
   16
   --
    2) 8 (4
       8
       -
       0
```

therefore 2 is the greatest common measure required.

Reason for the above process.

It appears from Art. (50) that every number, which measures 16 and 24, measures 8 also;

therefore every number, which measures 16, 24, and 18, measures 8 and 18;

therefore the greatest common measure of 16, 24, and 18, is the greatest common measure of 8 and 18.

But 2 is the greatest common measure of 8 and 18;

therefore 2 is the greatest common measure of 16, 24, and 18.

Ex. VII.

1. Find the greatest common measure of

(1) 16 and 72.
(2) 30 and 75.
(3) 63 and 99.
(4) 55 and 121.
(5) 128 and 324.
(6) 120 and 320.
(7) 272 and 425.
(8) 394 and 672.
(9) 720 and 860.

(10) 825 and 960.
(11) 775 and 1800.
(12) 856 and 936.
(13) 176 and 1000.
(14) 1236 and 1632.
(15) 6409 and 7395.
(16) 689 and 1573.
(17) 1729 and 5850.
(18) 5210 and 5718.
(19) 2023 and 7581.
(20) 468 and 1266.
(21) 2484 and 2628.
(22) 3444 and 2268.
(23) 5544 and 6552.
(24) 4067 and 2573.
(25) 10395 and 16819.
(26) 80934 and 110331.
(27) 1242 and 2323.
(28) 13536 and 23148.
(29) 42237 and 75582.
(30) 285714 and 999999.
(31) 10353 and 14877.
(32) 271469 and 30599.

2. Find the greatest common measure of

(1) 14, 18, and 24.
(2) 16, 24, 48, and 74.
(3) 13, 52, 416, and 78.
(4) 837, 1134, and 1347.
(5) 805, 1311, and 1978.
(6) 28, 84, 154, and 343.
(7) 504, 5292, and 1520.
(8) 396, 5184, and 6914.

LEAST COMMON MULTIPLE.

53. A COMMON MULTIPLE of two or more given numbers is a number which will contain each of the given numbers an exact number of times without a remainder. Thus, 144 is a common multiple of 3, 9, 18, and 24.

The LEAST COMMON MULTIPLE of two or more given numbers is the least number which will contain each of the given numbers an exact number of times without a remainder. Thus, 72 is the least common multiple of 3, 9, 18, and 24.

54. *To find the least common multiple of two numbers.*

RULE. Divide their product by their greatest common measure: the quotient will be the least common multiple of the numbers.

Ex. Find the least common multiple of 18 and 30.

Proceeding by the Rule given above,

```
18) 30 (1
    18
    --
    12) 18 (1
        12
        --
         6) 12 (2
            12
            --
             0
```

therefore 6 is the greatest common measure of 18 and 30.

$$\begin{array}{rr} & 18 \\ & 30 \\ \cline{2-2} 6 & \multicolumn{1}{|r}{540} \\ \cline{2-2} & 90 \end{array}$$

therefore 90 is the least common multiple of 18 and 30.

Reason for the above process.

$$18=3\times 6,\text{ and }30=5\times 6.$$

Since 3 and 5 are prime factors, it is clear that 6 is the greatest common measure of 18 and 30; therefore their least common multiple must contain 3, 6, and 5, as factors.

Now every multiple of 18 must contain 3 and 6 as factors; and every multiple of 30 must contain 5 and 6 as factors; therefore every number, which is a multiple of 18 and 30, must contain 3, 5, and 6 as factors; and the least number which so contains them is $3\times 5\times 6$, or 90.

Now, $90=(3\times 6)\times(5\times 6)$, divided by 6,
$=18\times 30$, divided by 6,
$=18\times 30$, divided by the greatest common measure of 18 and 30.

55. Hence it appears that the least common multiple of two numbers, which are prime to each other, or have no common measure but unity, is their product.

56. *To find the least common multiple of three or more numbers.*

Rule. Find the least common multiple of the first two numbers; then the least common multiple of that multiple and the third number, and so on. The last common multiple so found will be the least common multiple required.

Ex. Find the least common multiple of 9, 18, and 24.

Proceeding by the Rule given above,

Since 9 is the greatest common measure of 18 and 9, their least common multiple is clearly 18.

Now, to find the least common multiple of 18 and 24.

$$\begin{array}{l} 18)\,24\,(1 \\ \quad\;\underline{18} \\ \qquad 6)\,18\,(3 \\ \qquad\quad\;\underline{18} \\ \qquad\qquad 0 \end{array}$$

therefore 6 is the greatest common measure of 18 and 24;

therefore the least common multiple of 18 and 24 is equal to (18×24) divided by 6,

$$\begin{array}{r|r}
 & 24 \\
 & \underline{18} \\
 & 192 \\
 & 24 \\
\hline
6 & 432 \\
\hline
 & 72
\end{array}$$

therefore 72 is the least common multiple required.

Reason for the above process.

Every multiple of 9 and 18 is a multiple of their least common multiple 18; therefore every multiple of 9, 18, and 24 is a multiple of 18 and 24; and therefore the least common multiple of 9, 18, and 24 is the least common multiple of 18 and 24: but 72 is the least common multiple of 18 and 24; therefore 72 is the least common multiple of 9, 18, and 24.

57. *When the least common multiple of several numbers is required, the most convenient practical method is that given by the following Rule.*

Rule. Arrange the numbers in a line from left to right, with a comma placed between every two. Divide those numbers which have a common measure by that common measure, and place the quotients so obtained and the undivided numbers in a line beneath, separated as before. Proceed in the same way with the second line, and so on with those which follow, until a row of numbers is obtained in which there are no two numbers which have any common measure greater than unity. Then the continued product of all the divisors and the numbers in the last line will be the least common multiple required.

Note. It will in general be found advantageous to begin with the lowest prime number 2 as a divisor, and to repeat this as often as can be done; and then to proceed with the prime numbers 3, 5, &c. in the same way.

Ex. Find the least common multiple of 18, 28, 30, and 42.

Proceeding by the Rule given above,

$$\begin{array}{r|l}
2 & 18, 28, 30, 42 \\
\hline
2 & 9, 14, 15, 21 \\
\hline
3 & 9, 7, 15, 21 \\
\hline
7 & 3, 7, 5, 7 \\
\hline
 & 3, 1, 5, 1
\end{array}$$

therefore the least common multiple required

$$=2\times 2\times 3\times 7\times 3\times 5=1260.$$

Reason for the above process.

Since $18=2\times 3\times 3$; $28=2\times 2\times 7$; $30=2\times 3\times 5$; $42=2\times 3\times 7$; it is clear that the least common multiple of 18 and 28 must contain as a factor $2\times 2\times 3\times 3\times 7$; and this factor itself is evidently a common multiple of $2\times 3\times 3$, or 18, and of $2\times 2\times 7$, or 28; now the least number which contains $2\times 2\times 3\times 3\times 7$ as a factor, is the product of these numbers; therefore $2\times 2\times 3\times 3\times 7$ is the least common multiple of 18 and 28: also it is clear that the least common multiple of 18, 28 and 30, or of $2\times 2\times 3\times 3\times 7$ and 30, or of $2\times 2\times 3\times 3\times 7$ and $2\times 3\times 5$ must contain as a factor $2\times 2\times 3\times 3\times 7\times 5$, and this factor itself is evidently a common multiple of $2\times 3\times 3$ or 18, $2\times 2\times 7$ or 28, and $2\times 3\times 5$ or 30; hence it follows as before that $2\times 2\times 3\times 3\times 7\times 5$ is the least common multiple of 18, 28, and 30; again the least common multiple of $2\times 2\times 3\times 3\times 7\times 5$ and 42, or of $2\times 2\times 3\times 3\times 7\times 5$ and $2\times 3\times 7$ must contain $2\times 2\times 3\times 3\times 7\times 5$ as a factor, and this factor, as before, is evidently itself a common multiple of 18, 28, 30, and 42; now the least number which contains $2\times 2\times 3\times 3\times 7\times 5$ as a factor, is the product of these numbers.

Therefore this product, or 1260, is the least common multiple required.

Ex. VIII.

1. Find the least common multiple of

(1) 16 and 24. (2) 36 and 75. (3) 7 and 15.
(4) 28 and 35. (5) 319 and 407. (6) 333 and 504.
(7) 2961 and 799. (8) 7568 and 9504. (9) 4662 and 5476.
(10) 6327 and 23997. (11) 5415 and 30105.
(12) 15863 and 21489.

2. Find the least common multiple of

(1) 12, 8, and 9. (2) 8, 12, and 16.
(3) 6, 10, and 15. (4) 8, 12, and 20.
(5) 27, 24, and 15. (6) 12, 51, and 68.
(7) 19, 29, and 38. (8) 24, 48, 64, and 192.
(9) 63, 12, 84, and 14. (10) 5, 7, 9, 11, and 15.
(11) 6, 15, 24, and 25. (12) 12, 18, 30, 48, and 60.
(13) 15, 35, 63, and 72. (14) 9, 12, 14, and 210.
(15) 54, 81, 63, and 14. (16) 24, 10, 32, 45, and 25.

(17) 1, 2, 3, 4, 5, 6, 7, 8, and 9.
(18) 7, 8, 9, 18, 24, 72, and 144.
(19) 12, 20, 24, 54, 81, 63, and 14.
(20) 225, 255, 289, 1023, and 4095.

Ex. IX.

Miscellaneous Questions and Examples on the foregoing Articles.

I.

(1) Explain the principle of the common system of numerical notation. Multiply 603 by 48, and give the reasons for the several steps.

(2) Write at length the meaning of 9090909, and of 90909. Find their sum and difference, and explain fully the processes employed.

(3) Find the difference between the sum of 4715 added to itself 398 times, and the sum of 2017 added to itself 408 times.

(4) A person, whose age is 73, was 37 years old at the birth of his eldest son; what is the son's age?

(5) Explain the meaning of the terms 'vinculum', 'bracket'; and of the signs $+$, $-$, $=$, $\therefore$, $\times$.

Find the value of the following expression:

$$15 \times 37153 - 73474 - 67152 \div 4 + 40734 \times 2.$$

II.

(1) Define 'a Unit', 'Number', 'Arithmetic'. What is the difference between Abstract and Concrete numbers?

(2) The annual deaths in a town being 1 in 45, and in the country 1 in 50; in how many years will the number of deaths out of 18675 persons living in the town, and 79250 persons living in the country, amount together to 10000?

(3) Define 'Notation', 'Numeration'; express in numbers seven hundred thousand four hundred and nine billions.

(4) Find the value of

$$494871 - 94853 + (45079 - 3177) - (54312 - 3987) - (1763 + 231) + 379 \times 379.$$

(5) What number divided by 528 will give 36 for the quotient, and leave 44 as a remainder?

III.

(1) Define Multiplication, and Division. Shew that the product of two numbers is the same in whatever order the operation is performed.

(2) The Iliad contains 15683 lines, and the Æneid contains 9892 lines; how many days will it take a boy to read through both of them, at the rate of eighty-five lines a day?

(3) Explain what is meant by the greatest common measure, and by the least common multiple of two or more numbers; and shew that the product of two numbers is the product of their least common multiple into their greatest common measure. Find the least common multiple of 12, 16, 21, 52, and 70.

(4) Explain the meaning of the sign ÷, and find the value of

$$(7854-4913)\times 3-(20374-12530)\div 53-6+(395456-2364)\div 556.$$

(5) At a game of cricket A, B, and C together score 108 runs; B and C together score 90 runs, and A and C together score 51 runs; find the number of runs scored by each of them.

IV.

(1) Define Addition, and Subtraction. What is meant by a prime number? When are numbers said to be prime to each other? Give examples.

Explain the rule of *carrying* in the addition of numbers; exemplify it in the addition of 3864, 4768, and 15938.

(2) There are two numbers of which the product is 373625; the greater number is 875; find the sum and difference of the numbers.

(3) A father was 21 years old when his eldest son was born; how old will his son be when he is 50 years old, and what will be the father's age when the son is 50 years old?

(4) Write in figures one hundred millions, one hundred thousand, one hundred and one; and in words 1010101010. Express in figures M.DCCC.XL.

(5) When are numbers said to be 'composite'? Find the greatest number which can divide each of the two numbers 849 and 1132; also the least number which can be divided by each of them; explaining the process in each case.

V.

(1) Multiply 478 by 146, and test the result by casting out the nines. In what cases does this method of proof fail? Divide 4843 by 99, and prove the correctness of the operation by any test you please.

(2) What number multiplied by 86 will give the same product as 163 by 430?

(3) In the city of Prague, for every two persons who speak German only, three speak Tschech only, and seven both German and Tschech; and the whole population is 120000. How many speak German only, Tschech only, and both German and Tschech?

(4) A gentleman dies, and leaves his property thus: 10000 pounds to his widow; 15000 pounds to his eldest son, on the condition of his building a national school at a cost of 350 pounds; 5500 pounds to each of his four younger sons; 3750 pounds to each of his three daughters; 4563 pounds to different societies; and 599 pounds in legacies to his servants. What amount of property did he die possessed of?

(5) The quotient arising from the division of 9281 by a certain number is 17, and the remainder is 373. Find the divisor.

VI.

(1) Explain briefly the Roman method of Notation. Express 1563 and 9000 in Roman characters.

(2) Explain the terms 'factor', 'product', 'quotient'; shew by an example how the process of Division can be abridged, if the divisor terminate with cyphers.

(3) The remainder of a division is 97, the quotient 665, and the divisor 91 more than the sum of both. What is the dividend?

(4). Express in words the numbers 270130 and 26784; also write down in figures the number ten thousand, two hundred and thirty four; and find the least number which added to the last number will make it divisible by 8.

(5) A gentleman, whose age is 60, has two sons and a daughter; his age equals the sum of the ages of his children; two years since his age was double that of his eldest son; the sum of the ages of the father and the eldest son is seven times as great as that of the youngest son; find the ages of the children.

FRACTIONS.

58. If 1 represent any concrete quantity, as for instance 1 yard, it is divisible into parts: suppose the parts to be equal to each other, and the number of them 3; one of the parts would be denoted by $\frac{1}{3}$ (read *one-third*), two of them by $\frac{2}{3}$ (read *two-thirds*), three of them or the whole yard by $\frac{3}{3}$ or 1; if another equal portion of a second yard divided in the same manner as the first be added, the sum would be denoted by $\frac{4}{3}$; if two such portions were added, by $\frac{5}{3}$; and so on. Such expressions, representing any number of parts of a unit, that is, of the quantity which is denoted by 1, are termed BROKEN NUMBERS or FRACTIONS; we may therefore define a fraction thus:

59. DEF. A FRACTION denotes a part or parts of a unit; it is expressed by two numbers placed one above the other with a line drawn between them; the lower number is called the DENOMINATOR, and shews into how many equal parts the unit is divided; the upper is called the NUMERATOR, and shews how many of such parts are taken to form the fraction.

Thus $\frac{5}{6}$ denotes that the unit is divided into 6 equal parts, and that 5 of these parts are taken to form the fraction: so, if a yard were divided into 6 equal parts, and 5 of them were taken, then denoting one yard by 1, we should denote the parts taken by the fraction $\frac{5}{6}$. Again, $\frac{7}{6}$ denotes that the unit is divided into 6 equal parts, and that 7 such parts are taken to form the fraction; for instance, in the example before us, one whole yard would be taken, and also one of the equal parts of another yard divided in the same manner as the first.

60. A Fraction also represents the quotient of the numerator by the denominator.

Thus, $\frac{5}{6}$ represents $5 \div 6$; for we should obtain the same result, whether we divide *one* unit into 6 equal parts, and take 5 of such parts (which would be represented by $\frac{5}{6}$); or divide *five* units into 6 equal parts, and take 1 of such parts, which would be equivalent to $\frac{1}{6}^{\text{th}}$ part of 5 units, *i.e.* $5 \div 6$: hence $\frac{5}{6}$ and $5 \div 6$ will have the same meaning.

61. When fractions are denoted in the manner above explained, they are called VULGAR FRACTIONS.

Fractions, whose denominators are composed of 10, or 10 multiplied by itself, any number of times, are often denoted in a different manner; and when so denoted, they are called DECIMAL FRACTIONS.

VULGAR FRACTIONS.

62. In treating of the subject of Vulgar Fractions, it is usual to make the following distinctions:

(1) A PROPER FRACTION is one whose numerator is less than the denominator; thus, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{7}$ are proper fractions.

(2) An IMPROPER FRACTION is one whose numerator is equal to or greater than the denominator; thus, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{7}{5}$ are improper fractions.

(3) A SIMPLE FRACTION is one whose numerator and denominator are simple integer numbers; thus, $\frac{1}{3}$, $\frac{4}{5}$ are simple fractions.

(4) A MIXED NUMBER is composed of a whole number and a fraction; thus, $5\frac{1}{6}$, $7\frac{3}{4}$ are mixed numbers, representing respectively 5 units, together with $\frac{1}{6}$th of a unit; and 7 units, together with $\frac{3}{4}$ths of a unit.

(5) A COMPOUND FRACTION is a fraction of a fraction; thus, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{9}{10}$ are compound fractions.

(6) A COMPLEX FRACTION is one which has either a fraction or a mixed number in one or both terms of the fraction; thus, $\frac{\frac{2}{3}}{\frac{4}{7}}$, $\frac{2\frac{1}{2}}{3}$, $\frac{3}{4\frac{2}{3}}$, $\frac{2\frac{1}{7}}{5\frac{1}{6}}$, $\frac{\frac{2}{3} \text{ of } \frac{1}{2}}{2\frac{1}{6}}$ are complex fractions.

63. It is clear from what has been said, that every integer may be considered as a fraction whose denominator is 1; thus, $5=\frac{5}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 5 of such parts, that is 5 units, are taken.

64. *To multiply a fraction by a whole number, multiply the numerator of the fraction by it.*

Thus, $\frac{2}{7} \times 3 = \frac{6}{7}$.

Reason for the above process.

In $\frac{2}{7}$ the unit is divided into 7 equal parts, and 2 of those parts are taken: whereas in $\frac{6}{7}$ the unit is divided into 7 equal parts, and 6 of those parts are taken; *i.e.* 3 times as many parts are taken in $\frac{6}{7}$ as are taken in $\frac{2}{7}$, the value of each part being the same in each case.

Ex. X.

(1) Multiply $\frac{5}{12}$ separately by 3, 9, 12, 36.
(2) Multiply $\frac{41}{63}$ separately by 7, 15, 21, 45.

65. *To divide a fraction by a whole number, multiply the denominator by it.*

Thus, $\frac{2}{7} \div 3 = \frac{2}{7 \times 3} = \frac{2}{21}$.

Reason for the above process.

In the fraction $\frac{2}{7}$, the unit is divided into 7 equal parts, and 2 of those parts are taken; in the fraction $\frac{2}{21}$, the unit is divided into 21 equal parts, and 2 of such parts are taken: but since each part in the latter case is equal to one-third of each part in the former case, and the same number of parts are taken in each case, it is clear that $\frac{2}{21}$ represents one-third part of $\frac{2}{7}$, or $\frac{2}{7} \div 3$.

Ex. XI.

(1) Divide $\frac{5}{8}$ separately by 2, 3, 4, 5, 10.
(2) Divide $\frac{15}{587}$ separately by 11, 20, 25, 45.

66. *If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

Thus, if the numerator and denominator of the fraction $\frac{2}{7}$ be multiplied by 3, the fraction resulting will be $\frac{6}{21}$, which is of the same value as $\frac{2}{7}$.

Reason for the above process.

In the fraction $\frac{2}{7}$ the unit is divided into 7 equal parts, and 2 of those parts are taken; in the fraction $\frac{6}{21}$ the unit is divided into 21 equal parts, and 6 of such parts are taken. Now there are 3 times as many parts taken in the second fraction as there are in the first fraction; but 3 parts in the second fraction are only equal to 1 part in the first fraction; therefore the 6 parts taken in the second fraction equal the 2 parts taken in the first fraction; therefore $\frac{2}{7} = \frac{6}{21}$.

67. Hence it follows that a whole number may be converted into a vulgar fraction with any denominator, by multiplying the number by the required denominator for the numerator of the fraction, and placing the required denominator underneath;

$$\text{for } 6 = \tfrac{6}{1};$$

and to convert it into a fraction with a denominator 5 or 14, we have

$$6=\frac{6}{1}=\frac{6\times5}{1\times5}=\frac{30}{5},$$

$$6=\frac{6}{1}=\frac{6\times14}{1\times14}=\frac{84}{14}.$$

Ex. XII.

Reduce (1) 7, 9, and 11, to fractions with denominators 3, 7, and 22 respectively; and (2) 26, 109, 117, and 125, to fractions with denominators 2, 5, 13, 23, and 35 respectively.

68. *Multiplying the numerator of a fraction by any number, is the same in effect as dividing the denominator by it, and conversely.*

For if the numerator of the fraction $\frac{6}{8}$ be multiplied by 4, the resulting fraction is $\frac{24}{8}$; and if the denominator be divided by 4, the resulting fraction is $\frac{6}{2}$.

Now the fraction $\frac{24}{8}$ signifies that unity is divided into 8 equal parts, and that 24 such parts are taken; these are equivalent to 3 units: also $\frac{6}{2}$ signifies that unity is divided into 2 equal parts, and that 6 such parts are taken; these are equivalent to 3 units: hence $\frac{24}{8}$ and $\frac{6}{2}$ are equal. The proof of their equality may also be put in this form: that since the unit, in the case of the second fraction, is only divided into 2 equal parts, each part in that case is 4 times as great as each part in the case of the first fraction, where the unit is divided into 8 equal parts; and therefore 4 parts in the case of the first fraction are equal to 1 part in the case of the second; or the 24 parts denoted by the first are equal to the 6 denoted by the second; or, in other words, the fractions $\frac{24}{8}$ and $\frac{6}{2}$ are equal.

Again, if we divide the numerator of the fraction $\frac{6}{8}$ by 2, the resulting fraction is $\frac{3}{8}$; and if we multiply the denominator by 2, the resulting fraction is $\frac{6}{16}$.

Now, $\frac{3}{8}$ signifies that the unit is divided into 8 equal parts, and that 3 of such parts are taken; and $\frac{6}{16}$ signifies that the unit is divided into 16 equal parts, and that 6 of such parts are taken: but each part in $\frac{3}{8}$ is equal to 2 parts in $\frac{6}{16}$; and therefore $\frac{3}{8}$ is of the same value as $\frac{2\times3}{16}$, or $\frac{6}{16}$.

69. *To represent an improper fraction as a whole or mixed number.*

Rule. Divide the numerator by the denominator: if there be no remainder, the quotient will be a whole number; if there be a remainder, put down the quotient as the integral part, and the remainder as the numerator of the fractional part, and the given denominator as the denominator of the fractional part.

Ex. Reduce $\frac{25}{5}$ and $\frac{35}{6}$ to whole or mixed numbers.

By the Rule given above,

$$\tfrac{25}{5} = 5, \text{ a whole number;}$$
$$\tfrac{35}{6} = 5\tfrac{5}{6}.$$

Reason for the above process.

Since $\dfrac{25}{5} = \dfrac{5 \times 5}{5} = \dfrac{5}{5} \times 5$, (Art. 64),

and since $\frac{5}{5}$ signifies that the unit is divided into 5 equal parts, and that 5 of those parts are taken, which 5 parts are equal to the whole unit or 1; therefore $\frac{25}{5} = \frac{5}{5} \times 5 = 1 \times 5$, or 5.

Again, $\dfrac{35}{6} = \dfrac{30+5}{6} = \dfrac{6 \times 5 + 5}{6}$,

which equals $\dfrac{6 \times 5}{6}$ together with $\frac{5}{6}$, that is, $= 5$ together with $\frac{5}{6}$, by what has been said above; or, as it is written, $5\frac{5}{6}$.

Ex. XIII.

Express the following improper fractions as mixed or whole numbers:

(1) $\frac{15}{4}$. (2) $\frac{77}{6}$. (3) $\frac{49}{8}$. (4) $\frac{113}{9}$.
(5) $\frac{63}{7}$. (6) $\frac{443}{12}$. (7) $\frac{587}{13}$. (8) $\frac{183}{55}$.
(9) $\frac{751}{24}$. (10) $\frac{5801}{63}$. (11) $\frac{648}{108}$. (12) $\frac{5876}{157}$.
(13) $\frac{10000}{111}$. (14) $\frac{231750}{153}$. (15) $\frac{14264}{239}$. (16) $\frac{95950}{999}$.
(17) $\frac{25713}{1168}$. (18) $\frac{632625}{1741}$. (19) $\frac{978911}{36425}$. (20) $\frac{46325}{3724}$.

70. *To reduce a mixed number to an improper fraction.*

Rule. Multiply the integer by the denominator of the fraction, and to the product add the numerator of the fractional part; the result will be the required numerator, and the denominator of the fractional part the required denominator.

Ex. Convert $2\frac{4}{7}$ into an improper fraction.

Proceeding by the Rule given above,

$$2\tfrac{4}{7} = \frac{2 \times 7 + 4}{7} = \frac{18}{7}.$$

Reason for the above process.

$2\frac{4}{7}$ is meant to represent the integer 2 with the fraction $\frac{4}{7}$ added to it.

But 2 is the same as $\dfrac{2\times 7}{7}$ or $\dfrac{14}{7}$; and therefore $2\frac{4}{7}$ must be the same as $\frac{14}{7}$ increased by $\frac{4}{7}$, or as $\frac{18}{7}$: for $\frac{18}{7}$ denotes that unity is divided into 7 equal parts, and represents 14 such parts together with 4 such parts.

Ex. XIV.

Reduce the following mixed numbers to improper fractions:

(1) $2\frac{1}{3}$. (2) $5\frac{3}{7}$. (3) $4\frac{5}{9}$. (4) $7\frac{2}{5}$.
(5) $25\frac{11}{12}$. (6) $43\frac{7}{11}$. (7) $25\frac{1}{13}$. (8) $14\frac{13}{15}$.
(9) $2003\frac{1}{7}$. (10) $857\frac{11}{13}$. (11) $57\frac{31}{43}$. (12) $13\frac{83}{84}$.
(13) $3\frac{95}{224}$. (14) $26\frac{201}{202}$. (15) $164\frac{118}{441}$. (16) $106\frac{119}{351}$.
(17) $157\frac{122}{137}$. (18) $17\frac{2605}{2859}$. (19) $427\frac{5}{1107}$. (20) $100\frac{413}{720}$.

71. *To reduce a compound fraction to its equivalent simple fraction.*

Rule. Multiply the several numerators together for the numerator of the simple fraction, and the several denominators together for its denominator.

Ex. Convert $\frac{3}{5}$ of $\frac{7}{8}$ into a simple fraction.

Proceeding by the Rule given above,

$$\frac{3}{5}\text{ of }\frac{7}{8}=\frac{3\times 7}{5\times 8}=\frac{21}{40}.$$

Reason for the above process.

By $\frac{3}{5}$ of $\frac{7}{8}$, we mean $\frac{3}{5}$ths of that part of unity which is denoted by $\frac{7}{8}$: thus if unity be divided into 8 equal parts, and 7 of these be taken, and if each of these be again divided into 5 equal parts, and 3 of each set of parts be taken, then each of the parts will be one-fortieth part of the original unit, and the number of parts taken will be 3 × 7, or 21; the result therefore is $\dfrac{21}{40}$, or $\dfrac{3\times 7}{5\times 8}$; that is,

$$\frac{3}{5}\text{ of }\frac{7}{8}=\frac{3\times 7}{5\times 8}.$$

Note. In reducing compound fractions to simple ones, we may strike out factors common to one of the numerators and one of the denominators: for this is in fact simply dividing the numerator and denominator of the fraction by the same number. Art. (66).

Thus $\frac{3}{5}$ of $2\frac{1}{12}$ of $1\frac{1}{15} = \frac{3}{5}$ of $\frac{25}{12}$ of $\frac{16}{15}$

$$=\frac{3\times 25\times 16}{5\times 12\times 15}=\frac{3\times 5\times 5\times 4\times 4}{5\times 3\times 4\times 3\times 5}$$

(striking out the factors 3, 5, 5, 4 from the numerator and denominator)

$$=\frac{4}{3}.$$

Ex. XV.

Reduce the following compound fractions to simple ones:

(1) $\frac{2}{3}$ of $\frac{4}{5}$. (2) $\frac{6}{7}$ of $\frac{9}{10}$. (3) $\frac{2}{5}$ of $\frac{3}{4}$. (4) $\frac{5}{6}$ of $\frac{11}{12}$.

(5) $\frac{7}{8}$ of $\frac{5}{6}$ of 7. (6) $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{1}{15}$ of 28.

(7) $\frac{5}{11}$ of $2\frac{1}{2}$ of $\frac{5}{7}$ of $10\frac{1}{2}$. (8) $\frac{7}{9}$ of $12\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of 9.

(9) $\frac{5}{18}$ of $\frac{7}{8}$ of $\frac{36}{10}$ of $\frac{9}{4}$ of $\frac{3}{10}$ of 2 of $\frac{8}{27}$.

(10) $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{6}{7}$ of $70\frac{2}{9}$ of $\frac{3}{40}$ of $1\frac{7}{11}$ of 147.

72. **Def.** A Fraction is in its LOWEST TERMS, when its numerator and denominator are PRIME to each other.

Note. When the numerator and denominator of a fraction are not prime to each other, they have (Art. 44) a common factor greater than unity. If we divide each of them by this, there results a fraction *equal* to the former, but of which the terms, that is, the numerator and denominator are less, or *lower* than those of the original fraction; and it may be considered to be the same fraction in *lower* terms. When the numerator and denominator of a fraction are PRIME to each other, that is, have no common factor greater than unity, it is clear that its terms cannot be made lower by division of this kind, and on this account the fraction is said to be in its LOWEST TERMS.

73. *To reduce a fraction to its lowest terms.*

Rule. Divide the numerator and denominator by their greatest common measure.

Ex. 1. Reduce $\frac{6465}{7335}$ to its lowest terms.

First, find the greatest common measure of 6465 and 7335.

```
6465) 7335 (1
      6465
      ----
       870) 6465 (7
            6090
            ----
             375) 870 (2
                  750
                  ---
                  120) 375 (3
                       360
                       ---
                        15) 120 (8
                            120
                            ---
```

therefore 15 is the greatest common measure.

```
15) 6465 (431            15) 7335 (489
    60                       60
    --                       ---
     46                      133
     45                      120
     --                      ---
      15                      135
      15                      135
      --                      ---
```

therefore fraction in its lowest terms $=\frac{431}{489}$.

Reason for the above process.

If the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not altered (Art. 66); and the greatest number which will divide the numerator and denominator is their greatest common measure.

Note. Sometimes it is unnecessary to find the greatest common measure, as it is easier to bring the fraction to its lowest terms by successive divisions of the numerator and denominator by common factors, which are easily determined by inspection.

Ex. 2. Reduce $\frac{540}{750}$ to its lowest terms,

$\frac{540}{750}=\frac{54}{75}$, dividing numerator and denominator by 10,

$=\frac{18}{25}$, dividing numerator and denominator by 3.

Ex. XVI.

Reduce each of the following fractions to its lowest terms:

(1) $\frac{4}{8}$. (2) $\frac{10}{15}$. (3) $\frac{14}{21}$. (4) $\frac{18}{45}$.

(5) $\frac{56}{63}$. (6) $\frac{54}{81}$. (7) $\frac{63}{135}$. (8) $\frac{66}{121}$.

(9) $\frac{246}{342}$. (10) $\frac{247}{323}$. (11) $\frac{272}{425}$. (12) $\frac{330}{726}$.
(13) $\frac{825}{2709}$. (14) $\frac{630}{936}$. (15) $\frac{324}{612}$. (16) $\frac{936}{2368}$.
(17) $\frac{5184}{6912}$. (18) $\frac{3444}{3556}$. (19) $\frac{7845}{96780}$. (20) $\frac{2472}{3264}$.
(21) $\frac{625}{9000}$. (22) $\frac{81}{4872}$. (23) $\frac{1632}{2976}$. (24) $\frac{10265}{14371}$.
(25) $\frac{4301}{95897}$. (26) $\frac{55247}{74341}$. (27) $\frac{6093}{9174}$. (28) $\frac{10812}{22800}$.
(29) $\frac{25194}{88179}$. (30) $\frac{374192}{575680}$. (31) $\frac{114135}{220661}$. (32) $\frac{128352}{238368}$.

74. *To reduce fractions to equivalent ones with a common denominator.*

Rule. Find the least common multiple of the denominators: this will be the common denominator. Then divide the common multiple so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction.

Note 1. If the given fractions be in their *lowest* terms, the above rule will reduce them to others having the *least* common denominator; if the *least* common denominator be required, the given fractions should be reduced to their lowest terms before the rule be applied.

Ex. Reduce $\frac{5}{12}$, $\frac{9}{16}$, $\frac{11}{24}$, $\frac{17}{33}$, into equivalent fractions with a common denominator.

Proceeding by the Rule given above,

$$\begin{array}{r|l}
2 & 12,\ 16,\ 24,\ 33 \\ \hline
2 & 6,\ 8,\ 12,\ 33 \\ \hline
2 & 3,\ 4,\ 6,\ 33 \\ \hline
3 & 3,\ 2,\ 3,\ 33 \\ \hline
 & 1,\ 2,\ 1,\ 11
\end{array}$$

$$\text{therefore least common multiple} = 2 \times 2 \times 2 \times 3 \times 2 \times 11 = 528;$$

therefore the fractions become respectively,

$$\frac{5 \times 44}{12 \times 44} = \frac{220}{528} \left(\text{since } \frac{528}{12} = 44\right),$$

$$\frac{9 \times 33}{16 \times 33} = \frac{297}{528} \left(\text{since } \frac{528}{16} = 33\right),$$

$$\frac{11 \times 22}{24 \times 22} = \frac{242}{528} \left(\text{since } \frac{528}{24} = 22\right),$$

$$\frac{17 \times 16}{33 \times 16} = \frac{272}{528} \left(\text{since } \frac{528}{33} = 16\right),$$

or the fractions with a common denominator are

$$\frac{220}{528},\ \frac{297}{528},\ \frac{242}{528},\ \frac{272}{528}.$$

Reason for the above process.

The least common multiple of the denominators of the given fractions will evidently contain the denominator of any one of the fractions an exact number of times. If both the numerator and denominator of that fraction be multiplied by that number, the value of the fraction will not be altered (Art. 66); and the denominator will then be equal to the least common multiple of all the denominators. If this be done with all the fractions, they will evidently be, in like manner, reduced to others of the same value, and having the least common multiple of all the denominators for the denominator of each fraction.

Note 2. If the denominators have no common measure, we must then multiply each numerator into all the denominators, except its own, for a new numerator for each fraction, and all the denominators together for the common denominator.

Ex. Reduce $\frac{1}{5}$, $\frac{2}{7}$, $\frac{1}{9}$, to equivalent fractions with a common denominator.

The least common multiple of the denominators

$$=5\times7\times9;$$

therefore the fractions become

$$\frac{1\times7\times9}{5\times7\times9}=\frac{63}{315},$$

$$\frac{2\times5\times9}{7\times5\times9}=\frac{90}{315},$$

$$\frac{1\times5\times7}{9\times5\times7}=\frac{35}{315}.$$

or the fractions with a common denominator are

$$\frac{63}{315},\ \frac{90}{315},\ \text{and}\ \frac{35}{315}.$$

Ex. XVII.

Reduce the fractions in each of the following sets to equivalent fractions, having the least common denominator:

(1) $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

(2) $\frac{2}{5}$, and $\frac{7}{8}$.

(3) $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

(4) $\frac{2}{9}$, and $\frac{5}{27}$.

(5) $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{11}{28}$.

(6) $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{9}$.

(7) $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{17}{18}$.

(8) $\frac{5}{12}$, $\frac{7}{16}$, and $\frac{13}{24}$.

(9) $\frac{5}{6}$, $\frac{9}{10}$, and $\frac{14}{15}$.

(10) $\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{7}{10}$.

(11) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

(12) $\frac{1}{7}$, $\frac{4}{11}$, $\frac{7}{13}$, and $\frac{2}{3}$.

(13) $\frac{3}{5}$, $\frac{35}{80}$, and $\frac{14}{200}$.

(14) $\frac{7}{12}$, $\frac{6}{7}$, $\frac{20}{63}$, and $\frac{13}{84}$.

(15) $\frac{7}{9}$, $\frac{5}{11}$, $\frac{13}{18}$, $\frac{3}{22}$, and $\frac{1}{36}$.

(16) $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{9}{14}$, $\frac{3}{28}$, and $\frac{17}{32}$.

(17) $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{27}$, $\frac{8}{81}$, $\frac{16}{243}$, and $\frac{31}{729}$.

(18) $\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$, and $\frac{9}{10000}$.

(19) $\frac{31}{60}$, $\frac{17}{90}$, $\frac{13}{25}$, $\frac{1}{105}$, and $\frac{5}{9}$.

(20) $\frac{31}{54}$, $\frac{11}{28}$, $\frac{53}{63}$, and $\frac{3}{12}$.

Note 3. Whenever a comparison has to be made between fractions, in respect of their magnitudes, they must be reduced to equivalent ones with a common denominator; because then we shall have the unit divided, in the case of each fraction so obtained, into the same number of equal parts; and the respective numerators will shew us how many of such parts are taken in each case; or which is the greatest fraction, which the next, and so on.

Ex. Compare the values of $\frac{5}{27}$, $\frac{11}{24}$, $\frac{5}{6}$, $\frac{4}{15}$, and $\frac{3}{5}$.

First, to find the least common multiple of the denominators;

$$\begin{array}{r|l} 2 & 27,\ 24,\ 6,\ 15,\ 5 \\ \hline 3 & 27,\ 12,\ 3,\ 15,\ 5 \\ \hline 5 & \ 9,\ \ 4,\ 1,\ \ 5,\ 5 \\ \hline & \ 9,\ \ 4,\ 1,\ \ 1,\ 1 \end{array}$$

therefore the least common denominator

$$=2\times3\times5\times9\times4$$

$$=1080;$$

therefore the fractions become

$$\frac{5\times40}{27\times40}=\frac{200}{1080},$$

$$\frac{11\times45}{24\times45}=\frac{495}{1080},$$

$$\frac{5\times180}{6\times180}=\frac{900}{1080},$$

$$\frac{4\times72}{15\times72}=\frac{288}{1080},$$

$$\frac{3\times216}{5\times216}=\frac{648}{1080},$$

therefore $\frac{5}{6}$ is the greatest, $\frac{3}{5}$ the next, $\frac{11}{24}$ the next, $\frac{4}{15}$ the next, and $\frac{5}{27}$ the least.

Ex. XVIII.

1. Compare the values of

(1) $\frac{3}{5}$, $\frac{8}{9}$, and $\frac{7}{10}$.
(2) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
(3) $\frac{1}{5}$ of $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{4}{3}$ of $\frac{6}{7}$.
(4) $\frac{5}{12}$, $\frac{3}{16}$, $\frac{10}{21}$, and $\frac{31}{60}$.
(5) $\frac{3}{7}$, $\frac{7}{13}$, $\frac{9}{22}$, $\frac{8}{11}$, and $\frac{21}{26}$.
(6) $\frac{3}{7}$ of $\frac{5}{8}$ of 4, $\frac{2}{11}$ of $\frac{3}{5}$ of 5, $\frac{1}{6}$ of $\frac{1}{2}$ of $4\frac{3}{4}$, and $\frac{14}{28}$.
(7) $\frac{3}{8}$, $\frac{27}{32}$, $\frac{9}{16}$, $\frac{7}{10}$, and $\frac{27}{40}$.
(8) $\frac{15}{4}$, $3\frac{1}{3}$, and $\frac{2}{7}$ of $9\frac{2}{5}$.
(9) $\frac{6}{7}$, $\frac{13}{28}$, $1\frac{4}{9}$, $\frac{5}{8}$, and $\frac{29}{56}$.
(10) $\frac{8}{9}$, $\frac{3}{11}$, $\frac{7}{18}$, $\frac{9}{22}$, and $\frac{5}{36}$.
(11) $\frac{51}{76}$, $\frac{113}{152}$, $1\frac{1}{38}$, $\frac{401}{448}$, and $\frac{700}{748}$.
(12) $\frac{15}{4}$, $3\frac{1}{3}$, $\frac{2}{7}$ of $9\frac{2}{5}$, and $\frac{2}{7}$ of $\frac{5}{9}$ of $\frac{4}{5}$.

2. Find the greatest and least of the fractions

(1) $\frac{3}{4}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{1}{2}$.
(2) $\frac{11}{12}$, $\frac{29}{30}$, $\frac{17}{18}$, $\frac{7}{16}$, and $\frac{47}{48}$.

ADDITION OF VULGAR FRACTIONS.

75. **Rule.** Reduce the fractions to equivalent ones with their least common denominator; add all the new numerators together, and under their sum write the common denominator.

Ex. Find the sum of $\frac{7}{15}$, $\frac{10}{21}$, and $\frac{16}{35}$.

Proceeding by the Rule given above,

First, find the least common multiple of the denominators;

$$\begin{array}{r|l} 3 & 15,\ 21,\ 35 \\ \hline 5 & 5,\ 7,\ 35 \\ \hline 7 & 1,\ 7,\ 7 \\ \hline & 1,\ 1,\ 1 \end{array}$$

therefore the least common multiple $=3\times5\times7=105$;

therefore the fractions become

$$\frac{7\times7}{15\times7}=\frac{49}{105},$$

$$\frac{10\times5}{21\times5}=\frac{50}{105},$$

$$\frac{16\times3}{35\times3}=\frac{48}{105},$$

therefore their sum $=\dfrac{49+50+48}{105}=\dfrac{147}{105},$

$$=\frac{49}{35},$$

$$=1\tfrac{14}{15}.$$

Reason for the Rule.

In each of the equivalent fractions, we have unity divided into 105 equal parts, and those fractions represent respectively 49, 50, and 48 of such parts; therefore the sum of the fractions must represent $49+50+48$ or 147 such parts, that is, must be $\dfrac{147}{105}$.

Note 1. If the sum of the fractions be a fraction which is not in its lowest terms, reduce it to its lowest terms; and if the result be an improper fraction, then reduce it to a whole or mixed number: thus $\tfrac{147}{105}=\tfrac{49}{35}=1\tfrac{14}{35}$: the same remark applies to all results in Vulgar Fractions.

Note 2. Before applying the rule, reduce all fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Note 3. If any of the given numbers be whole or mixed numbers; the whole numbers may be added together as in simple addition, and the fractional parts by the Rule given above.

Ex. Find the sum of $\dfrac{3}{8}$, $3\tfrac{14}{15}$, $10\tfrac{2}{5}$, and $\dfrac{9}{22}$.

$$\frac{3}{8}+3\tfrac{14}{15}+10\tfrac{2}{5}+\frac{9}{22}=3+10+\frac{3}{8}+\frac{14}{15}+\frac{2}{5}+\frac{9}{22}$$

$$=13+\frac{3}{8}+\frac{14}{15}+\frac{2}{5}+\frac{9}{22}.$$

Now to find the sum of $\dfrac{3}{8}+\dfrac{14}{15}+\dfrac{2}{5}+\dfrac{9}{22}$.

First, find the least common multiple of the denominators;

$$\begin{array}{r|l} 2 & 8,\ 15,\ 5,\ 22 \\ \hline 5 & 4,\ 15,\ 5,\ 11 \\ \hline & 4,\ \ 3,\ 1,\ 11 \end{array}$$

therefore the least common multiple

$$=2\times5\times4\times3\times11=1320;$$

therefore the fractions become

$$\frac{3\times165}{8\times165}=\frac{495}{1320},$$

$$\frac{14\times88}{15\times88}=\frac{1232}{1320},$$

$$\frac{2\times264}{5\times264}=\frac{528}{1320},$$

$$\frac{9\times60}{22\times60}=\frac{540}{1320},$$

therefore the sum of the fractions

$$=\frac{495+1232+528+540}{1320}$$

$$=\frac{2795}{1320}$$

$$=\frac{559}{264},$$ dividing numerator and denominator by 5,

$$=2\tfrac{31}{264};$$

therefore the whole sum $=13+2\tfrac{31}{264}$,

$$=15\tfrac{31}{264}.$$

Ex. XIX.

1. Add together,

(1) $\frac{2}{3}$ and $\frac{5}{7}$. (2) $\frac{2}{9}$ and $\frac{4}{9}$. (3) $\frac{5}{6}$ and $\frac{7}{8}$.
(4) $\frac{1}{7}$ and $\frac{4}{21}$. (5) $\frac{3}{16}$ and $\frac{5}{64}$. (6) $\frac{9}{14}$ and $\frac{13}{21}$.
(7) $\frac{5}{12}$ and $\frac{7}{18}$. (8) $\frac{3}{24}$ and $\frac{21}{32}$. (9) $\frac{3}{5}$ and $2\frac{1}{6}$.
(10) $\frac{5}{12}$ and $\frac{7}{18}$. (11) $3\frac{5}{8}$ and $7\frac{2}{5}$. (12) $4\frac{3}{4}$ and $9\frac{11}{12}$.

2. Find the sum of

(1) $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$. (2) $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{1}{3}$.

(3) $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{1}{10}$. (4) $\frac{5}{12}$, $\frac{1}{8}$, and $\frac{7}{24}$.

(5) $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{12}$ (6) $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{7}{9}$.

(7) $\frac{1}{2}$, $\frac{5}{6}$, and $\frac{3}{21}$. (8) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

(9) $\frac{5}{7}$, $\frac{6}{45}$, and $\frac{9}{35}$. (10) $\frac{3}{6}$, $2\frac{1}{7}$, and $13\frac{3}{10}$.

(11) $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$, and $9\frac{3}{20}$. (12) $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$, and $\frac{2}{19}$.

(13) $\frac{7}{5}$, $\frac{4}{3}$, $\frac{1}{6}$, and $\frac{11}{30}$. (14) $\frac{10}{14}$, $\frac{2}{15}$, and $\frac{18}{70}$.

(15) $\frac{4}{5}$, $\frac{7}{10}$, $\frac{4}{7}$, and $\frac{2}{21}$. (16) $\frac{1}{8}$, $\frac{7}{12}$, $\frac{5}{9}$, and $\frac{9}{20}$.

(17) $\frac{1}{5}$, $6\frac{4}{5}$, and $\frac{4}{7}$ of $\frac{1}{3}$. (18) $100\frac{2}{5}$, $64\frac{5}{9}$, $\frac{3}{5}$ of 701.

(19) $261\frac{1}{3}$, $174\frac{3}{4}$, and $\frac{5}{6}$ of $10\frac{1}{2}$.

(20) $387\frac{1}{2}$, $285\frac{1}{4}$, $394\frac{1}{3}$, and $\frac{2}{5}$ of 3704.

3. Find the value of

(1) $\frac{11}{10}+\frac{11}{100}+\frac{11}{1000}+\frac{11}{10000}$.

(2) $\frac{11}{12}+\frac{17}{18}+\frac{29}{30}+\frac{47}{48}+\frac{59}{60}$.

(3) $\frac{1}{2}+\frac{1}{9}+\frac{3}{17}+\frac{29}{38}+\frac{33}{34}$.

(4) $2\frac{11}{17}+6\frac{21}{34}+\frac{16}{51}+\frac{1}{3}$ of $\frac{16}{17}+\frac{29}{68}+\frac{1}{3}$ of $2\frac{3}{4}$.

(5) $2\frac{2}{3}+3\frac{3}{4}+4\frac{4}{5}+5\frac{5}{6}+6\frac{6}{7}$.

(6) $1\frac{4}{7}+3\frac{1}{10}+\frac{13}{15}+7\frac{5}{16}+\frac{9}{28}+\frac{2}{7}$ of $\frac{1}{5}$.

(7) $5\frac{1}{6}+\frac{8}{9}$ of $\frac{6}{7}$ of $3\frac{1}{2}+9\frac{3}{10}+\frac{3}{7}$ of $\frac{5}{8}$ of 4.

(8) $\frac{5}{8}$ of $12+\frac{3}{5}$ of $\frac{8}{9}+3\frac{3}{8}$ of $1\frac{8}{9}$ of $\frac{14}{153}+\frac{13}{15}$ of $3\frac{3}{4}$ of $\frac{1}{81}$ of $1\frac{1}{26}$.

(9) $270\frac{3}{4}+650\frac{3}{20}+5000\frac{1}{4}+53\frac{4}{5}+1\frac{1}{20}$.

(10) $\frac{1}{2}$ of $\frac{3}{4}+\frac{7}{31}$ of $(1+\frac{15}{16})+\frac{31}{32}+\frac{63}{64}$ of $\{1+\frac{1}{2}\}$.

SUBTRACTION.

76. Rule. Reduce the fractions to their least common denominator, take the difference of the new numerators, and place the common denominator underneath.

Ex. Subtract $\frac{1}{2}$ from $\frac{7}{8}$.

Proceeding by the Rule given above, since 8 is clearly the least common multiple of the denominators, the equivalent fractions will be $\frac{4}{8}$ and $\frac{7}{8}$,

$$\text{and their difference} = \frac{7-4}{8} = \frac{3}{8}.$$

Reason for the Rule.

The unit in each of the equivalent fractions is divided into 8 equal parts, and there are 7 and 4 parts respectively taken, and therefore the difference must be 3 of such parts, or, in other words, the difference of the two fractions is $\frac{3}{8}$.

Note 1. Remember always, before applying the above Rule, to reduce fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Note 2. If either of the given fractions be a whole or mixed number, it is most convenient to take separately the difference of the integral parts and that of the fractional parts, and then add the two results together, as in the following examples.

Ex. 1. From $4\frac{3}{8}$ subtract $2\frac{1}{4}$.

Here $4-2=2$, and $\frac{3}{8}-\frac{1}{4}=\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$;

therefore the difference of $4\frac{3}{8}$ and $2\frac{1}{4}=2\frac{1}{8}$.

For the process expressed at length is

$$4+\tfrac{3}{8}-(2+\tfrac{1}{4}),$$

which $=4+\frac{3}{8}-2-\frac{1}{4}$, Art. (12),

or $=4-2+(\frac{3}{8}-\frac{1}{4})$

$$=2+\tfrac{1}{8}$$

$$=2\tfrac{1}{8}.$$

Ex. 2. Take $2\frac{3}{8}$ from $4\frac{1}{4}$.

Now $\frac{3}{8}$ cannot be taken from $\frac{1}{4}$, since it is the greater of the two; we therefore add 1 to $\frac{1}{4}$, and take $\frac{3}{8}$ from $1+\frac{1}{4}$ or $\frac{5}{4}$; and then, in order that the difference may not be altered, we add 1 to the 2.

Now $\frac{5}{4}-\frac{3}{8}=\frac{10}{8}-\frac{3}{8}=\frac{7}{8}$,

$$4-3=1;$$

therefore the difference of $4\frac{1}{4}$ and $2\frac{3}{8}=1\frac{7}{8}$.

For the process expressed at length is

$$4+\tfrac{1}{4}-(2+\tfrac{3}{8})$$

which $=4+1+\frac{1}{4}-(2+1+\frac{3}{8})$ (adding and subtracting 1),

$$=4+\tfrac{5}{4}-(3+\tfrac{3}{8})$$

$$=4-3+\tfrac{5}{4}-\tfrac{3}{8}$$

$$=1+\tfrac{10}{8}-\tfrac{3}{8}$$

$$=1+\tfrac{7}{8}$$

$$=1\tfrac{7}{8}.$$

Ex. XX.

1. Find the difference between

(1) $\frac{3}{4}$ and $\frac{1}{2}$. (2) $\frac{5}{8}$ and $\frac{1}{4}$. (3) $\frac{4}{9}$ and $\frac{3}{11}$.

(4) $\frac{7}{12}$ and $\frac{8}{15}$. (5) $\frac{17}{18}$ and $\frac{11}{12}$. (6) $\frac{8}{15}$ and $\frac{9}{20}$.

(7) $2\frac{3}{5}$ and $1\frac{1}{3}$. (8) $37\frac{4}{15}$ and $33\frac{5}{24}$. (9) $6\frac{3}{8}$ and $4\frac{1}{2}$.

(10) $13\frac{5}{12}$ and $9\frac{7}{18}$. (11) $50\frac{1}{16}$ and $47\frac{1}{24}$. (12) 42 and $30\frac{5}{12}$.

(13) $15\frac{32}{729}$ and $1\frac{16}{243}$.
(14) $90\frac{10}{111}$ and $25\frac{12}{125}$.
(15) 21 and $1\frac{179}{298}$.
(16) 125 and $\frac{2}{7}$ of 14.
(17) $46\frac{5}{8}$ and $15\frac{1}{6}$.
(18) $\frac{13}{56}$ and $\frac{5}{28}$ of $1\frac{1}{5}$.
(19) $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{3}{4}$ and $\frac{2}{9}$ of $\frac{3}{4}$.
(20) $\frac{3}{7}$ of $\frac{2}{9}$ of $\frac{4}{5}$ of $8\frac{3}{4}$ and $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{11}{18}$ of $1\frac{7}{11}$.

2. By how much does $\frac{5}{8}$ of $\frac{4}{10}-\frac{7}{9}$ of $\frac{4}{21}$ exceed $\frac{5}{6}$ of $\frac{2}{15}-\frac{3}{8}$ of $\frac{4}{18}$?

3. Add $\frac{8}{15}$ of $\frac{5}{6}$ to $2\frac{7}{8}$ and subtract $\frac{5}{9}$ from the result.

4. From the sum of $11\frac{2}{3}$ and $8\frac{7}{9}$ subtract $9\frac{19}{22}$.

5. By how much does the difference of $5\frac{23}{32}$ and $2\frac{3}{7}$ exceed the sum of $\frac{5}{8}+\frac{5}{14}+\frac{1}{6}$?

6. By how much does the sum of the fractions $1\frac{1}{26}$ and $\frac{9}{13}$ exceed their difference?

MULTIPLICATION.

77. **Rule.** Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

Ex. Multiply $\frac{3}{7}$ by $\frac{5}{8}$.

Proceeding by the Rule given above,

$$\frac{3\times5}{7\times8}=\frac{15}{56}.$$

Reason for the Rule.

If $\frac{3}{7}$ be multiplied by 5, the result is $\frac{15}{7}$, Art. (64).

But this result must be 8 times too large, since, instead of multiplying by 5, we have only to multiply by $\frac{5}{8}$, which is 8 times smaller than 5, or, in other words, is one-eighth part of 5. Consequently the product above, viz. $\frac{15}{7}$, must be divided by 8, and $\frac{15}{7}\div8=\frac{15}{56}$, Art. (65).

Note 1. The same reasoning will apply, whatever be the number of fractions which have to be multiplied together.

Note 2. Before applying the above Rule mixed numbers must be reduced to improper fractions.

Note 3. It has been shewn that a fraction is reduced to its lowest terms by dividing its numerator and denominator by their greatest common measure, or, in other words, by the product of those factors which are common to both: hence, in all cases of multiplication of fractions, it will be well to split up the numerators and denominators as much as possible into the factors which compose them; and then, after putting the several fractions under the form of one fraction, the sign of × being placed between each of the factors in the numerator and denominator, to cancel those factors which are common to both, before carrying into effect the final multiplication. Thus, in the following Examples:

Ex. 1. Multiply $\frac{3}{4}$ and $\frac{4}{5}$ together.

$$\text{Product} = \frac{3 \times 4}{4 \times 5}.$$

Now cancelling, *i.e.* dividing the numerator and denominator by the common factor 4, we see that

$$\text{product} = \frac{3}{5}.$$

Ex. 2. Multiply $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ together.

$$\text{Product} = \frac{1 \times 2 \times 3}{2 \times 3 \times 4},$$

(or cancelling, *i.e.* dividing numerator and denominator by the product of the common factors 2, 3,)

$$= \frac{1}{4}.$$

Ex. 3. Multiply $\frac{8}{9}$, $\frac{16}{24}$, $\frac{27}{30}$, $\frac{45}{60}$ together.

$$\begin{aligned} \text{Product} &= \frac{8 \times 16 \times 27 \times 45}{9 \times 24 \times 30 \times 60}, \\ &= \frac{8 \times 4 \times 4 \times 3 \times 9 \times 5 \times 9}{9 \times 3 \times 8 \times 5 \times 6 \times 5 \times 12}, \end{aligned}$$

(or cancelling, *i.e.* dividing the numerator and denominator by the product of the common factors 8, 3, 9, 5,)

$$\begin{aligned} \text{product} &= \frac{4 \times 4 \times 9}{6 \times 5 \times 12}, \\ &= \frac{4 \times 2 \times 2 \times 3 \times 3}{3 \times 2 \times 5 \times 3 \times 4}, \end{aligned}$$

(or cancelling, *i.e.* dividing numerator and denominator by the product of the common factors 4, 2, 3, 3,)

$$\text{product in its lowest terms} = \frac{2}{5}.$$

Ex. 4. Multiply $2\frac{1}{2}$, $3\frac{3}{8}$, $10\frac{1}{8}$, $20\frac{4}{9}$, and $5\frac{9}{23}$ together.

$$\text{Product} = 2\frac{1}{2} \times 3\frac{3}{8} \times 10\frac{1}{8} \times 20\frac{4}{9} \times 5\frac{9}{23},$$

$$= \frac{5}{2} \times \frac{27}{8} \times \frac{81}{8} \times \frac{184}{9} \times \frac{124}{23},$$

$$= \frac{5 \times 9 \times 3 \times 9 \times 9 \times 8 \times 23 \times 4 \times 31}{2 \times 2 \times 4 \times 8 \times 9 \times 23},$$

(or cancelling, *i.e.* dividing numerator and denominator by the product of the common factors 9, 8, 23, 4,)

$$\text{product} = \frac{5 \times 3 \times 9 \times 9 \times 31}{2 \times 2},$$

$$= \frac{37665}{4},$$

$$= 9416\frac{1}{4}.$$

Ex. XXI.

1. Multiply

(1) $\frac{5}{7}$ by $\frac{2}{3}$. (2) $\frac{8}{9}$ by $\frac{13}{15}$. (3) $\frac{3}{4}$ by $\frac{8}{9}$.

(4) $\frac{5}{12}$ by $\frac{8}{15}$. (5) $\frac{51}{60}$ by $\frac{16}{17}$. (6) $7\frac{1}{2}$ by $\frac{1}{3}$.

(7) $3\frac{7}{9}$ by $2\frac{2}{3}$. (8) $7\frac{1}{2}$ by $\frac{1}{5}$ of $\frac{4}{9}$. (9) 12 by $\frac{2}{3}$ of 5.

(10) $\frac{1}{2}$ of $\frac{2}{3}$ by $5\frac{3}{8}$ of 3. (11) $\frac{12}{13}$ of $3\frac{2}{3}$ by $1\frac{1}{33}$ of $\frac{39}{51}$ of $\frac{3}{8}$.

(12) $\frac{11}{15}$ of $1\frac{3}{22}$ of $\frac{1}{70}$ by $\frac{7}{12}$ of $37\frac{14}{35}$ of $3\frac{1}{8}$ of $\frac{1}{11}$.

(13) $\frac{3}{8}$ of $2\frac{1}{16}$ of $1\frac{1}{63}$ of $3\frac{5}{87}$ by $\frac{3}{11}$ of $\frac{10}{19}$.

(14) $5\frac{2}{19}$ of $3\frac{1}{4}$ of $\frac{8}{117}$ of 34 by $\frac{3}{194}$ of $\frac{9}{68}$ of $1\frac{1}{3}$ of 19.

2. Find the continued product of

(1) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. (2) $\frac{15}{64}$, $\frac{16}{27}$, $\frac{11}{10}$, $\frac{72}{385}$, and $\frac{21}{85}$.

(3) $1\frac{17}{15}$, $2\frac{2}{3}$ of $1\frac{7}{37}$, $\frac{33}{42}$, $\frac{19}{328}$, $5\frac{1}{22}$ of 49, and $\frac{3}{76}$.

(4) $\frac{5}{6}$, $2\frac{3}{4}$, $3\frac{5}{11}$, $5\frac{2}{19}$, and $6\frac{1}{194}$.

(5) $\frac{95}{116}$. $\frac{17}{21}$, $\frac{87}{153}$, $\frac{126}{190}$, and $\frac{4}{7}$ of $1\frac{2}{5}$.

(6) $\frac{147}{196}$, $\frac{1764}{1491}$, $\frac{355}{529}$, $\frac{207}{507}$, and $1\frac{110}{189}$.

DIVISION.

78. **Rule.** Invert the divisor, *i.e.* take its numerator as a denominator and its denominator as a numerator, and proceed as in Multiplication.

Ex. Divide $\frac{2}{11}$ by $\frac{3}{5}$.

Proceeding by the Rule given above,

$$\frac{2}{11}\div\frac{3}{5}=\frac{2}{11}\times\frac{5}{3}=\frac{10}{33}.$$

Reason for the Rule.

If $\frac{2}{11}$ be divided by 3, the result is $\frac{2}{11\times 3}$ or $\frac{2}{33}$, (Art. 65).

This result is 5 times too small, or, in other words, is only one-fifth part of the required quotient, since, instead of dividing by 3, we have to divide by $\frac{3}{5}$, which is only one-fifth part of 3; and the quotient of $\frac{2}{11}$ divided by $\frac{3}{5}$ must therefore be 5 times greater than if the divisor were 3. Hence the above result $\frac{2}{33}$ must be multiplied by 5 in order to give the true quotient.

Therefore, the quotient $=\frac{2}{33}\times 5=\frac{2\times 5}{33}=\frac{10}{33}$.

Note 1. Before applying this Rule, mixed numbers must be reduced to improper fractions, and compound fractions to simple ones, as in the following Examples:

Ex. 1. Divide $4\frac{1}{3}$ by $2\frac{3}{4}$.

$$\begin{aligned}4\tfrac{1}{3}\div 2\tfrac{3}{4}&=\frac{13}{3}\div\frac{11}{4}\\&=\frac{13}{3}\times\frac{4}{11}\\&=\frac{52}{33}\\&=1\tfrac{19}{33}.\end{aligned}$$

Ex. 2. Divide $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{15}{16}$ of 7.

$$\begin{aligned}&\frac{3}{4}\text{ of }\frac{7}{8}\div\frac{15}{16}\text{ of }7\\&=\frac{3\times 7}{4\times 8}\div\frac{15\times 7}{16\times 1}\\&=\frac{3\times 7}{4\times 8}\times\frac{16\times 1}{15\times 7}\\&=\frac{3\times 7\times 16}{4\times 8\times 15\times 7}\\&=\frac{3\times 7\times 4\times 4}{4\times 2\times 4\times 3\times 5\times 7}\\&=\frac{1}{10}.\end{aligned}$$

Note 2. **Complex Fractions** may by this Rule be reduced to simple ones.

Thus,
$$\frac{1\frac{3}{4}}{2\frac{1}{2}}=\frac{\frac{7}{4}}{\frac{5}{2}}=\frac{7}{4}\div\frac{5}{2}\text{ (Art. 60)}$$
$$=\frac{7}{4}\times\frac{2}{5}=\frac{7}{10}.$$

Or thus,
$$\frac{1\frac{3}{4}}{2\frac{1}{2}}=\frac{\frac{7}{4}}{\frac{5}{2}}=\frac{\frac{7}{4}\times4\times2}{\frac{5}{2}\times4\times2},$$

multiplying the numerator and denominator of the complex fraction by the product of the denominators of the simple fractions,
$$=\frac{14}{20}=\frac{7}{10}.$$

Again,
$$\frac{4\frac{1}{2}}{30}=\frac{\frac{9}{2}}{30},$$
$$=\frac{\frac{9}{2}}{\frac{30}{1}}=\frac{9}{2}\div\frac{30}{1}=\frac{9}{2}\times\frac{1}{30}=\frac{3\times3}{2\times3\times10}$$
$$=\frac{3}{20}.$$

Or thus,
$$\frac{4\frac{1}{2}}{30}=\frac{\frac{9}{2}}{\frac{30}{1}}=\frac{\frac{9}{2}\times2\times1}{\frac{30}{1}\times2\times1}$$
$$=\frac{9}{60}=\frac{3}{20}.$$

Again,
$$\frac{30}{4\frac{1}{2}}=\frac{30}{\frac{9}{2}}=\frac{\frac{30}{1}}{\frac{9}{2}}=\frac{30}{1}\div\frac{9}{2}$$
$$=\frac{30}{1}\times\frac{2}{9}=\frac{3\times10\times2}{3\times3}$$
$$=\frac{20}{3}=6\frac{2}{3}.$$

Or thus,
$$\frac{30}{4\frac{1}{2}}=\frac{\frac{30}{1}}{\frac{9}{2}}=\frac{\frac{30}{1}\times1\times2}{\frac{9}{2}\times1\times2}$$
$$=\frac{60}{9}=\frac{20}{3}=6\frac{2}{3}.$$

Ex. XXII.

1. Divide

(1) 3 by $\frac{3}{4}$. (2) $\frac{5}{8}$ by $\frac{1}{4}$. (3) $\frac{8}{9}$ by $\frac{13}{18}$.
(4) $\frac{13}{20}$ by $\frac{3}{5}$. (5) $\frac{51}{64}$ by $3\frac{1}{6}$. (6) $\frac{16}{45}$ by $\frac{14}{15}$.
(7) $2\frac{8}{21}$ by $4\frac{1}{6}$. (8) $\frac{69}{80}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. (9) $2\frac{1}{112}$ by $6\frac{2}{7}$ of $2\frac{1}{2}$.
(10) $3\frac{3}{4}$ of $3\frac{1}{3}$ of $\frac{1}{2}$ by 75.
(11) $3\frac{1}{5}$ of $5\frac{5}{8}$ of $3\frac{5}{9}$ by $9\frac{3}{4}$ of $\frac{8}{33}$ of $7\frac{1}{3}$. (12) 119 by $\frac{7}{9}$.

(13) $\frac{3}{8}$ of $\frac{6}{7}$ of $80\frac{1}{9}$ of 9 by $\frac{8}{7}$ of $\frac{3}{9}$ of $\frac{4}{5}$ of $8\frac{3}{4}$.

(14) $\frac{5}{3}$ of $\frac{4}{5}$ of $\frac{11}{13}$ of $1\frac{7}{11}$ by $\frac{6}{17}$ of $\frac{95}{114}$ of $\frac{33}{44}$ of $1\frac{6}{11}$.

2. Compare the product and quotient of $2\frac{1}{2}$ by $3\frac{7}{8}$.

3. Reduce to simple fractions the following complex fractions:

(1) $\dfrac{\frac{3}{4}}{1\frac{7}{8}}$. (2) $\dfrac{\frac{14}{27}}{2\frac{1}{3}}$. (3) $\dfrac{2\frac{11}{12}}{\frac{5}{9}}$. (4) $\dfrac{\frac{6}{11}}{\frac{10}{99}}$.

(5) $\dfrac{13\frac{1}{3}}{20}$. (6) $\dfrac{56}{1\frac{5}{9}}$. (7) $\dfrac{13\frac{9}{10}}{1\frac{11}{14}}$.

79. *Miscellaneous Examples in Fractions worked out.*

Ex. 1. What number added to $\frac{7}{8}+\frac{5}{12}$ will give $2\frac{1}{8}$?

This question in other words is the following: "What number will remain after $\frac{7}{8}+\frac{5}{12}$ has been subtracted from $2\frac{1}{8}$?"

Now
$$\begin{aligned}&2\tfrac{1}{8}-(\tfrac{7}{8}+\tfrac{5}{12})\\&=2\tfrac{1}{8}-\tfrac{7}{8}-\tfrac{5}{12}\\&=\tfrac{17}{8}-\tfrac{7}{8}-\tfrac{5}{12}\\&=\tfrac{10}{8}-\tfrac{5}{12}\\&=\tfrac{30-10}{24}=\tfrac{20}{24},\\&=\tfrac{5}{6}.\end{aligned}$$

Therefore the number required $=\frac{5}{6}$.

Note. It will be remembered, that all quantities within a vinculum are equally affected by any sign placed before the vinculum.

Thus in the above expression, $-(\frac{7}{8}+\frac{5}{12})$ means that the sum of $\frac{7}{8}$ and $\frac{5}{12}$ has to be subtracted from $2\frac{1}{8}$; whereas $-\frac{7}{8}+\frac{5}{12}$ would mean that $\frac{7}{8}$ had to be subtracted from $2\frac{1}{8}$, and then $\frac{5}{12}$ had to be added to the result.

Ex. 2. What number subtracted from $14\frac{3}{8}$ will leave $1\frac{3}{4}$ for a remainder?

$$\begin{aligned}\text{Number required}&=14\tfrac{3}{8}-1\tfrac{3}{4}\\&=(14+1+\tfrac{3}{8})-(1+1+\tfrac{3}{4})\\&=(14+\tfrac{11}{8})-(2+\tfrac{3}{4})\\&=14-2+(\tfrac{11}{8}-\tfrac{3}{4})\\&=12\tfrac{5}{8}.\end{aligned}$$

Or thus,
$$\begin{aligned}14\tfrac{3}{8}-1\tfrac{3}{4}&=13\tfrac{3}{8}-\tfrac{3}{4}=\tfrac{107}{8}-\tfrac{6}{8}=\tfrac{101}{8}\\&=12\tfrac{5}{8}.\end{aligned}$$

Ex. 3. What number multiplied by $1\frac{3}{8}$ will produce $14\frac{3}{4}$?

This question in other words is the following: "If $14\frac{3}{4}$ be divided by $1\frac{3}{8}$, what will the quotient be?"

But
$$\frac{14\frac{3}{4}}{1\frac{3}{8}}=\frac{\frac{59}{4}}{\frac{11}{8}},$$

$$=\frac{59}{4}\times\frac{8}{11}$$

$$=\frac{59\times2}{11}=\frac{118}{11}$$

$$=10\tfrac{8}{11}.$$

Therefore the number required $=10\tfrac{8}{11}$.

Ex. 4. What number divided by $1\tfrac{3}{8}$ will produce $10\tfrac{8}{11}$?

This question in other words is the following: "What is the product of $1\tfrac{3}{8}$ and $10\tfrac{8}{11}$?"

The product of $1\tfrac{3}{8}$ and $10\tfrac{8}{11}$

$$=\tfrac{11}{8}\times\tfrac{118}{11}$$

$$=\tfrac{118}{8}$$

$$=\tfrac{59}{4}$$

$$=14\tfrac{3}{4}.$$

Ex. 5. Reduce the expression

$$\left(\frac{3\frac{1}{3}}{7}+\frac{2}{10\frac{1}{2}}-\frac{5}{18}\text{ of }\frac{4}{7}\right)\times1\tfrac{3}{4}$$

to its simplest form.

$$\left(\frac{3\frac{1}{3}}{7}+\frac{2}{10\frac{1}{2}}-\frac{5}{18}\text{ of }\frac{4}{7}\right)\times1\tfrac{3}{4}$$

$$=\left(\frac{\frac{10}{3}}{\frac{7}{1}}+\frac{\frac{2}{1}}{\frac{21}{2}}-\frac{5\times4}{18\times7}\right)\times\tfrac{7}{4}$$

$$=(\tfrac{10}{21}+\tfrac{4}{21}-\tfrac{10}{63})\times\tfrac{7}{4}$$

$$=(\tfrac{14}{21}-\tfrac{10}{63})\times\tfrac{7}{4}$$

$$=\frac{42-10}{63}\times\tfrac{7}{4}$$

$$=\tfrac{32}{63}\times\tfrac{7}{4}$$

$$=\tfrac{8}{9}.$$

Ex. 6. Simplify the expression $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$.

$$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}=\frac{\frac{6+4+3}{12}}{\frac{1}{\frac{5}{2}}+\frac{1}{\frac{7}{2}}+\frac{1}{\frac{9}{2}}}$$

$$=\frac{\frac{13}{12}}{\frac{2}{5}+\frac{2}{7}+\frac{2}{9}}$$

$$=\frac{\frac{13}{12}}{\frac{126+90+70}{315}}$$

$$=\frac{\frac{13}{12}}{\frac{286}{315}}$$

$$=\frac{13}{12}\times\frac{315}{286}=\frac{13\times3\times105}{3\times4\times13\times22}$$

$$=\frac{105}{88}=1\tfrac{17}{88}.$$

Ex. 7. Divide $3\tfrac{1}{4}-\tfrac{5}{6}$ of $\tfrac{4}{15}$ by $21\tfrac{1}{5}+\tfrac{3}{10}+4\tfrac{1}{3}$ of 5.

$$3\tfrac{1}{4}-\tfrac{5}{6}\text{ of }\tfrac{4}{15}=\tfrac{13}{4}-\tfrac{2}{9}$$

$$=\frac{117-8}{36}$$

$$=\frac{109}{36},$$

$$21\tfrac{1}{5}+\frac{3}{10}+4\tfrac{1}{3}\text{ of }5=21\tfrac{1}{5}+\frac{3}{10}+\frac{13\times5}{3}$$

$$=21\tfrac{2}{10}+\frac{3}{10}+\frac{65}{3}$$

$$=21\tfrac{5}{10}+21\tfrac{2}{3}$$

$$=21+21+\tfrac{1}{2}+\tfrac{2}{3}$$

$$=21+21+\tfrac{7}{6}$$

$$=43\tfrac{1}{6}$$

$$=\frac{259}{6};$$

therefore the quotient required $=\dfrac{109}{36}\div\dfrac{259}{6}$

$$=\frac{109}{36}\times\frac{6}{259}$$

$$=\frac{109}{6\times6}\times\frac{6}{259}$$

$$=\frac{109}{1554}.$$

Ex. 8. Simplify the expression

$$\frac{1}{13}\text{ of }\frac{1}{\dfrac{1+\frac{1}{3}}{3+\frac{1}{4}}}.$$

Now, $\dfrac{1}{\dfrac{1+\frac{1}{3}}{3+\frac{1}{4}}}=\dfrac{1}{\dfrac{1+\frac{1}{3}}{\frac{13}{4}}}$

$$= \frac{1}{1+\frac{4}{39}}$$

$$= \frac{39}{39+4}$$

$$= \frac{39}{43};$$

therefore $\frac{1}{13}$ of $\frac{1}{1+\frac{\frac{1}{3}}{3+\frac{1}{4}}} = \frac{1}{13}$ of $\frac{39}{43}$

$$= \frac{3}{43}.$$

Ex. 9. Simplify $\left\{2\frac{3}{4}+\frac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}}-\frac{1\frac{2}{3}}{2\frac{1}{2}}\right\} \div 1\frac{77}{228}$.

The expression

$$= \left\{\frac{11}{4}+\frac{5}{2} \text{ of } \frac{\frac{7}{1}}{\frac{19}{5}}-\frac{\frac{5}{3}}{\frac{5}{2}}\right\} \div \frac{305}{228}$$

$$= \left\{\frac{11}{4}+\frac{5}{2} \times \frac{7}{1} \times \frac{5}{19}-\frac{5}{3} \times \frac{2}{5}\right\} \times \frac{228}{305}$$

$$= \left\{\frac{11}{4}+\frac{175}{38}-\frac{2}{3}\right\} \times \frac{228}{305}$$

(the least common multiple of 4, 38, and 3, $= 38 \times 2 \times 3$)

$$= \left\{\frac{11 \times 19 \times 3+175 \times 2 \times 3-2 \times 38 \times 2}{38 \times 2 \times 3}\right\} \times \frac{228}{305}$$

$$= \left\{\frac{627+1050-152}{228}\right\} \times \frac{228}{305}$$

$$= \frac{1677-152}{228} \times \frac{228}{305}$$

$$= \frac{1525}{305}$$

$$= 5.$$

Ex. XXIII.

Miscellaneous Questions and Examples on Arts. (58—79).

I.

1. Define a fraction; what is the distinction between a Vulgar and a Decimal fraction? How many different kinds of Vulgar fractions are there? Give an example of each kind.

2. Find the sum and difference of $\frac{2\frac{1}{2}}{5}$ of $7\frac{3}{4}$, and $1\frac{3}{4}$ divided by $2\frac{1}{8}$; and the sum of $5\frac{1}{8}$, $\frac{2}{3}$ of $3\frac{1}{2}$, and $\frac{3}{4} \div \frac{6}{7}$.

3. Simplify

(1) $\{\frac{3}{4}+\frac{7}{6}$ of $5\frac{1}{2}\}\times\{\frac{5}{6}+\frac{2}{3}+3\frac{3}{4}\}$. (2) $3\frac{1}{125}$ of $3\frac{4}{7}\div\frac{47}{345}$ of 9.

(3) $\dfrac{3\frac{3}{4}}{4\frac{2}{7}}-\dfrac{3\frac{2}{5}}{4\frac{1}{4}}+\dfrac{\frac{4}{7}}{2\frac{1}{2}}$. (4) $\dfrac{4\frac{1}{3}\times4\frac{1}{3}\times4\frac{1}{3}-1}{4\frac{1}{3}\times4\frac{1}{3}-1}$. (5) $3+\dfrac{1}{7+\frac{1}{16}}$.

4. Shew that the fraction $\dfrac{2+4+6}{3+5+7}$ lies between the greatest and least of the fractions $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{7}$.

5. The difference of two numbers is $15\frac{4}{35}$; the greater number is $20\frac{11}{15}$: find the smaller number.

II.

1. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction is not altered: prove this by means of an example.

2. What number subtracted from $41\frac{1}{4}$ leaves $19\frac{1}{2}$? and what number multiplied by $2\frac{4}{10}$ of $\frac{4}{13}$ produces $3\frac{1}{2}$ of $\frac{4}{5}$?

3. When is a fraction said to be in its *lowest* terms?

Reduce the fractions $\frac{93208}{13786}$ and $\frac{95469}{359784}$ to their lowest terms.

4. Simplify

(1) $\dfrac{2\frac{1}{2}}{3\frac{1}{4}}+\dfrac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{4}+\frac{5}{6}}-1\frac{2}{39}$. (2) $3\frac{3}{5}$ of $5\frac{1}{2}$ of $\frac{7}{9}-\frac{1}{3}$ of $\frac{5}{12}$.

(3) $(\frac{2}{19}+\frac{1}{18})\div(3-\frac{1}{3})\times(\frac{1}{3}+\frac{1}{5})$. (4) $\frac{3}{14}$ of $\dfrac{4\frac{5}{9}}{6\frac{1}{6}}$ of $\dfrac{6\frac{8}{11}}{11\frac{5}{7}}$.

5. Divide the product of $2\frac{2}{15}$ and $2\frac{5}{8}$ by the difference of $2\frac{3}{5}$ and $2\frac{1}{4}$. Explain why it is necessary in the addition and subtraction of fractions to reduce the fractions to a common denominator.

III.

1. Shew by an example that multiplying the numerator of a fraction by any number, is the same in effect as dividing the denominator by that number, and conversely.

2. Simplify

(1) $275\frac{1}{3}+62\frac{11}{126}+1031\frac{1}{5}+\frac{7}{8}$ of $4150\frac{1}{7}$. (2) $\frac{32}{51}\div\frac{112}{85}\times\frac{189}{207}\div1\frac{13}{23}$.

(3) $\dfrac{1}{3\frac{1}{5}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{5}{8}}{2}+\dfrac{\frac{4}{7}}{4\frac{4}{7}}$. (4) $\dfrac{4\frac{1}{4}-3\frac{3}{8}}{4\frac{1}{4}+3\frac{3}{8}}+\dfrac{3-2\frac{1}{3}}{4-3\frac{1}{4}}$.

3. Which is the greater, $\frac{1}{3}$ of 4 or $\frac{1}{4}$ of 5? and by how much?

4. Divide the sum of the fractions $\frac{3}{7}$ and $\frac{4}{13}$ by the product of $\frac{9}{11}$ and $\frac{13}{14}$; and reduce the result to its lowest terms.

5. What number is that, from which if you deduct $\frac{8}{9}-\frac{3}{7}$, and to the remainder add the quotient of $\frac{2}{15}$ divided by $2\frac{1}{2}$, the sum will be $1\frac{8}{63}$?

IV.

1. Define a Vulgar fraction; an improper fraction; and the terms numerator and denominator of a fraction.

Prove by means of an example the rule for the multiplication of fractions; and multiply the sum of $\frac{4}{7}$ of $\frac{1}{2}$ and $1\frac{1}{9}$ by the difference of $\frac{4}{11}$ and $\frac{1}{5}$.

2. Reduce to their most simple forms the following expressions:

(1) $\frac{3}{4} \times \frac{7}{11} \times 8\frac{1}{5} \div \frac{2}{5}$ths of $(7\frac{3}{4} + \frac{5}{2})$. (2) $\frac{1}{8} - \frac{1}{12} + \frac{1}{15} - \frac{1}{20}$.

(3) $\frac{27968}{37376}$. (4) $\frac{1}{13}$ of $(1 + 5\frac{1}{2}) + \frac{5}{6}$ of $\frac{1}{27}$ of $(7 - 2\frac{2}{5}) - \frac{1}{3}$.

(5) $\dfrac{\frac{10}{3} + \frac{5}{6} - \frac{20}{21}}{\frac{5}{6} - \frac{4}{7}}$.

3. What number added to $\frac{3}{4}$ of $(\frac{1}{3} + \frac{1}{5} - \frac{4}{15} + \frac{1}{9})$ makes $3\frac{1}{4}$? and what number divided by $\frac{1}{4}$ of $\frac{1}{6}$ of $\frac{1}{8}$ will give $\frac{64}{315}$?

4. If I pay away $\frac{1}{3}$ of my money, then $\frac{1}{2}$ of what remains, and then $\frac{1}{4}$ of what still remains; what fraction of the whole will be left?

5. Explain the method of 'comparing' fractions.

Compare the product and quotient of the sum and difference of $5\frac{1}{3}$ and $5\frac{1}{4}$.

V.

1. State the rules for multiplying and dividing one fraction by another; and prove them by means of an example.

Divide $\dfrac{2+3}{4+5}$ by $\dfrac{4+3\frac{1}{2}}{5+5\frac{1}{2}}$; and multiply the sum of $\frac{1}{2}$, $1\frac{3}{7}$, and $\frac{5}{6}$ by the difference of $\frac{4}{15}$ and $\frac{3}{20}$, and divide the product by $\frac{11}{18}$ of $1\frac{14}{15}$.

2. Reduce to their simplest forms

(1) $(\frac{5}{2} - \frac{2}{5}) \div (\frac{4}{3} - \frac{3}{4})$. (2) $\dfrac{\frac{41}{162} - \frac{9}{49} - \frac{3}{54}}{\frac{4}{9} + \frac{1}{2} - \frac{13}{14}}$.

(3) $\frac{2}{5}$ of $\frac{13}{16} - \dfrac{1\frac{2}{3}}{6\frac{2}{3}}$ of $\frac{19}{20} + \frac{3}{7}$ of $\dfrac{6\frac{5}{12}}{3\frac{2}{3}}$.

(4) $\dfrac{\frac{3}{5}}{\frac{8}{9}} + \dfrac{\frac{5}{8}}{2\frac{1}{2} \times 1\frac{1}{3}} \times \frac{1}{80}$. (5) $2\frac{1}{2} \times \dfrac{1}{3\frac{1}{3} + \dfrac{1}{4\frac{1}{4}}}$.

(6) $\dfrac{\frac{5}{7} \text{ of } \frac{3}{10} + \frac{1}{4} \text{ of } \frac{8}{21}}{\frac{2}{3} \text{ of } \frac{9}{14} - \frac{5}{6} \text{ of } \frac{2}{15}}$. (7) $\dfrac{11\frac{5}{7} - 7\frac{5}{11}}{3\frac{1}{2} + 5\frac{5}{22}}$.

3. What is meant by the symbol $\frac{2}{5}$?

Find the least fraction which added to the sum of $\frac{3}{4}$, $\frac{7}{6}$, and $\frac{29}{12}$, shall make the result an integer.

4. Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{5}{12}$, $\frac{4}{9}$ and $\frac{7}{20}$; the sum of the other two; and the difference of these sums.

5. A man has $\frac{3}{8}$ of an estate, he gives his son $\frac{1}{2}$ of his share; what portion of the estate has he then left?

VI.

1. State the rules for addition and subtraction of vulgar fractions; and prove them by means of an example.

2. Simplify

(1) $\frac{4}{5}$ of $\frac{1}{2}-\frac{2}{3}$ of $\frac{9}{17}+\frac{3}{5}$ of $1\frac{10}{17}$. (2) $\dfrac{2\frac{3}{4}+3\frac{2}{5}}{4\frac{1}{5}+5\frac{1}{4}}+\dfrac{3\frac{2}{3}}{10\frac{1}{2}}$.

(3) $\{\frac{5}{7}\times\frac{2}{9}\times13\frac{1}{2}\}\div\{\frac{1}{9}\times\frac{3}{7}+40\}$ (4) $\dfrac{2\frac{4}{11}}{2\frac{3}{5}}\div\dfrac{2\frac{7}{11}}{8\frac{7}{10}}$.

3. Define a *proper*, *mixed*, and *compound* fraction. Explain the method of reducing a compound fraction to a simple one.

Ex. $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{81}{141}$ of $1\frac{1}{5}$.

4. Shew by means of an example how a fraction is affected if the same number be added to its numerator and denominator.

5. Multiply $3\frac{1}{9}$ by $3\frac{1}{10}$, and divide $\dfrac{20\frac{3}{4}}{3}$ by $\dfrac{41\frac{1}{2}}{4}$, and find the difference between the sum and difference of these results.

6. What number added to $\frac{31}{35}+\frac{10}{21}$ will produce $3\frac{227}{420}$? and what number divided by $2\frac{1}{16}$ will produce $\frac{4}{15}$?

VII.

1. Shew from the nature of fractions that $\frac{2}{3}+\frac{5}{7}=\frac{29}{21}$; that $\frac{2}{3}$ of $\frac{5}{7}=\frac{10}{21}$; and that $\frac{2}{3}\div\frac{5}{7}=\frac{14}{15}$.

2. Simplify

(1) $\dfrac{\frac{3}{4}}{\frac{5}{4}}+\dfrac{2\frac{1}{4}-\frac{3}{7}}{5\frac{1}{4}+\frac{1}{14}}-\dfrac{2}{3\frac{4}{7}}$. (2) $2\frac{1}{6}+3\frac{3}{8}+\frac{9}{14}+\frac{1}{15}+6\frac{17}{20}$.

(3) $(3\frac{1}{4}$ of $4\frac{1}{3})\div(2\frac{1}{2}-\frac{1}{3})$ of $(3\frac{1}{2}-\frac{1}{4})$.

(4) $(\frac{1}{26}$ of $3\frac{1}{4})+(\frac{3}{5}\div\frac{27}{64})-\left(\dfrac{1}{1\frac{1}{7}}-\dfrac{1\frac{1}{3}}{3}\right)\div(2-\frac{5}{9})$.

3. Simplify $\dfrac{\frac{2}{3}\text{ of }\frac{5}{7}\text{ of }\frac{35}{9}}{\frac{1}{7}\text{ of }\frac{2}{3}\text{ of }\frac{84}{81}}$, and take the result from the sum of $10\frac{3}{4}$, $3\frac{9}{10}$, $7\frac{24}{25}$.

4. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, subtract the sum from 2, multiply the result by $\frac{2}{3}$ of $\frac{27}{40}$ of 8, and find what fraction this is of 99.

5. In a match of cricket, a side of 11 men made a certain number of runs, one obtained $\frac{1}{8}$th of the number, each of two others $\frac{1}{10}$th, and each of three others $\frac{1}{20}$th, the rest made up between them 126; which was the remainder of the score, and 4 of these last scored 5 times as many as the other. What was the whole number of runs, and the score of each man?

DECIMALS.

80. It has been stated that figures in the units' place retain their *intrinsic* values, while those to the *left* of the units' place increase *tenfold* at each step from the units' place; therefore, according to the same notation, as we proceed from the units' place to the *right*, every successive figure would decrease *ten-fold.* We can thus represent whole numbers or integers and fractions under a uniform notation by means of figures in the units' place and on each side of it; for instance, in the number 5673·2412, the figures on the left of the *dot* · represent *integers,* while those on the right of the dot denote *fractions.* The number written at length would stand thus,

$$5\times1000+6\times100+7\times10+3+\tfrac{2}{10}+\tfrac{4}{100}+\tfrac{1}{1000}+\tfrac{2}{10000}.$$

The dot is termed the decimal point, and all digits to the right of it are called DECIMALS, because they are fractions with either 10, 100, or 10 × 10, 1000, or 10 × 10 × 10, &c. as their respective denominators.

81. It may here be observed, that, when a number is multiplied by itself any number of times, the product is called a POWER of that number; being called the *second, third, fourth,* &c. power, according as the number is multiplied *once, twice, three* times, &c. by itself, that is, according as it is employed *twice, three* times, &c., as a factor.

82. It will be seen from what has been said, that DECIMALS are in fact fractions having either 10 or some power of 10, for their denominators. For this reason also they are called DECIMAL FRACTIONS, in contradistinction to VULGAR FRACTIONS, which, as we have seen, are represented by a different notation, and not limited in their denominators to 10, or powers of 10.

83. From the preceding observations, it appears that

First,
$$\cdot 2345=\frac{2}{10}+\frac{3}{100}+\frac{4}{1000}+\frac{5}{10000}$$

Now the least common multiple of the denominators of the fractions is 10000: therefore, reducing the several fractions to equivalent ones with their least common denominator, we get

$$\begin{aligned}\cdot 2345&=\frac{2}{10}\times\frac{1000}{1000}+\frac{3}{100}\times\frac{100}{100}+\frac{4}{1000}\times\frac{10}{10}+\frac{5}{10000}\\&=\frac{2000+300+40+5}{10000}\\&=\frac{2345}{10000}.\end{aligned}$$

Secondly, $${\cdot}00324 = \frac{0}{10} + \frac{0}{100} + \frac{3}{1000} + \frac{2}{10000} + \frac{4}{100000}$$

(the least common multiple of the denominators is 100000)

$$= \frac{0}{10} \times \frac{10000}{10000} + \frac{0}{100} \times \frac{1000}{1000} + \frac{3}{1000} \times \frac{100}{100} + \frac{2}{10000} \times \frac{10}{10} + \frac{4}{100000}$$

$$= \frac{300 + 20 + 4}{100000}$$

$$= \frac{324}{100000}.$$

Thirdly, $$56{\cdot}816 = 5 \times 10 + 6 + \tfrac{8}{10} + \tfrac{1}{100} + \tfrac{6}{1000}$$

(the least common multiple of the denominators is 1000)

$$= \frac{5 \times 10}{1} \times \frac{1000}{1000} + \frac{6}{1} \times \frac{1000}{1000} + \frac{8}{10} \times \frac{100}{100} + \frac{1}{100} \times \frac{10}{10} + \frac{6}{1000}$$

$$= \frac{50000 + 6000 + 800 + 10 + 6}{1000}$$

$$= \frac{56816}{1000}.$$

Hence, we infer that every decimal, and every number composed of integers and decimals, can be put down in the form of a vulgar fraction, with the figures comprising the decimal or those composing the integer and decimal part (the dot being in either case omitted) as a numerator, and with 1 followed by as many zeros as there are decimal places in the given number for the denominator.

84. Conversely, any fraction having 10 or any power of 10 for its denominator, as $\frac{56816}{1000}$, may be represented in the form 56·816.

For $$\frac{56816}{1000} = \frac{5 \times 10000 + 6 \times 1000 + 8 \times 100 + 1 \times 10 + 6}{1000}$$

$$= \frac{5 \times 10000}{1000} + \frac{6 \times 1000}{1000} + \frac{8 \times 100}{1000} + \frac{1 \times 10}{1000} + \frac{6}{1000}$$

$$= 5 \times 10 + 6 + \tfrac{8}{10} + \tfrac{1}{100} + \tfrac{6}{1000}$$

$= 56{\cdot}816$ (by the notation we have assumed).

85. Again, by what has been said above, it appears that

$${\cdot}327 = \frac{327}{1000}$$

$${\cdot}0327 = \frac{327}{10000}$$

$$\cdot 3270 = \frac{3270}{10000} = \frac{327}{1000}.$$

We see that ·327, ·0327, and ·3270 are respectively equivalent to fractions which have the same numerator, and the first and third of which have also the same denominator, while the denominator of the second is greater.

Consequently, ·327 is equal to ·3270, but ·0327 is less than either.

The value of a decimal is therefore not affected by *affixing* cyphers to the right of it; but its value is decreased by *prefixing* cyphers: which effect is exactly opposite to that which is produced by affixing and prefixing cyphers to integers.

86. Hence it appears that a decimal is *multiplied* by 10, if the decimal point be removed *one* place towards the *right* hand; by 100, if *two* places; by 1000, if *three* places; and so on: and conversely, a decimal is *divided* by 10, if the point be removed *one* place to the *left* hand; by 100, if *two* places; by 1000, if *three* places; and so on.

Thus

$$5\cdot 6 \times 10 = \tfrac{56}{10} \times 10 = 56.$$
$$5\cdot 6 \times 1000 = \tfrac{56}{10} \times 1000 = 5600.$$
$$5\cdot 6 \div 10 = \tfrac{56}{10} \times \tfrac{1}{10} = \tfrac{56}{100} = \cdot 56.$$
$$5\cdot 6 \div 1000 = \tfrac{56}{10} \times \tfrac{1}{1000} = \tfrac{56}{10000} = \cdot 0056.$$

87. The advantage arising from the use of decimals consists in this; viz. that the addition, subtraction, multiplication, and division of *decimal* fractions are much more easily performed than those of *vulgar* fractions; and although all vulgar fractions cannot be reduced to finite decimals, yet we can find decimals so near their true value, that the error arising from using the *decimal* instead of the *vulgar* fraction is not perceptible.

Ex. XXIV.

1. Convert the following decimals into vulgar fractions:

·1; ·3; ·31; ·311; ·31111; ·31111111.

2. Convert the following decimals into vulgar fractions in their lowest terms:

·5; ·25; ·35; ·05; ·005; ·256; ·0256; ·000256; ·00008125.

3. Express as vulgar fractions in their lowest terms:

·075; ·848; 3·02; 3·434; 343·4; ·03434; ·050005; 230·409; 2·30409; 2137·2; 91300·0008; 24·000625; 8213·7169125; ·00083276; 1·0000009; ·000000001.

4. Express as decimals,

$\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{53}{100}$; $\frac{7}{100}$; $\frac{3}{1000}$; $\frac{9178}{10000}$; $\frac{9178}{100}$; $\frac{9178}{100000}$; $\frac{91}{10000}$; $\frac{9}{100000}$; $\frac{5203}{10}$; $\frac{90}{100}$; $\frac{30142}{10000}$; $\frac{672819}{100000}$; $\frac{672819}{1000000000}$; $\frac{67281900}{10000}$.

5. Multiply

·7 separately by 10, 100, 1000, and by 100000;
·006 separately by 100, 10000, and by 10000000;
·0431 separately by 100, and by 1000000;
16·201 separately by 10, 1000, and by a million;
9·0016 by ten hundred thousand, and by 100.

6. Divide

·51 separately by 10, 1000, and by 100000;
·008 separately by 100, and by a million;
5·016 separately by 1000, and by 100000;
378·0186 separately by 1000, and by a million.

7. Express according to the decimal notation, five-tenths; seven-tenths; nineteen hundredths; twenty-eight hundredths; five thousandths; ninety-seven tenths; one millionth; fourteen and four-tenths; two hundred and eighty, and four ten-thousandths; seven and seven-thousandths; one hundred and one hundred-thousandths; one one-thousandth and one ten-millionth; five-billionths.

8. Express the following decimals in words:

·4; ·25; ·75; ·745; ·1; ·001; ·00001; 23·75; 2·375; ·2375; ·00002375; 1·000001; ·1000001; ·00000001.

ADDITION OF DECIMALS.

88. **Rule.** Place the numbers under each other, units under units, tens under tens, &c., one-tenths under one-tenths, &c.; so that the decimals be all under each other: add as in whole numbers, and place the decimal point in the sum under the decimal point above.

Ex. Add together 27·5037, ·042, 342, and 2·1.

Proceeding by the Rule given above,

$$\begin{array}{r@{}l} 27 & \cdot5037 \\ & \cdot042 \\ 342 & \cdot \\ 2 & \cdot1 \\ \hline 371 & \cdot6457 \end{array}$$

Note. The same method of explanation holds for the fundamental rules of decimals, which has been given at length in explaining the Rules for Simple Addition, Simple Subtraction, and the other fundamental rules in whole numbers.

Reason for the above process.

If we convert the decimals into fractions, and add them together as such, we obtain

$$27{\cdot}5037+{\cdot}042+342+2{\cdot}1,$$

$$=\frac{275037}{10000}+\frac{42}{1000}+\frac{342}{1}+\frac{21}{10};$$

(or reducing the fractions to a common denominator),

$$=\frac{275037}{10000}+\frac{420}{10000}+\frac{3420000}{10000}+\frac{21000}{10000}$$

$$=\frac{3716457}{10000}$$

$$=371{\cdot}6457,\ \text{(Art. 84).}$$

Ex. XXV.

1. Add together:

(1) ·234, 14·3812, ·01, 32·47, and ·00075.

(2) 232·15, 3·225, 21, ·0001, 34·005, and ·001304.

(3) 14·94, ·00857, 1·5, 5607·25, 530, and ·0057.

2. Express in one sum:

(1) ·08 + 165 + 1·327 + ·0003 + 2760·1 + 9.

(2) 346 + ·0027 + ·25 + ·186 + 72·505 + ·0014 + ·00004.

(3) 6·3084 + ·006 + 36·207 + ·0001 + 364 + ·008022.

(4) 725·1201 + 34·00076 + ·04 + 50·9 + 143·713.

(5) 67·8125 + 27·105 + 17·5 + ·000375 + 255 + 3·0125.

3. Add together:

(1) 2·0068, ·04137, ·987641, 1·0000009, 57, and 1·5; and prove the result.

(2) ·0003025, 29·99987, 143·2, 5·000025, 9000, and 3·4073; and verify the result.

(3) 21·74, ·075, 103·00375, ·0005495, and 4957·5; and verify the result.

(4) Five hundred, and nine-hundredths; three hundred and seventy-five; twenty thousand and eighty-four, and seventy-eight hundred-thousandths; eleven millions, two thousand, and two hundred and nine millionths; eleven thousand-millionths; one billion, and one billionth.

SUBTRACTION OF DECIMALS.

89. **Rule.** Place the less number under the greater, units under units, tens under tens, &c., one-tenths under one-tenths, &c.; suppose cyphers to be supplied if necessary in the upper line to the right of the decimal: then proceed as in Simple Subtraction of whole numbers, and place the decimal point under the decimal point above.

Ex. Subtract 5·473 from 6·23.

Proceeding by the Rule given above,

$$\begin{array}{r} 6{\cdot}23 \\ \underline{5{\cdot}473} \\ {\cdot}757 \end{array}$$

Reason for the above process.

If we convert the decimals into fractions, and subtract the one from the other as such, we obtain

$$\begin{aligned} 6{\cdot}23-5{\cdot}473 &= \frac{623}{100}-\frac{5473}{1000} \\ &= \frac{6230}{1000}-\frac{5473}{1000} \\ &= \frac{757}{1000} \\ &= {\cdot}757,\text{ Art. (84).} \end{aligned}$$

Ex. XXVI.

1. Find the difference between 2·1354 and 1·0436; 7·835 and 2·0005; 15·67 and 156·7; ·001 and ·0009; ·305 and ·000683.

2. Find the value of

(1) 213·5 − 1·8125.	(2) ·0516 − ·0094187.
(3) 603 − ·6584003.	(4) 17·5 − 13·0046.
(5) ·582 − ·09647.	(6) 9·233 − ·0536.

3. Take ·01 from ·1; 57·704 from 713·00683; 35·009876 from 56·078; 27·148 from 9816; and prove the truth of each result.

4. Required the difference between seven and seven tenths; also between seven tenths and seven millionths; also between seventy-four + three hundred and four thousandths and one hundred and seventy-four + one hundredths; and verify each result.

MULTIPLICATION OF DECIMALS.

90. **Rule.** Multiply the numbers together as if they were whole numbers, and point off in the product as many decimal places as there are decimal places in both the multiplicand and the multiplier; if there are not figures enough, supply the deficiency by prefixing cyphers.

Ex. 1. Multiply 5·34 by ·21.

Proceeding by the Rule given above,

$$\begin{array}{r} 5{\cdot}34 \\ \underline{{\cdot}21} \\ 534 \\ \underline{1068} \\ 11214 \end{array}$$

Now the number of decimal places in the multiplicand + the number of those in the multiplier = 2 + 2 = 4;

therefore product = 1·1214.

Ex. 2. Multiply 5·34 by ·0021.

$$\begin{array}{r} 5{\cdot}34 \\ \underline{{\cdot}0021} \\ 534 \\ \underline{1068} \\ 11214 \end{array}$$

We must have 6 decimal places in the product; but there are only 5 figures; and therefore we must prefix one zero, and place a point before it thus ·011214.

Reason for the above process.

$$5{\cdot}34 \times {\cdot}21 = \frac{534}{100} \times \frac{21}{100} = \frac{11214}{10000} = 1{\cdot}1214.$$

Again,

$$5{\cdot}34 \times {\cdot}0021 = \frac{534}{100} \times \frac{21}{10000} = \frac{11214}{1000000} = {\cdot}011214.$$

Ex. XXVII.

1. Multiply together:

(1) 3·8 and 42; ·38 and ·42; 3·8 and 4·2; ·038 and ·0042.

(2) 417 and ·417; ·417 and ·417; 71956 and ·000025.
(3) 2·052 and ·0031; 4·07 and ·916; 476 and ·00026.

2. Multiply (proving the truth of the result in each case)
(1) 81·4632 by ·0378. (2) 27·35 by 7·70071. (3) ·04375 by ·0754.

3. Find the product of
(1) ·0046 by 7·85. (2) ·00846 by ·00324. (3) ·314 by ·0021.
(4) ·009 by ·00846. (5) ·009207 by 6·056. (6) ·00948 by 29;
proving the truth of each result.

4. Find the continued product of 1, ·01, ·001, and 100; also of ·12, 1·2, ·012, and 120; and prove the truth of the results.

5. Find the value of
(1) $7{\cdot}6 \times {\cdot}071 \times 2{\cdot}1 \times 29$.
(2) ${\cdot}007 \times 700 \times 760{\cdot}3 \times {\cdot}00416 \times 100000$.

DIVISION OF DECIMALS.

91. ***First.*** ***When the number of decimal places in the dividend exceeds the number of decimal places in the divisor.***

Rule. Divide as in whole numbers, and mark off in the quotient a number of decimal places equal to the excess of the number of decimal places in the dividend over the number of decimal places in the divisor; if there are not figures sufficient, prefix cyphers as in Multiplication.

Ex. 1. Divide 1·1214 by 5·34.

Proceeding by the Rule given above,

5·34) 1·1214 (21
1068
534
534

Now the number of decimal places in the dividend − the number of decimal places in the divisor $=4-2=2$;

therefore the quotient = ·21.

Ex. 2. Divide ·011214 by 53·4.

53·4) ·011214 (21
1068
534
534

Now the number of decimal places in the dividend − the number of decimal places in the divisor

$$=6-1=5;$$

therefore we prefix three cyphers, and the quotient is ·00021.

Reason for the above process.

$$\begin{aligned}
&1\cdot1214 \div 5\cdot34\\
&=\frac{11214}{10000}\div\frac{534}{100}\\
&=\frac{11214}{10000}\times\frac{100}{534}\\
&=\frac{11214}{534}\times\frac{100}{10000}\\
&=\frac{21}{1}\times\frac{1}{100}
\end{aligned}$$

$$\left(\text{since } \frac{11214}{534}=21, \text{ and } \frac{100}{10000}=\frac{1}{100}\right),$$

$$\begin{aligned}
&=\frac{21}{100}\\
&=\cdot21.
\end{aligned}$$

Again,

$$\begin{aligned}
&\cdot011214 \div 53\cdot4\\
&=\frac{11214}{1000000}\div\frac{534}{10}\\
&=\frac{11214}{1000000}\times\frac{10}{534}\\
&=\frac{11214}{534}\times\frac{10}{1000000}\\
&=\frac{21}{1}\times\frac{1}{100000}\\
&=\frac{21}{100000}\\
&=\cdot00021.
\end{aligned}$$

92. *Secondly. When the number of decimal places in the dividend is less than the number of decimal places in the divisor.*

Rule. Affix cyphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor; the quotient up to this point of the division will be a whole number; if there be a remainder, and the division be carried on further, the figures in the quotient after this point will be decimals.

Ex. Divide 1121·4 by ·534.

Proceeding by the Rule given above,

$$\begin{array}{l}
\cdot534)\ 1121\cdot400\ (2100 \\
\underline{1068} \\
534 \\
\underline{534} \\
00
\end{array}$$

Reason for the above process.

$$\begin{aligned}
&1121\cdot4 \div \cdot534 \\
&= \frac{11214}{10} \div \frac{534}{1000} \\
&= \frac{11214}{10} \times \frac{1000}{534} \\
&= \frac{11214}{534} \times \frac{1000}{10} \\
&= 21 \times 100 \\
&= 2100.
\end{aligned}$$

Note. In order to prevent mistakes in the proof of examples in Division of Decimals, always contrive in the process to separate 10, 100, &c. in the two fractions from the other figures, as in the above examples; and be sure never to effect the multiplication if there be tens left in the denominator; nor, if there be tens left in the numerator, to effect it until the last step of the operation.

Ex. Divide 172·9 by ·142 to three places of decimals.

$$\begin{array}{l}
\cdot142)\ 172\cdot900000\ (1217\cdot605 \\
\underline{142} \\
309 \\
\underline{284} \\
250 \\
\underline{142} \\
1080 \\
\underline{994} \\
860 \\
\underline{852} \\
800 \\
\underline{710} \\
90
\end{array}$$

Here we must affix 5 cyphers to 172·9; for if we affix two according to the rule, the division up to that point will give the integral part of the quotient only, and therefore as the quotient is to be obtained to three places of decimals, we must affix three cyphers more, that is, we must affix five altogether.

Reason for the above process.

$$
\begin{aligned}
& 172{\cdot}9 \div {\cdot}142 \\
&= \frac{1729}{10} \div \frac{142}{1000} \\
&= \frac{1729}{142} \times \frac{1000}{10} \\
&= \frac{1729}{142} \times \frac{100000}{1000} \\
&= \frac{172900000}{142} \times \frac{1}{1000}
\end{aligned}
$$

Now $\frac{172900000}{142} = 1217605$...from above;

therefore the result $= \frac{1217605...}{1000}$

$= 1217{\cdot}605.$

Ex. XXVIII.

1. Divide, (proving the truth of each result by Fractions):

(1) 10·836 by 5·16, and 34·96818 by ·381.
(2) ·025075 by 1·003, and ·02916 by ·0012.
(3) ·00081 by 27, and 1·77089 by 4·735.
(4) 1 by ·1, by ·01, and by ·0001.
(5) 31·5 by ·126, and 5·2 by ·32.
(6) 3217 by ·0625, and ·03217 by 6250.
(7) 4·63638 by 81·34, and 15·4546 by ·019.
(8) ·429408 by 59·64, and 2147·04 by ·036.
(9) 12·6 by ·0012, and ·065341 by ·000475.
(10) 3·012 by ·0006, and 293916·669 by 541·283.
(11) 130·4 by ·0004 and by 4, and 46·634205 by 4807·65.
(12) 1·69 by 1·3, by ·13, by 13, and also by ·013.
(13) ·00281 by 1·405, by 1405, and by ·001405.
(14) 72·36 by 36 and by ·0036, and ·003 by 1·6.
(15) 6725402.3544 by 7089, and by ·7089.
(16) 10363284·75 by 396·25, and ·09844 by ·0046.

(17) 816 by ·0004, and ·0019610652875 by 2·38645.
(18) 18368830·5 by 2315, by 231·5, and by ·2315.
(19) ·00005 by 2·5, by 25, and by ·0000025.
(20) 684·1197 by 1200·21, and also by ·0120021.

2. Divide to four places of decimals each of the following, and prove the truth of the results by Fractions:

(1) 32·5 by 8·7; ·02 by 1·7; 1 by ·013.
(2) ·009384 by ·0063; 51846·734 by 1·02.
(3) 7380·964 by ·023; 6·5 by 3·42; 25 by 19.
(4) 176432·76 by ·01257; 7457·1345 by 6535496·2.
(5) 37·24 by 2·9; ·0719 by 27·53.

3. Find the quotient (verifying each result) of

(1) ·0029202 by 157, and by 1·57.
(2) 5005 by 1953125; of 50·05 by 195·3125; of ·05005 by ·0001953125.
(3) ($7\frac{1}{2}$ of $\frac{1}{5}+\frac{17}{25}$) by ·0005; of 31·008 by $\frac{428}{515}$ of $1\frac{1}{4}$ of $\frac{5253}{18190}$; ·7575 by $16\frac{2}{3}$.

93. *Certain Vulgar Fractions can be expressed accurately as Decimals.*

Rule. Reduce the fraction to its lowest terms; then place a dot after the numerator and affix cyphers for decimals; divide by the denominator, as in division of decimals, and the quotient will be the decimal required.

Ex. 1. Convert $\frac{3}{5}$ into a decimal.

$$\begin{array}{r|l} 5 & \underline{3{\cdot}0} \\ & \ \ {\cdot}6 \end{array}$$

There is one decimal place in the dividend and none in the divisor; therefore there is one decimal place in the quotient.

Note. In reducing any such fraction as $\frac{3}{50}$ or $\frac{3}{500}$ to a decimal, we may proceed in the same way as if we were reducing $\frac{3}{5}$; taking care however in the result to move the decimal point one place further to the left for each cypher cut off.

Thus
$$\frac{3}{5}=\cdot6,$$
$$\frac{3}{50}=\cdot06,$$
$$\frac{3}{500}=\cdot006,$$

for in fact, we divide by 5, and then by 10, 100, &c., according as the divisor is 50, 500, &c.

Ex. 2. Reduce $\frac{5}{16}$ to a decimal.

$$\begin{array}{l}
16)\ 5{\cdot}0000\ ({\cdot}3125\\
\quad\ \ \underline{4\,8}\\
\quad\ \ \ \ 20\\
\quad\ \ \ \ \underline{16}\\
\quad\ \ \ \ \ \ 40\\
\quad\ \ \ \ \ \ \underline{32}\\
\quad\ \ \ \ \ \ \ \ 80\\
\quad\ \ \ \ \ \ \ \ \underline{80}
\end{array}$$

or thus,

$$16\left\{\begin{array}{c}4\\4\\\end{array}\right.\begin{array}{|l}5{\cdot}00\\\hline 1{\cdot}2500\\\hline {\cdot}3125\end{array}$$

$\therefore \frac{5}{16} = {\cdot}3125$

Ex. 3. Convert $\frac{3}{512}$ and $\frac{3}{51200}$ into decimals.

Now $512 = 8 \times 64 = 8 \times 8 \times 8.$

$$\begin{array}{r|l}
8 & 3{\cdot}000\\\hline
8 & {\cdot}375000\\\hline
8 & {\cdot}046875000\\\hline
 & {\cdot}005859375
\end{array}$$

or $\frac{3}{512}$ is equivalent to ·005859375, and $\frac{3}{51200}$ is equivalent to ·00005859375.

Ex. 4. Convert $\frac{3}{5} + 3\frac{1}{8} + 2\frac{9}{40} + 6\frac{11}{125}$ into a decimal.

$$\tfrac{3}{5} + 3\tfrac{1}{8} + 2\tfrac{9}{40} + 6\tfrac{11}{125} = 11 + \tfrac{3}{5} + \tfrac{1}{8} + \tfrac{9}{40} + \tfrac{11}{125}.$$

$$\begin{array}{r|l}
5 & 3{\cdot}0\\\hline
 & {\cdot}6
\end{array}
\qquad
\begin{array}{r|l}
8 & 1{\cdot}000\\\hline
 & {\cdot}125
\end{array}
\qquad
\begin{array}{r|l}
4 & 9{\cdot}00\\\hline
 & 2{\cdot}25
\end{array}$$

$$\begin{array}{r|l}
5 & 11\\\hline
5 & 2{\cdot}20\\\hline
5 & {\cdot}440\\\hline
 & {\cdot}088
\end{array}
\qquad
\therefore \tfrac{9}{40} = {\cdot}225.$$

therefore $\frac{3}{5} = {\cdot}6,\ \frac{1}{8} = {\cdot}125,\ \frac{9}{40} = {\cdot}225,\ \frac{11}{125} = {\cdot}088;$

therefore the whole expression

$$\begin{aligned}
&= 11 + {\cdot}6 + {\cdot}125 + {\cdot}225 + {\cdot}088\\
&= 12{\cdot}038.
\end{aligned}$$

Ex. XXIX.

1. Reduce to decimals:

(1) $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{9}{25}$; $\frac{5}{16}$; $\frac{19}{20}$.
(2) $\frac{66}{128}$; $\frac{54}{125}$; $\frac{579}{200}$; $\frac{179}{128}$; $\frac{1}{160}$.
(3) $6\frac{11}{64}$; $\frac{57}{240}$; $\frac{13}{256}$; $\frac{3}{512}$; $15\frac{588}{78125}$.

2. Reduce to decimals:

(1) $3\frac{5}{8}$ of $\frac{1}{512}$. (2) $\frac{1}{2}+\frac{1}{4}+\frac{1}{16}+\frac{1}{32}$. (3) $\frac{1}{64}\times{\cdot}0064$.

(4) $\frac{3}{5}+{\cdot}061$. (5) $\frac{1}{2}+\frac{1}{5}-\frac{1}{8}$. (6) $\dfrac{47\frac{5}{8}}{94}$ of $\dfrac{11\frac{3}{4}}{7{\cdot}5}$.

(7) $\dfrac{7{\cdot}75}{9}$ of $\dfrac{2\frac{1}{2}}{2\frac{7}{9}}$ of $\frac{20}{31}$. (8) $5\frac{5}{840}+{\cdot}75$ of $\frac{6}{5}$ of $7\frac{1}{2}$.

(9) $3\frac{4}{25}+\frac{33}{110}+81\frac{37}{1000}+\dfrac{7\frac{3}{4}}{3\frac{7}{8}}$. (10) $\dfrac{247}{\frac{5}{7}}+\frac{1512}{108}+\dfrac{17}{7\frac{5}{9}}+200\frac{7}{16}+\dfrac{11}{62{\cdot}5}$.

Note. 10 is sometimes called the *first power* of 10,
10 × 10.. .... *second power* of 10,
10 × 10 × 10..................... *third power* of 10,
10 × 10 × 10 × 10 × 10......... *fifth power* of 10,
and so on; similarly of other numbers.

94. We have seen that, in order to convert a vulgar fraction into a decimal, after reducing the fraction to its lowest terms and affixing cyphers to the numerator, we have in fact to divide 10, or some multiple of 10 or of its powers, by the denominator of the fraction: now 10 = 2 × 5, and these are the only factors into which 10 can be broken up; therefore, when the fraction is in its lowest terms, if the denominator be not composed solely of the factors 2 and 5, or one of them, or of powers of 2 and 5, or one of them, then the division of the numerator by the denominator will never terminate. Decimals of this kind, that is, which never terminate, are called indeterminate decimals, and they are also called CIRCULATING, REPEATING, or RECURRING DECIMALS, from the fact that when a decimal does not terminate, the same figures must come round again, or recur, or be repeated: for since we always affix the same figure to the dividend, namely a cypher, whenever any former remainder recurs, the quotient will also recur. Now when we divide by any number, the remainder must always be less than that number, and therefore some remainder must recur before we have obtained a number of remainders equal to the number of units in the divisor.

95. PURE CIRCULATING DECIMALS are those which recur from the beginning: thus ·3333..., ·272727..., are pure circulating decimals.

Mixed Circulating Decimals are those which do not begin to recur, till after a certain number of figures.

Thus ·128888......, ·0113636......, are mixed circulating decimals.

The circulating part, or the part which is repeated, is called the **Period** or **Repetend.**

Pure and mixed circulating decimals are generally written down only to the end of the first period, a dot being placed over the first and last figures of that period.

Thus $\cdot\dot{3}$ represents the pure circulating decimal ·333...

$\cdot\dot{3}\dot{6}$ .... ·3636...

$\cdot\dot{6}3\dot{9}$ .... ·639639...

$\cdot 13\dot{8}$ .... mixed .... ·1388...

$\cdot 01\dot{3}\dot{6}$ .... ·0113636...

96. *Pure Circulating Decimals may be converted into their equivalent Vulgar Fractions by the following Rule.*

Rule. Make the period or repetend the numerator of the fraction, and for the denominator put down as many *nines* as there are figures in the period or repetend; this fraction, reduced to its lowest terms, will be the fraction required.

Note. The fraction is only reduced to its lowest terms for the sake of exhibiting it in its simplest form. It is not of course actually necessary so to reduce it.

Exs. Reduce the following pure circulating decimals, $\cdot\dot{3}$, $\cdot\dot{2}\dot{7}$, $\cdot\dot{8}5714\dot{2}$, to their respective equivalent vulgar fractions.

Proceeding by the Rule given above,

$$\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}.$$

$$\cdot\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}.$$

$$\begin{aligned}\cdot\dot{8}5714\dot{2} = \frac{857142}{999999} &= \frac{95238}{111111}\\ &= \frac{6 \times 15873}{7 \times 15873}\\ &= \frac{6}{7}.\end{aligned}$$

The truth of these results will appear from the following considerations.

Let the circulating decimal ·3333... be represented by a symbol x;

then $x = \cdot 3333$ ..

therefore $$10 \text{ times } x = 10 \text{ times } \cdot3333\ldots$$
$$= 3\cdot3333\ldots \quad \text{(Art. 86).}$$

Now 10 times x, diminished by 1 time x, will leave 9 times x,

and $$3\cdot3333\ldots - \cdot3333 = 3\cdot3333\ldots$$
$$\underline{-\ \cdot3333\ldots}$$
$$= 3$$
$$\text{or } 9 \text{ times } x = 3\,;$$

therefore 1 time x, that is x or $\cdot3333\ldots = \frac{3}{9} = \frac{1}{3}$.

Next, let the circulating decimal ·2727... be represented by x.

Then,
$$x = \cdot272727\ldots$$

here, since there are two figures in each period, we multiply by 100, and we have
$$100 \text{ times } x = 100 \text{ times } \cdot2727\ldots$$
$$= 27\cdot2727\ldots \quad \text{(Art. 86).}$$

Therefore 100 times x, diminished by 1 time x, will be equal to
$$27\cdot2727\ldots - \cdot2727\ldots$$
$$\text{or } 99 \text{ times } x = 27\,;$$

therefore x or $\cdot2727\ldots. = \dfrac{27}{99} = \dfrac{3}{11}$.

Next, let the recurring decimal $\cdot\dot{8}5714\dot{2}$ be represented by x.

Then, $$x = \cdot857142857142\ldots$$

here since there are six figures in each period, we multiply 1000000, and we have
$$1000000 \text{ times } x = 1000000 \text{ times } \cdot875142\ldots$$
$$= 875142\cdot875142\ldots\,;$$

therefore $$999999x = 857142,$$

or $$x = \frac{857142}{999999}\,;$$

which fraction, reduced to its lowest terms, $= \dfrac{6}{7}$.

Note 1. The object in each case is to multiply the recurring decimal by such a power of 10, as will bring out the period a whole number.

Note 2. The *powers* of numbers are often expressed by placing *a small figure* (equivalent to the number of factors and called the INDEX or EXPONENT of the power) at the right hand of the number, a little above the line.

Thus 10×10, or the *second* power of 10 is expressed by 10^2,
$10 \times 10 \times 10$, or the *third* power of 10 .... by 10^3,
$10 \times 10 \times 10 \times 10 \times 10$, or the *fifth* power of 10 .... by 10^5,
and so on.

97. *Mixed Circulating Decimals may be converted into their equivalent Vulgar Fractions by the following Rule.*

Rule. Subtract the figures which do not circulate from the figures taken to the end of the first period, as if both were whole numbers; make the result the numerator; and write down as many *nines* as there are figures in the circulating part, followed by as many *zeros* as there are figures in the non-circulating part, for the denominator.

Exs. Reduce the following mixed circulating Decimals, $\cdot 1\dot{4}$, $\cdot 013\dot{8}$, $\cdot 2\dot{4}1\dot{8}$, to their respective equivalent vulgar fractions.

Proceeding by the Rule given above,

$$\cdot 1\dot{4} = \frac{14-1}{90} = \frac{13}{90}.$$

$$\cdot 013\dot{8} = \frac{138-13}{9000} = \frac{125}{9000}$$

$$= \frac{1}{72}, \text{ in its lowest terms.}$$

$$\cdot 2\dot{4}1\dot{8} = \frac{2418-2}{9990} = \frac{2416}{9990}$$

$$= \frac{1208}{4995}.$$

The truth of these results will appear from the following considerations.

Let the mixed circulating decimal be represented by x in each of the above cases.

First, let $x = \cdot 1444...$

If, by multiplication, we change the decimal in such a manner that the non-circulating part is rendered a whole number, and also change it so that the non-circulating and circulating parts to the end of the first period are rendered a whole number, and then subtract the first result from the second, we shall get rid of the circulating part. Thus, multiplying first by 10 to get the 1 out as a whole number, and then by 100 to get the 14 out as a whole number, we have

$$10 \text{ times } x = 10 \text{ times } \cdot 1444...$$

$$= 1\cdot 444...$$

$$100 \text{ times } x = 14\cdot 444...;$$

therefore

$$100 \text{ times } x - 10 \text{ times } x$$

$$= 14{\cdot}444\ldots - 1{\cdot}444\ldots$$

$$\text{or } 90 \text{ times } x = 14{\cdot}444\ldots$$

$$- \ \ 1{\cdot}444\ldots$$

$$= 13$$

therefore

$$x = \frac{13}{90}.$$

Next, let $x = {\cdot}013888\ldots$

Here there are three places in the non-recurring part, and one in the recurring part; therefore multiplying first by 1000, and then by 10000, we have

$$1000 \text{ times } x = 1000 \times {\cdot}013888\ldots$$

$$= 13{\cdot}8888\ldots,$$

and

$$10000 \text{ times } x = 138{\cdot}8888\ldots;$$

therefore subtracting, as before,

$$9000 \text{ times } x = 138 - 13$$

$$= 125;$$

therefore

$$x = \frac{125}{9000}$$

$$= \frac{1}{72}.$$

Next, let $x = {\cdot}2418418\ldots$

Now we have one place in the non-recurring part, and three places in the recurring part; therefore multiplying first by 10, and then by 10000, we have

$$10 \text{ times } x = 2{\cdot}418418\ldots$$

$$10000 \text{ times } x = 2418{\cdot}418418\ldots;$$

therefore

$$9990 \text{ times } x = 2418 - 2$$

$$= 2416;$$

therefore

$$x = \frac{2416}{9990}$$

$$= \frac{1208}{4995}.$$

Ex. XXX.

1. Reduce the following vulgar fractions and mixed numbers to circulating decimals:

(1) $\frac{5}{9}$; $\frac{2}{11}$; $\frac{1}{37}$; $\frac{3}{7}$. (2) $\frac{17}{30}$; $\frac{368}{495}$; $\frac{16}{81}$; $15\frac{52}{333}$.

(3) $\frac{3231}{3520}$; $7\frac{962}{3367}$; $\frac{17}{99000}$. (4) $24\frac{83}{9768}$; $17\frac{13}{700}$; $2\frac{139868}{833325}$.

2. Find the vulgar fractions equivalent to the recurring decimals;

(1) $\cdot\dot{7}$; $\cdot 0\dot{7}$; $\cdot\dot{2}2\dot{7}$. (2) $\cdot\dot{5}8\dot{3}$; $\cdot\dot{1}3\dot{5}$; $\cdot 2\dot{6}\dot{3}$.

(3) $\cdot 00\dot{1}8\dot{5}$; $3\cdot\dot{0}2\dot{4}$; $\cdot 012\dot{3}\dot{6}$. (4) $\cdot\dot{1}4285\dot{7}$; $\cdot 39791\dot{6}$; $\cdot 382\dot{1}4285\dot{7}$.

(5) $\cdot\dot{3}0769\dot{2}$; $\cdot 6\dot{3}0769\dot{2}$; $2\cdot 7\dot{8}5714\dot{2}$. (6) $\cdot 342\dot{7}5\dot{3}$; $\cdot 03132\dot{1}3\dot{2}$; $8\cdot 0208\dot{3}$.

(7) $85\cdot 6080\dot{6}$; $3\cdot 6\dot{4}2857\dot{1}$; $127\cdot 000220\dot{9}\dot{5}$.

98. The value of the circulating decimal ·999... is found by Art. (96) to be $\frac{9}{9}$ or 1; but since the difference between 1 and ·9 = ·1, between 1 and ·99 = ·01, between 1 and ·999 = ·001, &c., it appears that however far we continue the recurring decimal, it can never at any stage be *actually* = 1. But the recurring decimal is considered = 1, because the difference between 1 and ·99... becomes less and less, the more figures we take in the decimal, which thus, in fact, approaches nearer to 1 than by any difference that can be assigned.

In like manner, it is in this sense that any vulgar fraction can be said to be the value of a circulating decimal; because there is no assignable difference between their values.

99. In arithmetical operations, where circulating decimals are concerned, and the result is only required to be true to a certain number of decimal places, it will be sufficient to carry on the circulating part to two or three decimal places more than the number required: taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5; because, for instance, if we have the mixed decimal ·6288, and stop at ·628, it is clear that ·628 is less, and ·629 is greater than the true value of the decimal: but ·628 is less than the true value by ·000888,... and ·629 is greater than the true value by ·000111...

Now ·000111...is less than ·000888...

Therefore ·629 is nearer the true value than ·628.

Ex. 1. Add together $\cdot 3\dot{3}$, $\cdot 0\dot{4}3\dot{2}$, $2\cdot 3\dot{4}\dot{5}$, so as to be correct to 5 places of decimals.

$$\begin{array}{r} \cdot 3333333 \\ \cdot 0432432 \\ 2\cdot 3454546 \\ \hline 2\cdot 7220311 \end{array}$$

Ans. 2·72203.

Ex. 2. Subtract $\cdot 291\dot{6}$ from $\cdot 98958\dot{3}$, so as to be correct to 5 places of decimals.

$$\begin{array}{r} \cdot 9895833 \\ \underline{\cdot 2916667} \\ \cdot 6979166 \end{array} \qquad \textit{Ans.}\ \cdot 69791.$$

Note. This method may be advantageously applied in the Addition and Subtraction of circulating decimals. In the Multiplication and Division, however, of circulating decimals, it is always preferable to reduce the circulating decimals to Vulgar Fractions, and having found the product or quotient as a Vulgar Fraction, then, if necessary, to reduce the result to a decimal.

Ex. XXXI.

(1) Find the value (correct to 6 places of decimals) of

1. $2\cdot\dot{4}1\dot{8} + 1\cdot 1\dot{6} + 3\cdot\dot{0}0\dot{9} + \cdot 735\dot{4} + 24\cdot 042$.
2. $234\cdot\dot{6} + 9\cdot 92\dot{8} + \cdot\dot{0}12345678\dot{9} + \cdot\dot{0}04\dot{4} + 456$.
3. $6\cdot 45 - \cdot\dot{3}$; and $7\cdot\dot{7}\dot{2} - 6\cdot 0\dot{4}\dot{5}$; and $309 - \cdot 94\dot{7}2\dot{4}$.

(2) Express the sum of $\frac{49}{52}$, $\frac{278}{390}$, and $\frac{7}{12}$, and the difference of $18\frac{1}{12}$ and $4\frac{5}{14}$, as recurring decimals.

(3) Multiply

1. $2\cdot\dot{3}$ by $5\cdot\dot{6}$; $\cdot 7\dot{5}7\dot{5}$ by $\cdot 36\dot{6}$. 2. $\cdot 40\dot{6}$ by 62; 825 by $\cdot\dot{3}\dot{6}$.
3. $7\cdot\dot{5}\dot{2}$ by $48\cdot\dot{3}$; 368 by $\cdot\dot{6}$. 4. $3\cdot\dot{1}4\dot{5}$ by $\cdot 4\dot{2}9\dot{7}$; $20\frac{4}{7}$ by $\cdot\dot{8}\dot{4}$.

(4) Divide

1. $195\cdot 0\dot{2}$ by 4; $\cdot 3\dot{7}59\dot{2}$ by $\cdot 0\dot{5}$. 2. 54 by $\cdot 1\dot{7}$; $13\cdot\dot{2}$ by $5\cdot\dot{6}$.
3. $411\cdot 3\dot{5}1\dot{9}$ by $58\cdot 7\dot{6}4\dot{5}$; $2\cdot 1659\dot{5}$ by $\cdot 0\dot{4}$; $\cdot 6559\dot{9}0\dot{3}$ by $48\cdot 76$.

Ex. XXXII.

Miscellaneous Questions and Examples on Arts. (80—99).

I.

(1) Define a Decimal; and shew how its value is affected by affixing and prefixing cyphers. Reduce ·0625, and 3·14159 to fractions; and express the difference between $20\frac{5}{14}$ and $17\frac{1}{12}$ as a decimal.

(2) Find the value of $10\frac{3}{8} + 1\frac{5}{40} + \frac{7}{10} + \frac{13}{16}$ both by vulgar fractions, and by decimals; and shew that the results coincide.

(3) Find the sum, difference, product, and quotient of 573·005 and ·000754; and of 1·015 and ·01015, and prove the truth of each result.

(4) If a vulgar fraction, being converted into a decimal, do not terminate, prove that it must recur. What must be the limit to the number of figures in the recurring part? Is $\frac{3}{6144}$ convertible into a terminating decimal?

(5) Simplify 1. $2\frac{1}{3} + 72\frac{5}{8} + 316\frac{1}{6} + 2{\cdot}875$. 2. ${\cdot}026649 \div 2\frac{15}{16}$.

3. $\dfrac{1 - {\cdot}05}{5 + {\cdot}5} \times \dfrac{3 - {\cdot}8}{3{\cdot}8} \div \dfrac{1}{10}$. 4. $\{{\cdot}18 + {\cdot}009\} \div {\cdot}016$.

(6) Divide $\dfrac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\dfrac{7\frac{3}{11}}{174\frac{3}{17}}$; reduce the quotient to the form $1{\cdot}0\dot{7}1428\dot{5}$. Divide $91{\cdot}\dot{8}6\dot{3}$ by $87{\cdot}5\dot{6}$.

II.

(1) Write down in a decimal form seven hundred thousand four hundred and nine billionths. Express 12·1345 as a fraction, and $\frac{32546}{10000000}$ as a decimal.

(2) State the effect as regards the decimal point of multiplying and dividing a decimal by any given power of 10. Write down in words the meaning of 397008·405009; multiply it by 1000, and also divide it by 1000; and write down the meaning of each result in words.

(3) What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46?

(4) Multiply 1·05 by 10·5; and reduce the result to a fraction in its lowest terms. Divide ·8727588 by 1620; find the value of $\dfrac{{\cdot}0003 \times {\cdot}004}{{\cdot}006}$; reduce $\frac{1}{16} + \frac{9}{400} - \frac{13}{625}$ to a decimal.

(5) Simplify, expressing each result in a decimal form,

1. $\frac{1}{10000}$ of $\frac{21}{23}$. 2. $(2\frac{1}{2} + 6) \div (3\frac{1}{2} - \frac{1}{8})$.

3. $\dfrac{4{\cdot}4 + \frac{3}{5}}{7{\cdot}375 + \frac{3}{4} - \frac{1}{8}}$. 4. $2\frac{1}{3000} + 1\frac{5}{1600} + 5\frac{1}{6000} + 2{\cdot}000875$.

(6) Find a number which multiplied into 3132·458 will give a product which differs only in the 7th decimal place from 7823·6572.

III.

(1) Divide 684·1197 by 1200·21, and also by ·0120021; and 594·27 by ·047 to three places of decimals, and explain fully how the position of the decimal point is determined in each of the quotients.

(2) Simplify, expressing each result in a fractional and decimal form;

1. $\dfrac{{\cdot}015 \times 2{\cdot}1}{{\cdot}035}$. 2. $\dfrac{3\frac{1}{2} - {\cdot}04}{5 - {\cdot}0625}$.

3. $\frac{3}{5} + {\cdot}14 + \frac{3}{4}$ of $1{\cdot}0784$. 4. $(\frac{1}{4} - \frac{1}{9}) \times (\frac{3}{5} + 1\frac{1}{2})$.

(3) What is meant by a 'Recurring Decimal'? What kind of vulgar fractions produce such decimals? State the rules for reducing any recurring decimal to a vulgar fraction. Multiply $5{\cdot}\dot{8}\dot{1}$ by $\cdot 458\dot{3}$, and divide $1{\cdot}1\dot{3}$ by $\cdot 00\dot{0}13\dot{2}$. Is $\frac{91}{560}$ reducible to a recurring decimal?

(4) Shew that if $1\frac{1}{12}$, $2\frac{2}{15}$, $3\frac{3}{20}$, $4\frac{4}{27}$ be added together, (1) as fractions, and (2) as decimals, the results coincide.

(5) A man walked in 4 days 60 miles; in each of the three first days he walked an equal distance, in the fourth day he walked 13·95 miles; find the amount of his daily walking.

(6) A person has ·1875 part of a mine, he sells $\cdot 1\dot{7}$ part of his share; what fractional part of the mine has he still left?

IV.

(1) State the Rules for the Addition and Subtraction of Decimals. Add together 1·23, ·123, ·0123, ·00123, and 123; and find the vulgar fraction corresponding to the result. Find the fraction equivalent to $31{\cdot}457\dot{4}5\dot{7}$; and subtract it from the fraction $\frac{494}{15}$.

(2) Write down in figures the number, three millions six thousand and five. Also write down in words the signification of the same figures when the last is marked off as a decimal.

(3) Compare the values of 5 × ·05, 1·5 × ·75, and 2·625 ÷ 5.

(4) Find the product of $\cdot 014\dot{7}\dot{1}4\dot{7}$ by $\cdot 33\dot{3}$; and the quotients of $1269\dot{3}$ by $19{\cdot}\dot{3}\dot{9}$; of 132790 by ·245 : of ·014904 by $3\frac{6}{25}$; of 61061 by 3·05; and of 6106·1 by 305000.

(5) Shew that the decimal ·90437532 is more nearly represented by ·90438 than by ·90437; and find the value of

$$16\times\left\{\frac{1}{5}-\frac{1}{3\times5^{3}}+\frac{1}{5\times5^{5}}-\frac{1}{7\times5^{7}}+\&c.\right\}-\frac{4}{239}$$

accurately to 5 places of decimals.

(6) A person sold ·15 of an estate to one person, and then $\frac{5}{17}$ of the remainder to another person. What part of the estate did he still retain?

V.

(1) Express $\frac{1}{2}(6\frac{1}{2}+2\frac{2}{3}-3)$, $\frac{8978}{3125}$, and also the product of $3\frac{5}{8}$ and $(3\frac{1}{4}-\frac{2}{3})$ of $\frac{6}{7}$ as decimals.

(2) Simplify

1. $\dfrac{4{\cdot}255\times{\cdot}032}{{\cdot}00016}$

2. $(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10})\div(\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{15})$.

3. $(\frac{2}{11}\text{ of }35\frac{1}{5}-3\frac{1}{8})+(2{\cdot}5625+7\frac{1}{4})$. 4. $59\dot{3}\div1{\cdot}78\times{\cdot}\dot{3}\dot{6}\div{\cdot}072$.

(3) State at length the advantages which decimals possess over vulgar fractions; what disadvantages have they?

Shew whether $\frac{22}{7}$ or $\frac{333}{106}$ is nearer to the number 3·14159.

(4) Find the value of $1+\frac{1}{1}+\frac{1}{1\times2}+\frac{1}{1\times2\times3}+$&c., to 7 places of decimals; and also of

$$\frac{1}{10^3}\times\left(1-\frac{3}{10^2}+\frac{3\times4}{1\times2}\times\frac{1}{10^4}+\frac{3\times4\times5}{1\times2\times3}\times\frac{1}{10^6}\right)$$

expressing it (1) as a decimal, and (2) as a fraction.

(5) Find the Earth's equatoreal diameter in miles, supposing the Sun's diameter, which is 111·454 times as great as the equatoreal diameter of the Earth, to be 883345 miles.

(6) In what sense is a vulgar fraction said to be the value of a recurring decimal? Explain how a sufficient degree of accuracy may be obtained in the addition and subtraction of circulating decimals to any given number of decimal places, without converting the decimals into fractions. Ex. Find the sum of $\cdot12\dot{5}$, $4{\cdot}\dot{1}6\dot{3}$, and $9{\cdot}45\dot{7}$, correct to 5 places of decimals.

VI.

(1) Prove the Rule for Multiplication of decimals by means of the example 404·04 multiplied by ·030303. Multiply $\cdot\dot{3}4\dot{5}$ by $\frac{\cdot111}{4\cdot3}$; and divide ·04813489963 by ·6593, and by ·006593.

(2) Explain the meaning of 7^2, and 7^3; and find what vulgar fraction is equivalent to the sum of 20·5 and 2·05 divided by the difference.

(3) Reduce to their lowest terms $\frac{123\cdot48}{1033\cdot2}$, and $\frac{36\cdot595}{5\cdot7980}$.

(4) Shew that $\frac{\cdot375\times\cdot375-\cdot025\times\cdot025}{\cdot375-\cdot025}=\frac{2}{5}$, and that

$$3+\cfrac{1}{7+\cfrac{1}{16}}=\cdot314159\text{ nearly.}$$

Reduce $\cdot12931\dot{3}\dot{1}$ to its equivalent vulgar fraction.

(5) What decimal added to the sum of $1\frac{7}{24}$, $\frac{5}{8}$, and $\frac{13}{36}$ will make the sum total equal to 3?

(6) The quotient being $2\frac{11}{12}$ and the divisor ·15, find the dividend.

CONCRETE NUMBERS.

TABLES.

100. Our operations hitherto have been carried on with regard only to abstract numbers, or concrete numbers of one denomination. It is evident that if concrete numbers were all of one denomination; if, for instance, shillings were the only units of money, yards of length, years of time, and so on, such numbers would be subject to the common rules for abstract numbers. Again, if the concrete numbers were of different denominations, and those denominations differed from each other by 10 or multiples of 10, then all operations with such concrete numbers could be carried on by the rules which have been given for Decimals. But generally with concrete numbers such a relation does not hold between the different denominations, and therefore it is necessary to commit to memory tables, which connect the different units of money together, the different units of length together, the different units of time together, and so on.

We shall now put down some of the most useful of these tables, with a few brief remarks on each.

TABLE OF MONEY.

2 Farthings	make	1 Half-penny.
2 Half-pence	......	1 Penny.
12 Pence	....	1 Shilling.
20 Shillings	....	1 Pound.

Pounds, shillings, pence, and farthings were formerly denoted by *£*, *s*, *d*, and *q* respectively, these letters being the first letters of the Latin words *libra*, *solidus*, *denarius*, and *quadrans*, the Latin names of certain Roman coins or sums of money. *£*, *s*, *d* are still the abbreviated forms for pounds, shillings, and pence respectively; but $\frac{1}{4}$ annexed to pence denotes 1 farthing, $\frac{1}{2}$ denotes a half-penny, $\frac{3}{4}$ denotes three farthings; shewing that one farthing, two farthings, and three farthings are respectively $\frac{1}{4}$th, $\frac{2}{4}$ths or $\frac{1}{2}$, and $\frac{3}{4}$ths of the concrete unit, one penny.

The following coins are at present in common use in England:

Copper Coins.

A Farthing, the coin of least value.
A Half-penny = 2 Farthings.
A Penny ...... = 4 Farthings.

Silver Coins.

Threepenny-piece = 3 Pence.
~~Fourpenny-piece = 4 Pence.~~
A Sixpence......... = 6 Pence.
A Shilling .... = 12 Pence.
A Florin............ = 2 Shillings.
A Half-Crown ... = 2 Shillings and 6 Pence.
~~A Crown .... = 5 shillings.~~

Gold Coins.

A Half-Sovereign = 10 Shillings.
A Sovereign ...... = 20 Shillings.

The following coins have been in use at various periods in England, but with the exception of the first two, which are used under different names, they are now obsolete:

Silver Coins.

A Groat = 4 Pence.
A Tester = 6 Pence.

Gold Coins.

	£	*s.*	*d.*
A Noble............ =	0	6	8
An Angel .... =	0	10	0
A Half-Guinea ... =	0	10	6
A Mark or Merk =	0	13	4
A Guinea .... =	1	1	0
A Carolus .... =	1	3	0
A Jacobus .... =	1	5	0
A Moidore......... =	1	7	0

Note. The office at which coin is made and stamped, so as to pass or become current for legal money, is called *the Mint.*

The *standard* of gold coin in this kingdom is 22 parts of *pure gold,* and 2 parts of *copper,* melted together. From a pound Troy of standard gold there are coined at the Mint $46\frac{29}{40}$ sovereigns, or £46. 14*s*. 6*d*.: therefore the Mint price of gold is $\frac{1}{12}$ of £46. 14*s*. 6*d*. or £3. 17*s*. 10½*d*. per ounce standard, (12 ounces Troy = 1 pound Troy).

The *standard* of silver coin is 37 parts of *pure silver* and 3 parts of *copper*. From a pound Troy of standard silver are coined 66 shillings. Therefore the Mint price of silver is 5*s*. 6*d*. per ounce standard.

In the copper coinage, 24 pence are coined from 1 pound Avoirdupois of copper. Therefore 1 penny should weigh $\frac{1}{24}$th of a pound Avoirdupois.

The *copper* coinage is not, according to the present law, a *legal tender* for more than 12*d*.; nor is the *silver* coinage for more than 40*s*.; the *gold* coinage being the standard of this country.

MEASURES OF WEIGHT.

TABLE OF TROY WEIGHT.

101. This table derives its name probably from *Troyes* in France, the first city in Europe where it was adopted. It seems to have been brought thither from Egypt. It has also been derived from *Troy-novant*, the monkish name for London. It is used in weighing gold, silver, diamonds, and other articles of a costly nature; also in determining specific gravities; and generally in philosophical investigations.

The different units are grains (written grs.), pennyweights (dwts.), ounces (oz.), and pounds (lbs. or ℔s.), and they are connected thus:

24 Grains	make	1 Pennyweight	1 dwt.
20 Pennyweights		1 Ounce	1 oz.
12 Ounces		1 Pound	1 lb. or ℔.

Note 1. As the origin of weights, a grain of wheat was taken from the middle of the ear, and being well dried, was used as a weight, and called '*a grain.*'

Note 2. Diamonds and other precious stones are weighed by '*Carats*,' each carat weighing about 3⅛ grains. The term 'carat' applied to gold has a relative meaning only; any quantity of pure gold, or of gold alloyed with some other metal, being supposed to be divided into 24 equal parts (carats); if the gold be pure, it is said to be 24 carats fine; if 22 parts be pure gold and 2 parts alloy, it is said to be 22 carats fine.

Standard gold is 22 carats fine: jewellers' gold is 18 carats fine.

TABLE OF APOTHECARIES' WEIGHT.

102. Apothecaries' weight only differs from Troy weight in the subdivisions of the pound, which is the same in both. This table is used in mixing medicines. The different units are grains (grs.), scruples (℈), drams (ʒ), ounces (℥), pounds (lbs. or ℔s.), and they are connected thus:

20 Grains	make 1 Scruple	1 sc. or 1 ℈.
3 Scruples	1 Dram	1 dr. or 1 ʒ.
8 Drams	1 Ounce	1 oz. or 1 ℥.
12 Ounces	1 Pound	1 lb. or ℔.

TABLE OF AVOIRDUPOIS WEIGHT.

103. Avoirdupois weight derives its name from *Avoirs* (goods or chattels), and *Poids* (weight). It is used in weighing all heavy articles, which are coarse and drossy, or subject to waste, as butter, meat, and the like, and all objects of commerce, with the exception of medicines, gold, silver, and some precious stones. The different units are drams (drs.), ounces (oz.), pounds (lbs.), quarters (qrs.), hundredweights (cwts.), tons (tons), and they are connected thus:

16 Drams	make 1 Ounce	1 oz.
16 Ounces	1 Pound	1 lb.
28 Pounds	1 Quarter	1 qr.
4 Quarters	1 Hundredweight	1 cwt.
20 Hundredweights	1 Ton	1 Ton.

In general, 1 Stone (1 st.) = 14 lbs. Avoirdupois, but for butchers' meat or fish, 1 Stone = 8 lbs.; 1 Firkin of Butter = 56 lbs.; 1 Fodder of Lead = 19½ cwt.; 1 Great Pound of Silk = 24 ounces; 1 Pack of Wool = 240 pounds.

1 lb. Avoirdupois weighs 7000 grains Troy;
1 lb. Troy weighs 5760 grains Troy;
therefore 1 lb. Avoirdupois $= \frac{7000}{5760}$ of 1 lb. Troy
$= \frac{700}{576}$ of 1 lb. Troy
$= \frac{175}{144}$ of 1 lb. Troy
= 14 oz. 11 dwt. 16 grs. Troy
= 1 lb. 2 oz. 11 dwt. 16 grs. Troy.

MEASURES OF LENGTH.

TABLE OF LINEAL MEASURE.

104. In this measure, which is used to measure distances, lengths, breadths, heights, depths, and the like, of places or things:

3 Barley-corns (in length)	make	1 Inch,	which is written	1 in.
12 Inches		1 Foot,		1 ft.
3 Feet		1 Yard,		1 yd.
6 Feet		1 Fathom,		1 fth.
5½ Yards		1 Rod, Pole or Perch		1 po.
40 Poles		1 Furlong,		1 fur.
8 Furlongs		1 Mile,		1 m.
3 Miles		1 League,		1 lea.
69⅑ Miles		1 Degree,		1 deg. or 1°

Note. A grain of Barley, or a Barley-corn, is supposed to have been the original element of Lineal Measure.

The following measurements may be added, as useful in certain cases:

4 Inches make 1 Hand (used in measuring horses),
22 Yards make 1 Chain } used in measuring land,
100 Links make 1 Chain }
a Palm = 3 inches, a Span = 9 inches, a Cubit = 18 inches,
a Pace = 5 feet, 1 Geographical Mile = $\frac{1}{60}$th of a degree,
a Line = $\frac{1}{12}$th of an inch.

TABLE OF CLOTH MEASURE.

105. In this measure, which is used by linen and woollen drapers:

2¼ Inches	make	1 Nail.	
4 Nails		1 Quarter	1 qr.
4 Quarters		1 Yard	1 yd.
5 Quarters		1 English Ell.	
6 Quarters		1 French Ell.	
3 Quarters		1 Flemish Ell.	

MEASURES OF SURFACE.

TABLE OF SQUARE MEASURE.

106. This measure is used to measure all kinds of superficies, such as land, paving, flooring, in fact everything in which length and breadth are to be taken into account.

DEF. A SQUARE is a four-sided figure, whose sides are equal, each side being perpendicular to the adjacent sides.

A square inch is a square, each of whose sides is an inch in length; a square yard is a square, each of whose sides is a yard in length.

144 Square Inches make 1 Square Foot...1 sq. ft. or 1 ft.
9 Square Feet .... 1 Square Yard...1 sq. yd. or 1 yd.
30¼ Square Yards......... 1 Square Pole...1 sq. po. or 1 po.
40 Square Poles .... 1 Square Rood...1 ro.
4 Roods .... 1 Acre ....1 ac.

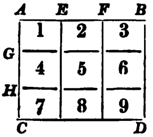
25000 Square Links = 1 Rood.
100000 .... = 1 Acre.
10 Square Chains = 1 Acre.

Note. This table is formed from the table for lineal measure, by multiplying each lineal dimension by itself.

The truth of the above table will appear from the following considerations.

Suppose *AB* and *AC* to be lineal yards placed perpendicular to each other.

Then by definition *ABCD* is a square yard. If *AE, EF, FB, AG, GH, HC* = 1 lineal foot each, it appears from the figure that there are 9 squares in the square yard, and that each square is 1 square foot.



The same explanation holds good of the other dimensions.

The following measurements may be added:

A Rod of Brickwork...... = 272¼ Square Feet.

(*The work is supposed to be* 14 *in., or rather more than a brick-and-a-half, thick.*)

A Square of Flooring .... = 100 Square Feet.
A Yard of Land .... = 30 Acres.
A Hide of Land .... = 100 Acres.

MEASURES OF SOLIDITY.

TABLE OF SOLID OR CUBIC MEASURE.

107. This measure is used to measure all kinds of solids, or figures which consist of three dimensions, length, breadth, and depth or thickness.

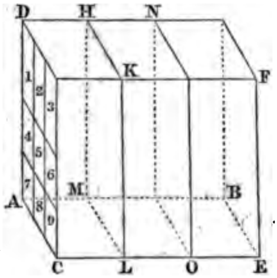
Def. A cube is a solid figure contained by six equal squares; for instance, a die is a cube.

A cubic inch is a cube whose side is a square inch.
A cubic yard .... square yard.
12 × 12 × 12 or 1728 cubic inches make 1 cubic foot.
3 × 3 × 3 or 27 cubic feet .... 1 cubic yard.

Note. This table is formed from the table for lineal measure by multiplying each lineal dimension by itself twice.

The truth of the above table will appear from the following considerations.

If *AB*, *AC*, and *AD* be perpendicular to each other, and each of them a lineal yard in length, then the figure *DE* is a cubic yard.



Suppose *DH* a lineal foot, and *HKLM* a plane drawn parallel to side *DC*.

By last table there are 9 square feet in side *DC*. There will therefore be 9 cubic feet in the solid figure *DL*.

Similarly if another lineal foot *HN* were taken, and a plane *NO* were drawn parallel to *HL*, there would be 9 cubic feet contained in the solid figure *HO*.

Similarly, there would be 9 cubic feet in the solid figure *NE*.

Therefore, there are 27 cubic feet in the solid figure *DE*, or in 1 cubic yard.

The following measurements may be added:

A Load of rough Timber = 40 cubic feet.
A Load of squared Timber = 50 cubic feet.
A Ton of Shipping = 42 cubic feet.

MEASURES OF CAPACITY.

TABLE OF WINE MEASURE.

108. In this measure, by which wines and all liquids, with the exception of malt liquors and water, are measured:

4 Gills	make	1 Pint	1 pt.
2 Pints		1 Quart	1 qt.
4 Quarts		1 Gallon	1 gal.
10 Gallons		1 Anker	1 ank.
18 Gallons		1 Runlet	1 run.
42 Gallons		1 Tierce	1 tier.
2 Tierces		1 Puncheon	1 pun.
63 Gallons		1 Hogshead	1 hhd.
2 Hogsheads		1 Pipe	1 pipe.
2 Pipes		1 Tun	1 tun.

TABLE OF ALE AND BEER MEASURE.

109. In this measure, by which all malt liquors and water are measured:

2 Pints	make	1 Quart	1 qt.
4 Quarts		1 Gallon	1 gal.
9 Gallons		1 Firkin	1 fir.
18 Gallons		1 Kilderkin	1 kil.
36 Gallons		1 Barrel	1 bar.
1½ Barrels or 54 Gallons		1 Hogshead	1 hhd.
2 Hogsheads		1 Butt	1 butt.
2 Butts		1 Tun	1 tun.

TABLE OF CORN OR DRY MEASURE.

110. In this measure, by which all dry commodities, as corn, and the like, which are not usually heaped above the measure, are measured:

2 Quarts	make	1 Pottle	1 pot.
2 Pottles		1 Gallon	1 gal.
2 Gallons		1 Peck	1 pk.
4 Pecks		1 Bushel	1 bus.
2 Bushels		1 Strike	1 str.
4 Bushels		1 Coomb	1 coomb.
2 Coombs or 8 Bushels		1 Quarter	1 qr.
5 Quarters		1 Load	1 load.
2 Loads or 10 Quarters		1 Last	1 last.

TABLE OF COAL MEASURE.

111. In this measure, which is not much used now, as coals are sold by weight:

4 Pecks make 1 Bushel.
3 Bushels...... 1 Sack.
36 Bushels...... 1 Chaldron.

MEASURES OF NUMBER.

TABLE OF NUMBER.

112.
12 Units .... make 1 Dozen.
12 Dozen...................... 1 Gross.
20 Units .... 1 Score.
120 Units .... 1 Long Hundred.
24 Sheets of Paper ...... 1 Quire.
20 Quires...................... 1 Ream.
10 Reams.................... 1 Bale.

MEASURES OF TIME.

TABLE OF TIME.

113.
1 Second is written thus 1″.
60 Seconds make 1 Minute .... 1′.
60 Minutes ...... 1 Hour............... 1 hr.
24 Hours .... 1 Day .... 1 day.
7 Days .... 1 Week .... 1 wk.

A year is divided into 12 months, called Calendar Months, the number of days in each of which are easily remembered by means of the following lines:

> Thirty days hath September,
> April, June, and November:
> February has twenty-eight alone,
> And all the rest have thirty-one:
> But leap-year coming once in four,
> February then has one day more.

A day, or rather a *mean solar day,* which is divided into 24 equal portions, called *mean solar hours,* is the standard unit for the measurement of time, and it is the mean or average time which elapses between two successive transits of the Sun across the meridian of any place.

The time between the Sun's leaving a certain point in the *Ecliptic* and its return to that point consists of 365·242218 mean *solar days*, or 365 days, 5 hours, 48 minutes, 47½ seconds, very nearly, and is called a *solar year*. Therefore the *civil* or *common* year, which contains 365 days, is about ¼th of a day less than the *solar* year; and this error would of course in time be very considerable, and cause great confusion.

Julius Cæsar, in order to correct this error, enacted that every 4th year should consist of 366 days; this was called *Leap or Bissextile year*. In that year February had 29 days, the extra day being called '*the Intercalary*' day.

But the solar year contains 365·242218 days, and the Julian year contains 365.25 or 365¼ days.

Now $365{\cdot}25 - 365{\cdot}242218 = {\cdot}007782.$

Therefore in one year, taken according to the Julian calculation, the Sun would have returned to the same place in the Ecliptic ·007782 of a day before the end of the Julian year.

Therefore in 400 years the Sun would have come to the same place in the Ecliptic ·007782 × 400 or 3·1128 days before the end of the Julian year; and in 1257 years would have come to the same place, ·007782 × 1257 or 9·7819, or about 10 days before the end of the Julian year. Accordingly, the vernal equinox which, in the year 325 at the council of Nice, fell on the 21st of March, in the year 1582 (that is, 1257 years later) happened on the 11th of March; therefore Pope Gregory caused 10 days to be omitted in that year, making the 15th of October immediately succeed the 4th, so that in the next year the vernal equinox again fell on the 21st of March; and to prevent the recurrence of the error, ordered that for the future in every 400 years, 3 of the leap years should be omitted, viz. those which complete a century, the numbers expressing which century, are *not* divisible by 4; thus 1600 and 2000 are leap years, because 16 and 20 are exactly divisible by 4; but 1700, 1800, and 1900 are not leap years, because 17, 18, and 19 are not exactly divisible by 4.

This Gregorian style, which is called the *new style*, was adopted in England on the 2nd of September 1752, when the error amounted to 11 days.

The Julian calculation is called the *old style:* thus Old Michaelmas and Old Christmas take place 12 days after New Michaelmas and New Christmas.

In Russia, they still calculate according to the *old style*, but in the other countries of Europe the new style is used. Sir Harry Nicolas in his Chronology gives the dates at which the new style was adopted in different countries. Of course it was almost immediately adopted by most of the Roman Catholic courts of Europe.

TABLE OF ANGULAR MEASURE.

114. 1 Second is written 1 sec. or 1″.
60 Seconds make 1 Minute .... 1 min. or 1′.
60 Minutes ...... 1 Degree .... 1 deg. or 1°.
90 Degrees .... 1 Right Angle... 1 rt. ang. or 90°.

The circumference of every circle is considered to be divided into 360 equal parts, each of which is often called a degree, as it subtends an *angle* of 1° at the centre of the circle.

115. An Act of Parliament "FOR ASCERTAINING AND ESTABLISHING UNIFORMITY OF WEIGHTS AND MEASURES," in this kingdom, came into operation on the 1st of January, 1826.

It is thereby enacted,

First; That the *brass Standard Yard of* 1760, then in custody of the Clerk of the House of Commons, shall be the *Imperial Standard Yard,* (the brass being at the temperature of 62° by Fahrenheit's thermometer); and that this Imperial Standard Yard shall be the unit or only standard measure of extension, wherefrom or whereby all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be divided, computed, and ascertained; and that the *thirty-sixth* part of this yard shall be *an Inch.*

Now the length of a *Pendulum* vibrating *seconds* in the latitude of *London,* in a vacuum, and at the level of the sea, is found to be 39·1393 such inches, *i.e.* 39 such inches and 1393 ten-thousandths of another such inch.

This affords the means of recovering the Imperial Standard Yard should it be lost. In fact, the brass Standard Yard of 1760 was destroyed or rendered useless by the fire at the House of Commons in 1834.

Secondly; That the *brass weight of one Pound Troy of the year* 1758, then in the custody of the same officer, shall continue the unit or *Standard Measure of Weight,* from which all other weights shall be derived, computed and ascertained; that 5760 grains shall be contained in the Imperial Standard Troy Pound, and 7000 such grains in the Avoirdupois Pound.

Now the weight of a *cubic inch* of distilled water is 252·458 grains Troy, the barometer being at 30 inches and the thermometer at 62°. This affords the means of recovering the Imperial Standard Pound should it be lost. In fact, the brass weight of 1758 was destroyed or lost at the above-mentioned fire.

Thirdly; That the *Standard Measure of Capacity* for Liquids and Dry Goods shall be "the *Imperial Standard Gallon,*" containing 10 Pounds Avoirdupois weight of distilled water, weighed in air at a temperature of 62° Fahrenheit's thermometer, and the barometer being at 30 inches.

Now this weight fills 277·274 cubic inches, therefore the Imperial Standard Gallon contains 277·274 cubic inches.

The *Imperial Bushel,* consisting of *eight* gallons, will consequently be 2218·192 cubic inches.

REDUCTION.

116. Reduction is the method of expressing numbers of a superior denomination in units of a lower denomination, and conversely. Thus £1 is of the same value as 240*d.*, and £21 as 5040*d.*, and conversely; and the process, by which we ascertain this to be so, is termed *Reduction.*

First. *To express a number of a higher denomination in units of a lower denomination.*

Rule. "Multiply the number of the highest denomination in the proposed quantity by the number of units of the next lower denomination contained in one unit of the highest, and to the product add the number of that lower denomination, if there be any in the proposed quantity; repeat this process for each succeeding denomination till the required one is arrived at."

Ex. 1. How many pence are there in £23. 15*s.*?

Proceeding by the Rule given above,

$$\begin{array}{l} \pounds 23\,.\,15s. \\ \underline{\;\;20\;\;} \\ 460+15 \text{ or } 475s. \\ \qquad\qquad\;\;\underline{\;12\;} \\ \qquad\qquad\;\;5700d. \end{array}$$

or £23. 15*s.* = 5700*d.*

Reason for the Process.

There are 20 shillings in £1.

Therefore there are (23 × 20) *s.* or 460*s.* in £23, and so there are 460*s.* + 15*s.*, or 475*s.* in £23. 15*s.*

Again, since there are 12 pence in 1*s.*; therefore there are (475 × 12)*d.*, or 5700*d.* in 475*s.* *i.e.* in £23. 15*s.*

Ex. 2. Reduce 2 tons, 7 cwt., 3 qrs., 24 lbs. into lbs.

tons . cwt. . qrs. . lbs.
2 . 7 . 3 . 24
20
40 + 7 or 47 cwt.
4
188 + 3 or 191 qrs.
28
1528
382
5348 + 24
or 5372 lbs.

Ex. 3. Reduce 27 acres, 1 rood, 32 poles, into poles.

acres . roods . poles
27 . 1 . 32
4
108 + 1 = 109 ro.
40
4360 + 32
= 4392 poles.

Ex. 4. Reduce 73 days, 21 hours, 10 minutes, 9 seconds, to seconds.

days . hrs. . min. . sec.
73 . 21 . 10 . 9
24
292
146
1752 + 21 = 1773 hrs.
60
106380 + 10
= 106390 min.
60
6383400 + 9
= 6383409 sec.

7

Ex. 5. How many inches are there in 106 miles, 6 furlongs, 25 perches, and 2½ yards?

$$\begin{array}{r}
\text{miles} \quad \text{fur.} \quad \text{per.} \quad \text{yds.} \\
106 \;.\; 6 \;.\; 25 \;.\; 2\tfrac{1}{2} \\
\underline{8} \\
848+6=854 \text{ fur.} \\
\underline{40} \\
34160+25 \text{ per.} \\
=34185 \\
\underline{5\tfrac{1}{2}} \\
170925 \\
\underline{17092\tfrac{1}{2}} \\
188017\tfrac{1}{2}+2\tfrac{1}{2} \text{ yds.} \\
=188020 \\
\underline{36} \\
1128120 \\
\underline{564060} \\
6768720 \text{ in.}
\end{array}$$

Secondly. *To express a number of inferior denomination in units of a higher denomination.*

Rule. "Divide the given number by the number of units which connect that denomination with the next higher, and the remainder, if any, will be the number of surplus units of the lower denomination. Carry on this process, till you arrive at the denomination required."

Ex. 1. How many pounds and shillings are there in 5700 pence?

Proceeding by the Rule given above,

$$\begin{array}{r|l}
12 & 5700 \\ \hline
2,0 & 47,5 \\ \hline
 & £23\,.\,15s.
\end{array}$$

In dividing 475 by 20 we cut off the 0 and 5 by Art. (43).

Reason for the above process.

Since 12 pence = 1 shilling; therefore in any given number of pence, for every 12 pence there is 1 shilling, so that in 5700*d.* or (12 × 475)*d.* there are 475*s.*

Again, since 20*s.* = £1; therefore in any given number of shillings, for every 20 shillings there is £1.

Hence, in 475*s*., or (20 × 23 + 15)*s*. there are £23, and 15*s*. over.

Note. Since each of the above Rules is the converse of the other, the accuracy of any result obtained by either of them may be tested by working the result back again by the other rule.

Ex. 2. In 272668 inches how many miles &c. are there? Verify the result.

In this Example it will be convenient to bring the inches to half-yards and the half-yards to poles. In a half-yard there are 18 or 3 × 6 inches, and in a pole there are 5½ yards or 11 half-yards.

$$\begin{array}{r|l}
18\left\{\begin{array}{r}3\\6\end{array}\right. & \left.\begin{array}{l}272668-1\\ \hline 90889-1\end{array}\right\}\text{ 4 in.}\\ \cline{2-2}
11 & 15148-1 \text{ half-yard or 18 inches}\\ \cline{2-2}
4,0 & 137,7-17 \text{ po.}\\ \cline{2-2}
8 & 34-2 \text{ fur.}\\ \cline{2-2}
 & \quad 4
\end{array}$$

therefore the answer is 4 miles, 2 fur., 17 po., 22 in.

$$\begin{array}{lllll}
 & \text{miles} & \text{fur.} & \text{poles} & \text{in.}\\
\textbf{Proof} & 4\ . & 2\ . & 17\ . & 22
\end{array}$$

$$\begin{array}{r}
\underline{8}\\
34 \text{ furlongs}\\
\underline{40}\\
1360+17\\
=1377 \text{ poles}\\
1377\\
\underline{11}\\
15147 \text{ half-yards}\\
\underline{18}\\
121176\\
15147\\
\underline{22}\\
272668 \text{ inches.}
\end{array}$$

Ex. 3. Reduce 5813456 pounds to tons, and prove the correctness of the result.

$$
\begin{array}{r|l}
 & \quad\text{lbs.}\\
28\left\{\begin{array}{r}4\\7\end{array}\right. & \left.\begin{array}{l}5813456-0\\ \hline 1453364-3\end{array}\right\}12\text{ lbs.}\\ \hline
4 & \;207623-3\text{ qrs.}\\ \hline
2{,}0 & \;\;5190{,}5-5\text{ cwt.}\\ \hline
 & \;\;2595
\end{array}
$$

therefore the answer is 2595 tons, 5 cwt., 3 qrs., 12 lbs.

Proof

$$
\begin{array}{r}
\text{tons}\quad\text{cwt.}\;\text{qrs.}\;\text{lbs.}\\
2595\;.\;5\;.\;3\;.\;12\\
20\\ \hline
51905\text{ cwt.}\\
4\phantom{\text{ cwt.}}\\ \hline
207623\text{ qrs.}\\
28\phantom{\text{ qrs.}}\\ \hline
1660984\phantom{\text{ lbs.}}\\
415246\phantom{\text{ lbs.}}\\
12\phantom{\text{ lbs.}}\\ \hline
5813456\text{ lbs.}
\end{array}
$$

Ex. 4. How many grains of gold are contained in 9 lbs., 11 oz., 13 dwts., 20 grs.? Prove the result.

$$
\begin{array}{l}
\text{lbs.}\;\;\text{oz.}\;\;\text{dwts.}\;\;\text{grs.}\\
\;\;9\;.\;11\;.\;13\;.\;20\\
\underline{\;12\;}\\
108+11=119\text{ oz. in 9 lbs., 11 oz.}\\
\qquad\quad\underline{\;20\;}\\
\qquad\;\;2380+13\text{ or }2393\text{ dwts. in 9 lbs., 11 oz., 13 dwts.}\\
\qquad\qquad\qquad\quad\underline{\;24\;}\\
\qquad\qquad\qquad\;\;9572\\
\qquad\qquad\qquad\underline{4786\;\;}\\
\qquad\qquad\qquad57432+20
\end{array}
$$

or 57452 grs. in 9 lbs., 11 oz., 13 dwts., 20 grs.

Proof

$$
\begin{array}{r|l}
24\left\{\begin{array}{r}4\\6\end{array}\right. & \left.\begin{array}{l}57452-0\\ \hline 14363-5\end{array}\right\}2\text{ grs.}\\ \hline
2{,}0 & \;239{,}3\\ \hline
12 & \;119-13\text{ dwts.}\\ \hline
 & \quad 9-11\text{ oz.}
\end{array}
$$

therefore in 57452 grs., there are 9 lbs., 11 oz., 13 dwts., 20 grs.

Ex. 5. Reduce 49 acres, 28 poles, 10 yards, 8 feet, 112 inches, to inches. Prove the result.

$$\begin{array}{l}
\text{ac.} \quad \text{po.} \quad \text{yds.} \quad \text{ft.} \quad \text{in.} \\
49 \;.\; 28 \;.\; 10 \;.\; 8 \;.\; 112 \\
\underline{\quad 4} \\
196 \text{ ro.} \\
\underline{\quad 40} \\
7840+28 \\
\quad =7868 \text{ poles} \\
\underline{\qquad 30\tfrac{1}{4}} \\
236040 \\
\underline{\quad 1967} \\
238007+10 \\
\quad =238017 \text{ yards} \\
\underline{\qquad 9} \\
2142153+8 \\
\quad =2142161 \text{ feet} \\
\underline{\qquad 144} \\
\quad 8568644 \\
\; 8568644 \\
\underline{2142161\qquad} \\
308471184+112 \\
=308471296 \text{ inches}
\end{array}$$

Proof.

$$\begin{array}{rr|ll}
 & & \text{sq. in.} & \\
144 & \begin{cases}12\\12\end{cases} & \begin{array}{l}\underline{308471296-4}\\ \underline{\;25705941-9}\end{array} & \left.\vphantom{\begin{array}{l}1\\1\end{array}}\right\} 112 \text{ sq. in.} \\
 & 9 & \underline{\;2142161-8 \text{ sq. ft.}} & \\
 & & \;238017 &
\end{array}$$

Now, since $30\frac{1}{4}$ or $\frac{121}{4}$ sq. yds. = 1 sq. po., we multiply by 4, which reduces the sq. yds. into quarters of sq. yds., and then divide that result by 121, or 11 × 11, which brings it into sq. poles.

$$\begin{array}{rr|ll}
 & & 238017 & \\
 & & \underline{\qquad 4} & \\
121 & \begin{cases}11\\11\end{cases} & \begin{array}{l}\underline{952068-7}\\ \underline{\;86551-3}\end{array} & \left.\begin{array}{l}\vphantom{1}\\ \vphantom{1}\end{array}\right\} \begin{array}{l}\text{40 quarters of sq. yds.}\\ \text{or 10 sq. yds.}\end{array} \\
 & 4{,}0 & \underline{\;786{,}8-28 \text{ sq. po.}} & \\
 & 4 & \underline{\;196} & \\
 & & \;49 &
\end{array}$$

Therefore in 308471206 sq. in., there are 49 ac., 28 sq. po., 10 sq. yds., 8 sq. ft., 112 sq. in.

Ex. 6. How many half-guineas are there in 537 half-crowns?

Here both the Rules are requisite.

By Rule 1,

$$537 \text{ half-crowns} = (537 \times 5) \text{ sixpences} = 2685 \text{ sixpences.}$$

Next, to find how many half-guineas there are in 2685 sixpences.

By Rule 2,

$$21\left\{\begin{array}{c|c} 3 & 2685-0 \\ \cline{2-2} 7 & 895-6 \\ \cline{2-2} \multicolumn{1}{c}{} & \multicolumn{1}{c}{127} \end{array}\right\} 18 \text{ sixpences;}$$

therefore in 537 half-crowns, there are 127 half-guineas and 18 sixpences.

Ex. XXXIII.

(1) Reduce (verifying each result);

1. £57 to pence; and 613 guineas to farthings.
2. £15. 12*s*. to pence; and 5000 guineas to pence.
3. 8*s*. $4\frac{1}{2}$*d*. to half-pence; and £1. 0*s*. $3\frac{3}{4}$*d*. to farthings.
4. £83. 15*s*. $6\frac{1}{4}$*d*. to farthings; and £393. 0*s*. $11\frac{1}{2}$*d*. to half-pence.
5. 738 half-crowns to farthings; and 570 crowns to fourpenny pieces.
6. 2673 half-guineas to farthings; and $22\frac{1}{2}$ guineas to sixpences.

(2) Find the number of pounds in 5673542 farthings, and prove the truth of the result.

(3) How many half-crowns, how many sixpences, and how many fourpences, are there in 25 pounds?

(4) In 6300 fourpences, how many half-crowns are there, and how many half-guineas?

(5) In 351 seven-shilling-pieces, how many half-guineas are there, and how many moidores?

(6) Reduce, verifying the result in each case, the following:

1. 59 lbs., 7 oz., 14 dwts., 19 grs. to grains; and 37400157 grs. to lbs.
2. 56332005 scrs. to lbs. Troy; and 536 lbs. to drams and scruples.
3. 7 tons, 15 cwt., 2 qrs., 16 lbs. to ounces; and 7563241 drs. to tons.
4. 5838297 oz. to tons; and 33 tons, 17 cwt., 3 qrs., 27 lbs., 15 drs. to drams.

5. 17 lbs. 2 ℥., 2 ℈. to grains; and 34678 grs. Apoth. to oz. Troy.
6. 375 cwt., 2 qrs., 15 lbs. to stones; and 573421 stones to tons.
7. 3 m., 7 fur., 8 po. to yards; and 573 miles to inches.
8. 1364428 in. to leagues; and 74 m., 3 fur., 4 yds. to inches.
9. 4 lea., 2 m., 2 in. to barleycorns; and 50 m., 3 po. to yards.
10. 7 fur., 200 yds. to chains; and 6 cubits, 1 span to feet.
11. 84 yds., 1 qr. to nails; and 56 Eng. ells, 1 qr. to nails.
12. 83 Fr. ells, 3 qrs. to nails; and 73 Fl. ells, 1 qr. to nails.
13. 35 ac., 2 ro. to poles; and 56 ac., 2 ro. to yards.
14. 3 ro., 37 po., 26 yds. to inches; and 3 ac., 30 po. to feet.
15. 15 ac., 3 ro. to links; and 50000 po. to acres.
16. 29 cub. yds. to feet; and 158279 cub. in. to yards.
17. 17 cub. yds., 1001 cub. in. to inches; and 26 cub. yds., 19 cub. ft. to inches.
18. 563 gals. to pints; and 365843 gills to gallons.
19. 5 pipes, 1 hhd., 35 gals. to pints; and 487634 gills to tierces.
20. 6 hhds., 1 bar. of beer to pints; and 2307621 pints of wine to hhds.
21. 760 bus., 3 pks. to quarts; and 2 qrs., 1 coomb, 3 pks. to gallons.
22. 3659712 pints to loads; and 7 lds., 1 qr., 2 bus. to pecks.
23. 250 chaldrons to bushels; and 186043 pks. to chaldrons.
24. 56 reams, 19 quires to sheets; and 52073 sheets of paper to reams.
25. 36 wks., 5 d., 17 hrs. to seconds; and 1 mo. of 30 days, 23 hrs., 59 sec. to seconds.

(7) How many barrels, gallons, quarts, and pints are there in 1336381 half-pints?

(8) One year being equivalent to 365 days, 6 hours, find how many seconds there are in 27 years, 245 days.

(9) From 9 o'clock P.M., Aug. 5, 1852, to 6 o'clock A.M., March 3, 1853, how many hours are there, and how many seconds?

(10) In England there are 50535 square miles; in Wales, 8125 square miles; in Scotland, 29167 square miles: how many square acres do they all contain?

COMPOUND ADDITION.

117. Compound Addition is the method of collecting several numbers of the same kind, but containing different denominations of that kind, into one sum.

Rule. "Arrange the numbers, so that those of the same denomination may be under each other in the same column, and draw a line below them. Add the numbers of the lowest denomination together, and find by reduction how many units of the next higher denomination are contained in this sum. Set down the remainder, if any, under the column just added, and carry the quotient to the next column: proceed thus with all the columns."

Ex. 1. Add together £2. 4*s.* 7½*d.*, £3. 5*s.* 10¼*d.*, £15. 15*s.*, and £33. 12*s.* 11½*d.*

Proceeding by the Rule given above,

£.	*s.*	*d.*
2	4	7½
3	5	10¼
15	15	0
33	12	11½
£54	18	5¼

Reason for the above process.

The sum of 2 farthings, 1 farthing, and 2 farthings, = 5 farthings, = 1 penny, and 1 farthing; we therefore put down ¼, that is, one farthing, and carry 1 penny to the column of pence. Then

$$(1+11+10+7)d. = 29d. = (12 \times 2+5)d.$$

or 2 shillings, and 5 pence; we therefore put down 5*d.*, and carry on the 2 to the column of shillings.

Then $(2+12+15+5+4)s = 38s. = (20 \times 1+18)s. =$ £1., and 18*s.*; we therefore put down 18*s.*, and carry on the 1 pound to the column of pounds. Then $(1+33+15+3+2)$ pounds = £54.

Therefore the result is £54. 18*s.* 5¼*d.*

Note. The method of proof is the same as that in Simple Addition.

Ex. 2. Add together 34 tons, 15 cwt., 1 qr., 14 lbs.; 42 tons, 3 cwt., 18 lbs.; 18 tons, 19 cwt., 3 qrs.; 7 cwt., 6 lbs.; 2 qrs., 19 lbs.; and 3 tons, 7 lbs.

	tons.	cwt.	qrs.	lbs.
	34	15	1	14
	42	3	0	18
	18	19	3	0
	0	7	0	6
	0	0	2	19
	3	0	0	7
Ans.	99	6	0	8

Ex. XXXIV.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1)	1	7	6	(2)	25	17	0	(3)	33	16	3¾
	6	0	3		63	15	10		67	0	7½
	5	11	4		24	19	8		73	19	10¾
	8	8	8		81	17	11		29	9	9¼
	2	1	11		57	0	3		47	16	8½

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(4)	5	17	10½	(5)	63	15	2¼	(6)	528	14	11¾
	36	0	11		83	8	9¾		854	19	4
	7	3	4¼		41	0	11¾		578	18	9½
	73	19	8¾		6	7	10½		507	0	0¾
	30	14	5¼		76	17	1¾		859	14	11½

	tons.	cwt.	qrs.	lbs.		oz.	drs.	sc.	grs.		ac.	ro.	po.
(7)	16	17	2	25	(8)	22	3	2	19	(9)	82	2	24
	13	10	0	20		56	0	1	10		18	3	14
	17	15	2	19		3	2	2	11		20	1	27
	84	0	3	27		15	6	1	9		56	0	0
	11	11	1	11		79	4	1	10		45	3	30

(10) Find the sum of £28. 14*s.* 6¾*d.*, £27. 18*s.* 4¼*d.*, £79. 12*s.* 6*d.*, £19. 18*s.* 10½*d.*, and £85. 14*s.* 3¾*d.*; also of £678. 10*s.* 2*d.*, £325. 6*s.* 5*d.*, £487. 18*s.* 9*d.*, £507. 0*s.* 11*d.*, and £779. 10*s.* 8*d.*; also of £568. 10*s.* 3½*d.*, £259. 19*s.* 5½*d.*, £188. 11*s.* 4½*d.*, £157. 9*s.* 3¾*d.*, £13. 13*s.* 5¼*d.*, and £779. 8*s.* 8¾*d.*; also of £941. 14*s.* 2*d.*, £888. 17. 9¾*d.*, £309. 19*s.* 10½*d.*, £679. 2*s.* 11¾*d.*, £455. 16*s.*, and £447. 0*s.* 7¾*d.*; also of £3966. 16*s.* 9¾*d.*, £2. 11*s.* 7¾*d.*, £3795. 0*s.* 2¾*d.*, £37. 17*s.* 0¾*d.*, £48. 0*s.* 0¾*d.*, and £59000. 14*s.* 6¾*d.*; also of £6491, £3651. 10*s.* 3¼*d.*, £8000. 0*s.* 11¾*d.*, £5510. 19*s.* 10½*d.*, £50430. 12*s.* 1¼*d.*, £316. 14*s.* 5¾*d.*, and £4850. 18*s.* 4*d.*; also of £306217. 13*s.* 9¾*d.*, £55. 0*s.* 9*d.*, £450812. 15*s.* 2½*d.*, £9837. 1*s.* 5½*d.*, and £2939. 3*s.* 11¾*d.*: and prove the result in each case.

(11) Add together 2 lbs., 9 oz., 1 dwt., 23 grs.; 8 lbs., 6 oz., 4 dwts., 20 grs.; 1 lb., 10 oz., 5 dwts., 12 grs.; 14 lbs., 11 oz., 14 dwts., 19 grs.; and 21 lbs., 8 oz., 13 dwts., 11 grs.: also 22 lbs., 7 dwts., 15 grs.; 15 lbs., 11 oz., 18 grs.; 34 lbs., 9 oz., 12 dwts.; 74 lbs., 1 oz., 1 dwt., 20 grs.; and 46 lbs., 11 oz., 16 dwts., 19 grs.: also 1740 oz., 9 dwts., 19 grs.; 4179 oz., 11 dwts., 14 grs.; 8497 oz., 12 dwts., 22 grs.; 5629 oz., 19 dwts., 17 grs.; and 1038 oz., 4 dwts., 14 grs.: verify each result.

(12) Add together 3 drs., 2 scr., 19 grs.; 2 drs., 2 scr., 11 grs.; 7 drs.,

17 grs.; 6 drs., 1 scr., 9 grs.; and 5 drs., 1 scr., 13 grs.; also 10 lbs., 8 oz., 4 drs., 1 scr.; 66 lbs., 10 oz., 2 drs.; 19 lbs., 9 oz., 3 drs., 2 scr.; 55 lbs., 6 drs.; and 79 lbs., 11 oz., 4 drs., 1 scr.: also 13 lbs., 6 oz., 7 drs., 2 scr., 17 grs.; 19 lbs., 11 oz., 1 scr., 18 grs.; 36 lbs., 3 oz., 2 scr., 19 grs.; 6 oz., 7 drs., 7 grs.; and 176 lbs., 96 grs.: explain the process in each case.

(13) Find the aggregate of 18 lbs., 14 oz., 6 drs.; 9 lbs., 6 oz., 15 drs.; 45 lbs., 9 oz., 8 drs.; 9 lbs., 15 oz., 4 drs.; and 14 lbs., 12 oz., 12 drs.: also of 1 cwt., 2 qrs., 26 lbs., 10 oz.; 11 cwt., 18 lbs., 9 oz.; 13 cwt., 3 qrs., 17 lbs., 14 oz.; 7 cwt., 1 qr., 25 lbs., 9 oz.; and 19 cwt., 2 qrs., 19 lbs., 14 oz.: also of 306 tons, 15 cwt., 2 qrs., 15 lbs.; 731 tons, 6 cwt., 3 qrs., 24 lbs.; 279 tons, 7 cwt., 10 lbs.; 896 tons, 9 cwt., 1 qr., 17 lbs.; and 10 cwt., 2 qrs., 16 lbs.: also of 23 tons, 12 cwt., 15 lbs., 12 oz.; 58 tons, 17 cwt., 1 qr., 10 oz.; 67 tons, 3 qrs., 15 oz.; 19 cwt., 27 lbs.; and 3 tons, 13 lbs., 13 oz.: prove the results.

(14) Find the sum of 11 yds., 2 ft., 9 in.; 27 yds., 1 ft., 3 in.; 36 yds., 2 ft., 10 in.; 48 yds., 2 ft., 11 in.; and 51 yds., 1 ft., 8 in.: also of 26 m., 7 fur., 23 po., 3 yds.; 22 m., 5 fur., 27 po., 5 yds.; 37 m., 4 fur., 3 yds.; 86 m., 6 fur., 38 po., 3 yds.; and 25 m., 1 fur., 29 po., 2½ yds.: also of 14 m., 7 fur., 23 po., 2½ yds., 2 ft., 11 in.; 12 m., 5 fur., 1 yd., 2 ft., 3 in.; 27 m., 2 fur., 13 po., 3½ yds., 1 ft., 10 in.; 36 m., 6 fur., 33 po., 4½ yds., 2 ft., 6 in.; and 75 m., 1 fur., 21 po., 3 yds., 1 ft., 7 in.: also of 2 lea., 1 m., 3 fur., 103 yds.; 67 lea., 3 fur., 157 yds.; 11 lea., 1 m., 93 yds.; 9 lea., 2 m., 5 fur., 87 yds.; and 34 lea., 2 m., 7 fur., 198 yds.

(15) Find the sum of 43 yds., 2 qrs., 3 na.; 37 yds., 2 qrs., 1 na.; 23 yds., 3 qrs., 2 na.; 41 yds., 2 qrs., 2 na.; and 38 yds., 2 qrs., 3 na.: and of 11 Eng. ells, 2 qrs., 3 na.; 13 Eng. ells, 2 qrs., 1 na.; 39 Eng. ells, 4 qrs., 2 na.; 37 Eng. ells, 4 qrs., 3 na.: and 79 Eng. ells, 3 na.: and prove each result.

(16) Find the sum of 25 ac., 2 ro., 16 po.; 30 ac., 2 ro., 25 po.; 26 ac., 2 ro., 35 po.; 63 ac., 1 ro., 31 po.; and 34 ac., 2 ro., 29 po.: also of 5 ac., 2 ro., 15 po., 25¼ sq. yds., 101 sq. in.; 9 ac., 1 ro., 35 po., 12½ sq. yds., 87 sq. in.; 42 ac., 3 ro., 24 po., 23¾ sq. yds., 57 sq. in.; 12 ac., 2 ro., 5 po., 13¾ sq. yds., 23 sq. in.; and 17 ac., 24 po., 30 sq. yds., 113 sq. in.: explain each process.

(17) Find the sum of 3 c. yds., 23 c. ft., 171 c. in.; 17 c. yds., 17 c. ft., 31 c. in.; 28 c. yds., 26 c. ft., 1000 c. in.; and 34 c. yds., 23 c. ft., 1101 c. in.: also of 12 po., 18 sq. yds., 7 sq. ft., 35 sq. in.; 13 po., 24¾ sq. yds., 8 sq. ft., 63 sq. in.; 14 po., 29¼ sq. yds., 5 sq. ft., 131 sq. in.; 15 po., 19 sq. yds., 3 sq. ft., 126 sq. in.; and 16 po., 28¼ sq. yds., 130 sq. in.

(18) Add together 39 gals., 3 qts., 1 pt.; 48 gals., 2 qts., 1 pt.;

56 gals., 1 pt.; 74 gals., 3 qts.; and 84 gals., 3 qts., 1 pt.: also 2 pipes, 42 gals., 3 qts.; 36 gals., 1 qt.; 5 pipes, 48 gals.; 12 pipes, 53 gals., 3 qts.; and 27 pipes, 2 qts., of wine: also 19 hhds., 10 gals., 3 pts.; 29 hhds., 50 gals., 7 pts.; 116 hhds., 46 gals., 5 pts.; 2 hhds., 2 pts.; and 235 hhds., 1 bar., 3 qts., of beer.

(19) Add together 14 qrs., 6 bus., 3 pks., 7 pts.; 37 qrs., 5 bus., 13 pts.; 43 qrs., 2 pks., 14 pts.; 57 qrs., 7 bus., 3 pks., 12 pts.; and 106 qrs., 4 bus., 13 pts.: also 37 lds., 37 bus., 2 pks.; 92 lds., 24 bus., 3 pks.; 136 lds., 28 bus., 1 pk.; 157 lds., 36 bus., 2 pks.; 540 lds., 1 pk.; and 736 lds., 39 bus.

(20) Add together 4 mo., 3 w., 5 d., 23 h., 46 m.; 5 mo., 1 d., 17 h., 57 m.; 6 mo., 2 w., 1 h.; 1 w., 6 d., 23 h., 59 m.; and 11 mo., 1 w., 58 m.: also 7 yrs., 28 w., 3 s.; 26 yrs., 5 w., 5 d.; 58 yrs., 6 d., 23 h., 59 s.; 43 w., 23 h., 50 m., 12 s.; and 124 yrs., 14 w., 19 h., 37 s.

(21) When *B* was born, *A*'s age was 2 yrs., 9 mo., 3 w., 4 d.; when *C* was born, *B*'s age was 13 yrs., and 3 d.; when *D* was born, *C*'s age was 9 mo., 2 w., 3 d., 23 h.; when *E* was born, *D*'s age was 6 yrs., 11 mo., 23 hrs.; when *F* was born, *E*'s age was 7 yrs., 3 w., 5 d., 15 h. What was *A*'s age on *F*'s 5th birth-day?

118. If *other fractions* of a penny, as well as those which denote farthings, be involved, the process is exactly the same as the above; those fractions being first added together by the ordinary rule of Addition of Fractions. For example, add together, £11. 4*s*. $5\frac{1}{8}d.$; £12. 2*s*. $7\frac{1}{10}d.$; £4. 7*s*. $3\frac{5}{6}d.$; £5. 3*s*. $2\frac{3}{8}d.$; and £6. 10*s*. $0\frac{1}{2}d.$

£	*s.*	*d.*
11	4	$5\frac{1}{8}$
12	2	$7\frac{1}{10}$
4	7	$3\frac{5}{6}$
5	3	$2\frac{3}{8}$
6	10	$0\frac{1}{2}$
£39	7	$6\frac{14}{15}$

Now $\left(\frac{1}{8}+\frac{1}{10}+\frac{5}{6}+\frac{3}{8}+\frac{1}{2}\right)d.$

$$=\left(\frac{4}{8}+\frac{1}{10}+\frac{5}{6}+\frac{1}{2}\right)d.=\left(\frac{1}{2}+\frac{1}{2}+\frac{3}{30}+\frac{25}{30}\right)d.$$

$$=\left(1+\frac{28}{30}\right)d.=1\frac{14}{15}d.$$

we therefore put down $\frac{14}{15}d.$, carry on 1 to the column of pence, and proceed by Rule, Art. (117).

Ex. XXXV.

(1) Add together £2. 0*s*. $7\frac{2}{3}d.$; £12. 16*s*. $0\frac{3}{4}d.$; £4. 14*s*. $8\frac{5}{6}d.$; £10. 0*s*. $0\frac{1}{2}d.$; £1. 7*s*. $5\frac{3}{8}d.$; and £14. 15*s*. $7\frac{7}{12}d.$

(2) Find the sum of £20. 16*s*. $5\frac{1}{9}d.$; £14. 15*s*. $0\frac{2}{7}d.$; £5. 13*s*. $8\frac{13}{21}d.$; £33. 19*s*. $1\frac{1}{3}d.$; and £18. 3*s*. $4\frac{5}{7}d.$

(3) Find the sum of £1. 3*s*. $6\frac{2}{3}d.$; £2. 4*s*. $7\frac{9}{10}d.$; £3. 5*s*. $8\frac{5}{6}d.$; £4. 9*s*. $1\frac{13}{15}d.$; and £6. 16*s*. $6\frac{19}{25}d.$

(4) Add together £23. 6*s.* $0\frac{7}{8}$*d.*; £4. 0*s.* $9\frac{1}{2}$*d.*; £57. 17*s.* $8\frac{15}{16}$*d.*; £96. 19*s.* $11\frac{5}{12}$*d.*; and £157. 7*s.* $7\frac{11}{24}$*d.*

(5) Add together £273. 16*s.* $7\frac{19}{20}$*d.*; £370. 11*s.* $3\frac{4}{5}$*d.*; £621. 13*s.* $9\frac{1}{4}$*d.*; £197. 4*s.* $11\frac{3}{8}$*d.*; and $5\frac{9}{10}$*d.*

COMPOUND SUBTRACTION.

119. Compound Subtraction is the method of finding the difference between two numbers of the same kind, but containing different denominations of that kind.

Rule. "Place the less number below the greater, so that the numbers of the same denomination may be under each other in the same column, and draw a line below them. Begin at the right hand, and subtract if possible each number of the lower line from that which stands above it, and set the remainder underneath. But when any number in the lower line is greater than the number above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination; subtract as before, and carry one to the number of the next higher denomination in the lower line: proceed thus throughout the columns."

Ex. 1. Subtract £88. 18*s.* $8\frac{1}{2}$*d.* from £146. 19*s.* $6\frac{1}{4}$*d.*

Proceeding by the Rule given above,

$$\begin{array}{r@{\;.\;}r@{\;.\;}l} \pounds. & s. & d. \\ 146 & 19 & 6\frac{1}{4} \\ 88 & 18 & 8\frac{1}{2} \\ \hline \pounds 58 & 0 & 9\frac{3}{4} \end{array}$$

Reason for the above process.

Since $\frac{1}{2}$*d.* is greater than $\frac{1}{4}$*d.*, we add to $\frac{1}{4}$*d.* 4 farthings or 1 penny, thus raising it to 5 farthings; and when 2 farthings are subtracted from 5 farthings, we have 3 farthings left; we therefore place down $\frac{3}{4}$*d.*: and in order to increase the lower number equally with the upper number, we add 1 penny to the 8 pence.

Now 9 pence cannot be taken from 6 pence; we therefore add 12 pence or 1*s.* to 6 pence, thus raising the latter to 18*d.*: we take the 9*d.* from 18*d.*, and put down the remainder 9*d.*; then adding 1*s.* to 18*s.*, the latter becomes 19*s.*: 19*s.* taken from 19*s.* leave no remainder: we then subtract £88. from £146., as though they were abstract numbers. It is manifest that in this process, whenever we add to the upper line, we also add a number of the same value to the lower line, so that the final difference is not altered.

Ex. 2. Subtract 106 lbs., 11 oz., 16 dwts., from 144 lbs., 8 oz., 14 dwts.

$$\begin{array}{rrr} \text{lb.} & \text{oz.} & \text{dwts.} \\ 144\,. & 8\,. & 14 \\ 106\,. & 11\,. & 16 \\ \hline 37\,. & 8\,. & 18 \end{array}$$

Ex. XXXVI.

(1)
$$\begin{array}{rrr} £. & s. & d. \\ 43\,. & 11\,. & 5 \\ 23\,. & 2\,. & 7 \\ \hline \end{array}$$

(2)
$$\begin{array}{rrr} £. & s. & d. \\ 149\,. & 4\,. & 6\frac{3}{4} \\ 86\,. & 13\,. & 2\frac{3}{4} \\ \hline \end{array}$$

(3)
$$\begin{array}{rrr} 309\,. & 13\,. & 11\frac{1}{4} \\ 119\,. & 19\,. & 10\frac{3}{4} \\ \hline \end{array}$$

(4)
$$\begin{array}{rrr} 5875\,. & 0\,. & 0 \\ 4986\,. & 19\,. & 9\frac{1}{2} \\ \hline \end{array}$$

(5)
$$\begin{array}{rrr} 343\,. & 18\,. & 5\frac{1}{4} \\ 11\,. & 18\,. & 5\frac{3}{4} \\ \hline \end{array}$$

(6)
$$\begin{array}{rrr} 663\,. & 5\,. & 11\frac{1}{2} \\ 349\,. & 19\,. & 9\frac{3}{4} \\ \hline \end{array}$$

(7)
$$\begin{array}{rrrr} \text{cwt.} & \text{qr.} & \text{lbs.} & \text{oz.} \\ 63\,. & 0\,. & 18\,. & 1 \\ 58\,. & 1\,. & 12\,. & 10 \\ \hline \end{array}$$

(8)
$$\begin{array}{rrr} \text{fur.} & \text{po.} & \text{yds.} \\ 14\,. & 34\,. & 5 \\ 1\,. & 38\,. & 4 \\ \hline \end{array}$$

(9)
$$\begin{array}{rrr} \text{ac.} & \text{ro.} & \text{po.} \\ 63\,. & 1\,. & 29 \\ 57\,. & 2\,. & 38 \\ \hline \end{array}$$

(10)
$$\begin{array}{rrrr} \text{qrs.} & \text{bus.} & \text{pk.} & \text{gal.} \\ 64\,. & 3\,. & 1\,. & 0 \\ 8\,. & 5\,. & 3\,. & 1 \\ \hline \end{array}$$

(11) Subtract £456. 15*s*. 11¾*d*. from £534. 13*s*. 10½*d*.; and prove the result.

(12) Find the difference between the following numbers, and verify the results:

1. 426 lbs., 8 oz., 1 dwt., 7 grs., and 388 lbs., 3 oz., 11 dwts., 21 grs.
2. 5836 lbs., and 4976 lbs., 7 oz., 13 dwts., 19 grs.
3. 26 tons, 2 qrs., 23 lbs., and 19 tons, 3 cwt., 3 qrs., 18 lbs.
4. 806 tons, 14 cwt., 7 lbs., and 789 tons, 16 lbs.
5. 144 lbs., 9 oz., 4 drs., 1 scr., and 129 lbs., 7 drs., 3 scr.

6. 418 yds., 1 qr., 1 na., and 387 yds., 3 qrs., 3 na.
7. 15 yds., 1 ft., 5 in., and 13 yds., 2 ft., 7 in.
8. 99 yds., and 87 yds., 1 ft., 11 in.
9. 13 m., 6 fur., 35 po., $3\frac{1}{2}$ yds., and 12 m., 38 po., 4 yds.
10. 35 lea., 4 fur., 23 po., 4 yds., 1 ft., and 28 lea., 5 fur., 39 po., $4\frac{1}{2}$ yds., 2 ft.
11. 56 ac., 2 ro., 34 po., and 48 ac., 3 ro., 38 po.
12. 3 ro., 28 po., 27 sq. yds., 7 sq. ft., and 1 ro., 39 po., $28\frac{1}{4}$ sq. yds., 8 sq. ft.
13. 37 cub. yds., 18 cub. ft., 857 cub. in., and 35 cub. yds., 24 cub. ft., 1280 cub. in.
14. 203 tuns, 19 gals., 3 qts., 1 pt., of wine, and 187 tuns, 1 hhd., 29 gals., 2 qts.
15. 83 bar., 2 fir., 7 gals., of beer, and 77 bar., 2 fir., 8 gals., 29 qts.
16. 23 lds., 2 qrs., 5 bus., 3 pks., and 18 lds., 2 qrs., 6 bus.
17. 216 yrs., 9 mo., 2 w., 4 d., and 217 yrs.
18. The latitude of St. Peter's at Rome is 41°, 53′, 54″ north, and that of St. Paul's at London is 51°, 30′, 49″ north. Find the difference of their latitude.
19. What sum added to £947. 19*s.* $7\frac{3}{4}$*d.* will make £1000?
20. A furnished house is worth £4759. 10*s.* $9\frac{1}{4}$*d.*; unfurnished, it is worth £1494. 11*s.* $9\frac{3}{4}$*d.* By how much does the value of the furniture exceed the value of the house?

120. If *other fractions* of a penny than those which denote farthings be involved, we must apply Rule, Art. (76), in order to find the difference of the fractions, and then proceed by Rule, Art. (119).

Ex. 1. Subtract £9. 14*s.* $6\frac{1}{3}$*d.* from £14. 0*s.* $5\frac{1}{2}$*d.*

$$\begin{array}{rrr} \pounds. & s. & d. \\ 14\ . & 0\ . & 5\frac{1}{2} \\ 9\ . & 14\ . & 6\frac{1}{3} \\ \hline \pounds 4\ . & 5\ . & 11\frac{1}{6} \end{array} \qquad \left(\frac{1}{2}-\frac{1}{3}\right)d.=\frac{3-2}{6}d.=\frac{1}{6}d.$$

Ex. 2. Subtract £7. 15*s.* $7\frac{11}{12}$*d.* from £10. 0*s.* $0\frac{3}{8}$*d.*

$\frac{11}{12}$ is greater than $\frac{3}{8}$, therefore we add 1 to $\frac{3}{8}$, which makes it $\frac{11}{8}$.

$$\begin{array}{rrr} \pounds. & s. & d. \\ 10\ . & 0\ . & 0\frac{3}{8} \\ 7\ . & 15\ . & 7\frac{11}{12} \\ \hline \pounds 2\ . & 4\ . & 4\frac{11}{24} \end{array}$$

Now $\frac{11}{8}-\frac{11}{12}=\frac{33-22}{24}=\frac{11}{24}$.

We must repay the 1*d.* by adding 1*d.* to 7*d.*

Ex. XXXVII.

Find the difference between

(1) £3. 13*s*. $9\frac{3}{4}$*d*., and £2. 15*s*. $5\frac{1}{6}$*d*.
(2) £20., and £15. 15*s*. $0\frac{6}{7}$*d*.
(3) £23. 13*s*. $7\frac{1}{5}$*d*., and £19. 19*s*. $7\frac{3}{5}$*d*.
(4) £416. 10*s*. $5\frac{1}{2}$*d*., and £305. 11*s*. $9\frac{5}{8}$*d*.
(5) £2163. 1*s*. $7\frac{5}{12}$*d*., and £364. 2*s*. $5\frac{13}{16}$*d*.

COMPOUND MULTIPLICATION.

121. Compound Multiplication is the method of finding the amount of any proposed compound number, that is, of any number composed of different denominations, but all of the same kind, when it is repeated a given number of times.

Rule. "Place the multiplier under the lowest denomination of the multiplicand; multiply the number of the lowest denomination by the multiplier, and find the number of units of the next denomination contained in this first product; if there be a remainder, place it down, adding on the number of units just found to the second product; for this second product, multiply the number of the next denomination in the multiplicand by the multipler, and after carrying on to it the above-mentioned number of units, proceed with the result as with the first product; carry this operation through with all the different denominations of the multiplicand."

Ex. Multiply £56. 4*s*. $6\frac{1}{2}$*d*. by 5.

Proceeding by the Rule given above,

$$\begin{array}{rcrcr} \pounds. & & s. & & d. \\ 56 & . & 4 & . & 6\frac{1}{2} \\ & & & & 5 \\ \hline \pounds 281 & . & 2 & . & 8\frac{1}{2} \end{array}$$

Reason for the above process.

$\frac{1}{2}$*d*. multiplied by 5 is the same as $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2})d.=5$ half-pence $=2\frac{1}{2}d.$; we therefore put down $\frac{1}{2}$*d*., and carry on 2*d*. to the denomination of pence:

6*d*. multiplied by $5=30d.$; therefore $(2+6\times 5)d.=32d.=(2\times 12+8)d.$ $=2s.+8d.$; we therefore put down 8*d*., and carry on 2*s*. to the denomination of shillings:

4*s.* multiplied by $5 = 20s.$; therefore $(2 + 4 \times 5)s. = 22s. = (20 + 2)s. = \pounds 1 + 2s.$; we therefore put down 2*s.*, and carry on £1 to the denomination of pounds:

Now by Simple Multiplication $\pounds 56 \times 5 = \pounds 280$; therefore $\pounds(1 + 56 \times 5) = \pounds(1 + 280) = \pounds 281$.

Therefore the total amount is £281. 2*s.* 8½*d.*

122. When the multiplier exceeds 12 it will be the easiest method to split the multiplier into factors, or into factors and parts: thus $15 = 3 \times 5$; $17 = 3 \times 5 + 2$; $23 = 4 \times 5 + 3$; $240 = 4 \times 6 \times 10$: and so on.

Ex. Multiply £55. 12*s.* 9¼*d.* by 23.

$$\begin{array}{rrrl}
\pounds. & s. & d. & \\
55\ . & 12\ . & 9\frac{1}{4} & \\
 & & 4 & \\
\hline
222\ . & 11\ . & 1 & = \text{value of } \pounds 55.\ 12s.\ 9\tfrac{1}{4}d. \text{ multiplied by } 4. \\
 & & 5 & \\
\hline
1112\ . & 15\ . & 5 & = \text{value of } \pounds 222.\ 11s.\ 1d. \text{ multiplied by } 5, \text{ or of} \\
 & & & \quad \pounds 55.\ 12s.\ 9\tfrac{1}{4}d. \text{ multiplied by } (4 \times 5, \text{ or } 20). \\
166\ . & 18\ . & 3\frac{3}{4} & = \text{value of } \pounds 55.\ 12s.\ 9\tfrac{1}{4}d. \text{ multiplied by } 3. \\
\hline
\pounds 1279\ . & 13\ . & 8\frac{3}{4} & = \text{value of } \pounds 55.\ 12s.\ 9\tfrac{1}{4}d. \text{ multiplied by } (20 + 3), \text{ or } 23.
\end{array}$$

Note 1. When the multiplicand contains farthings, if one of the factors of the multiplier be even, it will often be advantageous to use it first, as the farthings may disappear.

Note 2. Should the multiplier consist of *many* factors, it will be found in that case convenient to reduce the multiplicand to the lowest denomination contained in it, then to multiply this result by the multiplier, and then to reduce the result back again.

Ex. XXXVIII.

Multiply

(1) £11. 13*s.* 6*d.* separately by 2 and 5.
(2) £2. 18*s.* 7½*d.* separately by 4 and 6.
(3) £1. 16*s.* 6¼*d.* separately by 7 and 9.
(4) £2. 15*s.* 2¾*d.* separately by 5 and 8.
(5) £3. 16*s.* 0¾*d.* separately by 11 and 12.
(6) £7. 19*s.* 7½*d.* separately by 10 and 12.
(7) £347. 15*s.* 9¾*d.* separately by 3 and 11.
(8) £583. 0*s.* 10*d.* separately by 13 and 16.

(9) £1875. 13*s*. $8\frac{3}{4}d$. separately by 21 and 64.
(10) £721. 0*s*. $5\frac{1}{4}d$. separately by 81 and 96.
(11) £5072. 12*s*. $8\frac{1}{2}d$. separately by 112 and 128.
(12) £1100. 11*s*. $9\frac{1}{4}d$. separately by 62, 82, and 93.
(13) £2579. 0*s*. $0\frac{3}{4}d$. separately by 147, 155, 474, and 2331.
(14) 86 lbs., 7 oz., 16 dwts., 11 grs. separately by 8 and 36.
(15) 3 tons, 27 lbs., 13 oz. separately by 11 and 76.
(16) 45 lbs., 7 oz., 3 drs., 2 sc. separately by 12 and 68.
(17) 67 yds., 1 qr., 2 na. separately by 9 and 53.
(18) 70 yds., 2 ft., 10 in. separately by 7 and 29.
(19) 67 ro., 38 po., 27 yds., 2 ft. separately by 11 and 112.
(20) 380 ac., 3 ro., 32 po. separately by 12 and 106.
(21) 57 gals., 3 qts. separately by 10 and 257.
(22) 76 qrs., 5 bus., 2 pks. separately by 13 and 240.
(23) 5 wks., 6 d., 18 h., 14 m. separately by 11 and 339.
(24) 84 hhds., 43 gals., 1 pt. of wine separately by 27 and 364.
(25) 43 bar., 13 gals., 1 qt., 1 pt. of beer separately by 39 and 764.

(26) A person buys 67 lambs at £1. 0*s*. $9\frac{1}{2}d$. each; 73 sheep at £2. 2*s*. $11\frac{1}{4}d$. each; 12 cows at the average of £37. 0*s*. $2\frac{3}{4}d$. for every 3 of them; and 17 horses at 37 guineas each: the expenses of getting them all home amount to $17\frac{1}{2}$ guineas. What money must he draw from his bankers to pay for the whole outlay?

(27) There are 7 chests of drawers: in each chest there are 18 drawers; and in each drawer 8 divisions; and in each division there is placed £16. 6*s*. 8*d*. How much money is deposited in the chests?

123. If the multiplicand contain, instead of farthings, some *other fraction* of a penny, the process is exactly the same as the above: thus, Ex. 1, if we had to multiply £22. 15*s*. $4\frac{5}{8}d$. by 43;

$$43 = 5 \times 8 + 3$$

$$\begin{array}{rrrl}
\text{£.} & s. & d. & \\
22\ . & 15\ . & 4\frac{5}{8} & \\
 & & 8 & \\
\cline{1-3}
182\ . & 3\ . & 1 & \quad \text{for } \frac{5}{8}d. \times 8 = \frac{40}{8}d. = 5d. \\
 & & 5 & \\
\cline{1-3}
910\ . & 15\ . & 5 & \\
68\ . & 6\ . & 1\frac{7}{8} & \quad \text{for } \frac{5}{8}d. \times 3 = \frac{15}{8}d. = 1\frac{7}{8}d. \\
\cline{1-3}
\text{£}979\ . & 1\ . & 6\frac{7}{8} &
\end{array}$$

Ex. 2. Multiply £36. 10*s*. $0\frac{10}{17}$*d*. by 231.

$$231 = 7 \times 33 = 7 \times 3 \times 11.$$

$$\begin{array}{rrrl}
\text{£.} & \text{s.} & \text{d.} & \\
36 & 10 & 0\frac{10}{17} & \\
 & & 7 & \\
\hline
255 & 10 & 4\frac{2}{17} & \text{for } \frac{10}{17}d. \times 7 = \frac{70}{17}d. = 4\frac{2}{17}d. \\
 & & 3 & \\
\hline
766 & 11 & 0\frac{6}{17} & \text{for } \frac{2}{17}d. \times 3 = \frac{6}{17}d. \\
 & & 11 & \\
\hline
\text{£}8432 & 1 & 3\frac{15}{17} & \text{for } \frac{6}{17}d. \times 11 = \frac{66}{17}d. = 3\frac{15}{17}d.
\end{array}$$

Ex. XXXIX.

(1) Multiply £6. 12*s*. $8\frac{5}{6}$*d*. separately by 3, 11, and 57.
(2) Multiply £75. 13*s*. $9\frac{1}{5}$*d*. separately by 4, 15, and 88.
(3) Multiply £709. 17*s*. $11\frac{1}{12}$*d*. separately by 6, 26, and 120.
(4) Multiply £525. 14*s*. $0\frac{5}{8}$*d*. separately by 42, 44, and 163.
(5) Multiply £125. 19*s*. $11\frac{1}{16}$*d*. separately by 48, 144, and 577.

COMPOUND DIVISION.

124. COMPOUND DIVISION is the method of dividing a compound number, that is, a number composed of several denominations, but all of the same kind, into as many equal parts as the divisor contains units; and also of finding how often one compound number is contained in another of the same kind.

When the Divisor is an abstract number.

RULE. "Place the numbers as in Simple Division: then find how often the divisor is contained in the highest denomination of the dividend; put this number down in the quotient; multiply as in Simple Division and subtract; if there be a remainder, reduce that remainder to the next inferior denomination, adding to it the number of that denomination in the dividend, and repeat the division: carry on this process through the whole dividend."

Ex. 1. Divide £199. 6*s*. 8*d*. by 130.

Proceeding by the Rule given above,

$$\begin{array}{l}
\qquad\quad\ \ \pounds \qquad s. \quad d. \\
130)\ 199\ .\ 6\ .\ 8\ (1\pounds \\
\qquad\ \ \underline{130} \\
\qquad\ \ \ \ 69 \\
\qquad\ \ \ \ \underline{20} \\
130)\ 1386\ (10s. \\
\qquad\ \ \underline{130} \\
\qquad\ \ \ \ \ 86 \\
\qquad\ \ \ \ \ \underline{12} \\
130)\ 1040\ (8d. \\
\qquad\ \ \underline{1040}
\end{array}$$

Therefore the answer is £1. 10*s*. 8*d*.

Reason for the above process.

We first subtract £1 taken 130 times, from £199. 6*s*. 8*d*., and there remains £69. 6*s*. 8*d*.

Now £69. 6*s*. 8*d*. = 1386*s*. 8*d*.; from this amount we subtract 10*s*. taken 130 times, and there remains 86*s*. 8*d*.

Again, 86*s*. 8*d*. = 1040*d*.; from this amount we subtract 8*d*. taken 130 times, and nothing remains.

Therefore £1. 10*s*. 8*d*. is contained 130 times in £199. 6*s*. 8*d*.

Ex. 2. Divide £1076. 4*s*. $3\frac{3}{4}d$. by 527.

$$\begin{array}{l}
\qquad\quad\ \ \pounds. \qquad s. \quad d. \\
527)\ 1076\ .\ 4\ .\ 3\tfrac{3}{4}\ (2\pounds \\
\qquad\ \ \underline{1054} \\
\qquad\ \ \ \ \ 22 \\
\qquad\ \ \ \ \ \underline{20} \\
\qquad\ \ \ \ 444\ (0s. \\
\qquad\ \ \ \ \ \ \underline{12} \\
527)\ 5331\ (10d. \\
\qquad\ \ \underline{527} \\
\qquad\ \ \ \ \ 61 \\
\qquad\ \ \ \ \ \ \underline{4} \\
527)\ 247\ (0q.
\end{array}$$

Therefore the result is £2. 0*s*. 10*d*. and there remains 247 farthings to be divided by 527, which division will clearly not give so much as one farthing.

Therefore the quotient is £2. 0*s*. 10*d*. $0\frac{247}{527}$*q*.

Note. When the divisor is not greater than 12, the division can be easily performed in one line: thus for example, divide £8. 18*s*. 6*d*. by 12.

$$\begin{array}{r|ccccc} & \pounds. & & s. & & d. \\ 12 & 8 & . & 18 & . & 6 \\ \cline{2-6} & & & 14 & . & 10\frac{1}{2} \end{array}$$

Since we cannot divide 8 by 12, we reduce the £8 to shillings, and adding in the term 18*s*., we have to divide 178*s*. by 12 ; we obtain 14*s*., with remainder 10*s*.; and since 10*s*. = 120*d*.; therefore, adding in the term 6*d*., we have to divide 126*d*. by 12 ; we obtain 10*d*., with remainder 6*d*.: and since 6*d*. = 24*q*., we divide 24*q*. by 12, and thus we obtain 2*q*., or ½*d*.

Ex. XL.

(1) £7. 16*s*. 2*d*. ÷ 2. (2) £6. 8*s*. 9*d*. ÷ 3.
(3) £75. 8*s*. 6½*d*. ÷ 2. (4) £19. 14*s*. 9¾*d*. ÷ 3.
(5) £245. 14*s*. 8*d*. ÷ 4. (6) £86. 16*s*. 5½*d*. ÷ 5.
(7) £82. 13*s*. 7½*d*. ÷ 6. (8) £435. 17*s*. 2¼*d*. ÷ 7.
(9) £409. 6*s*. 2*d*. ÷ 8. (10) £605. 0*s*. 1½*d*. ÷ 9.
(11) £386. 16*s*. 5¼*d*. ÷ 11. (12) £473. 14*s*. 6*d*. ÷ 12.
(13) £33. 18*s*. 6*d*. ÷ 23. (14) £39. 1*s*. 5½*d*. ÷ 31.
(15) £12. 18*s*. 4½*d*. ÷ 39. (16) £165. 15*s*. 8¾*d*. ÷ 139.
(17) £62. 1*s*. 7½*d*. ÷ 198. (18) £2728. ÷ 744.
(19) £1288. 1*s*. 8*d*. ÷ 754. (20) £37. 3*s*. 1*d*. ÷ 74.
(21) £492710. 1*s*. 8*d*. ÷ 6352. (22) £162. 3*s*. 6*d*. ÷ 156.
(23) £130264. 9*s*. 6*d*. ÷ 9416. (24) £1746 ÷ 2737.
(25) £2648. 18*s*. 9*d*. ÷ 357. (26) £344. 0*s*. 8¼*d*. ÷ 129.
(27) 1283 cwt., 4 lbs. ÷ 75. (28) 178 cwt., 3 qrs., 14 lbs. ÷ 53.
(29) 206 mo. of 28 days, 4 d. ÷ 26. (30) 684 d., 8 h., 9 m. ÷ 47.
(31) 15 cwt., 27 lb., 11 oz. ÷ 456. (32) 76 cwt. ÷ 963.
(33) 75 ac., 3 ro., 39 po. ÷ 26. (34) 13 ac., 1 ro. ÷ 147.
(35) 91 yds., 2 qrs., 1 na. ÷ 903. (36) 97 qrs., 3 bus., 3 pks. ÷ 107.
(37) £12 ÷ ·000625 ; and £36 ÷ ·001875.

125. It may sometimes be found convenient to break up the divisor into factors : thus,

Ex. 1. Divide £37. 14*s*. by 24.

$$24 = 4 \times 6.$$

$$24\left\{\begin{array}{r|ccccc} & \pounds. & & s. & & d. \\ 4 & 37 & . & 14 & . & 0 \\ \cline{2-6} 6 & 9 & . & 8 & . & 6 \\ \cline{2-6} & \pounds1 & . & 11 & . & 5 \end{array}\right.$$

Ex. 2. Divide £131. 2s. 8½d. by 48, and also by its factors 6 and 8, and shew that the results coincide.

$$
\begin{array}{rl}
 & \;\;\text{£.}\quad\; s.\quad d.\\
48) & 131\ .\ 2\ .\ 8\frac{1}{2}\ (2\text{£}.\\
 & \underline{96}\\
 & 35\\
 & \underline{\;20}\\
48) & 702\ (14s.\\
 & \underline{48}\\
 & 222\\
 & \underline{192}\\
 & \;\;30\\
 & \underline{\;\;12}\\
48) & 368\ (7d.\\
 & \underline{336}\\
 & \;\;32\\
 & \underline{\;\;\;4}\\
48\ ) & 130\ (2q.\\
 & \underline{\;96}\\
 & \;\;34
\end{array}
$$

Therefore the quotient is £2. 14s. $7\frac{1}{2}d.\ \frac{17}{24}q.$

Now, dividing by the factors 6 and 8, we get

$$
\begin{array}{r|l}
 & \;\text{£.}\qquad s.\quad\; d.\\
6 & 131\ .\ \;\;2\ .\ 8\frac{1}{2}\\ \hline
8 & \;\;21\ .\ 17\ .\ 1\frac{1}{4}\frac{2}{3}\\ \hline
 & \;\;\;\;2\ .\ 14\ .\ 7\frac{1}{2}\frac{17}{24}
\end{array}
\qquad \left(\frac{5\frac{2}{3}}{8}=\frac{\frac{17}{3}}{8}=\frac{17}{24}\right).
$$

Ex. XLI.

In the following examples, divide by the numbers themselves, and then by any factors composing them; and shew that the results are the same.

(1) £440. 16s. 9½d. ÷ 15.
(2) £678. 19s. 9¾d. ÷ 32.
(3) £123. 13s. 0¾d. ÷ 99.
(4) £236. 17s. ÷ 96.
(5) £371. 2s. 9¼d. ÷ 18.
(6) £315. 11s. 7¼d. ÷ 42.
(7) £972. 15s. 10½d. ÷ 132.
(8) £860. 11s. 1¼d. ÷ 198.
(9) £2016. 2s. 2¼d. ÷ 108.
(10) £3363. 0s. 11¾d. ÷ 528.

126. If the Divisor be 10, 100, 1000, &c., the operation of Division is usually performed, by pointing off as decimals, one, two, three, &c. figures accordingly at the right hand of the dividend.

Thus, Ex. 1: Divide £5362. 10*s*. by 100.

Long Method.

£. *s.*
100) 5362 . 10 (53£.
500
362
300
62
20
1240 + 10 = 1250*s*.

100) 1250 (12*s*.
100
250
200
50
12
100) 600 (6*d*.
600

Usual Method.

£. *s.*
53·62 . 10
20
1240 + 10*s*.
= 12·50*s*.
12
6·00*d*.

Therefore the quotient is £53. 12*s*. 6*d*.

Reason for the above process.

$$\text{£}5362.\ 10s. \div 100 = \frac{\text{£}5362}{100} + \frac{10s.}{100}$$

$$= \text{£}53{\cdot}62 + \frac{10s.}{100} = \text{£}53 + \frac{\text{£}62}{100} + \frac{10s.}{100}$$

$$= \text{£}53 + \frac{(62 \times 20)s.}{100} + \frac{10s.}{100}$$

$$= \text{£}53 + \frac{(1240 + 10)s.}{100}$$

$$= \text{£}53 + \frac{1250s.}{100}$$

$$= \text{£}53 + 12{\cdot}50s.$$

$$= \text{£}53 + 12s. + \frac{50s.}{100}$$

$$= \text{£}53 + 12s. + \frac{(50 \times 12)d.}{100}$$

= £53 + 12*s*. + 6*d*.
= £53. 12*s*. 6*d*.

Ex. 2. Divide £1668. 15*s*. by 1500.

$$1500 = 3 \times 5 \times 100;$$

first divide by the factors 3 and 5, and then by 100: it will be found best in all cases of this kind to do so.

$$\begin{array}{r|rcl}
 & \text{£.} & & s. \\
3 & 1668 & . & 15 \\
\hline
5 & 556 & . & 5 \\
\hline
 & \text{£}1{\cdot}11 & . & 5 \\
 & 20 & & \\
 & 2{\cdot}25s. & & \\
 & 12 & & \\
 & 3{\cdot}00d. & &
\end{array}$$

Therefore the quotient is £1. 2*s*. 3*d*.

Ex. XLII.

(1) £396. 9*s*. 2*d*. ÷ 10.
(2) £1787. 10*s*. ÷ 100.
(3) £2025. ÷ 1000.
(4) £1447. 18*s*. 4*d*. ÷ 1000.
(5) £262. 10*s*. ÷ 2400.
(6) £26380. 4*s*. 2*d*. ÷ 25000.
(7) 21 ac., 3 ro., 17 perches × ·02; and £375. 3*s*. × ·0507.
(8) 24 ac., 3 ro., 10 perches × 112, and × 11·2.

127. *When the divisor and dividend are both compound numbers of the same kind.*

Rule. "Reduce both numbers to the same denomination: divide as in Simple Division, and the result will be the answer required."

Ex. How often is 5*s*. $3\frac{3}{4}$*d*. contained in £15. 8*s*. 9*d*.?

Proceeding by the above Rule,

$$\begin{array}{rcl}
s. & & d. \\
5 & . & 3\frac{3}{4} \\
12 & & \\
\hline
63 & & \\
4 & & \\
\hline
255 & &
\end{array}
\qquad\qquad
\begin{array}{rcccl}
\text{£.} & & s. & & d. \\
15 & . & 18 & . & 9 \\
20 & & & & \\
\hline
318 & & & & \\
12 & & & & \\
\hline
3825 & & & & \\
4 & & & & \\
\hline
15300 & & & &
\end{array}$$

$$\begin{array}{l}
255)\ 15300\ (60 \\
\underline{1530}
\end{array}$$

Therefore 60 is the answer.

Reason for the above process.

$$5s.\ 3\tfrac{3}{4}d. = 255 \text{ farthings},$$

$$\pounds 15.\ 18s.\ 9d. = 15300 \text{ farthings};$$

and 255 farthings subtracted 60 times from 15300 farthings leave no remainder.

Ex. XLIII.

(1) £2. 12*s.* 3*d.* ÷ 1*s.* $4\frac{1}{2}$*d.*
(2) £55. 18*s.* $10\frac{1}{4}$*d* ÷ £2. 8*s.* $7\frac{3}{4}$*d.*
(3) £160. 4*s.* $8\frac{1}{4}$*d.* ÷ £1. 10*s.* $6\frac{1}{4}$*d.*
(4) £401. 4*s.* 3*d.* ÷ £2. 11*s.* $5\frac{1}{4}$*d.*
(5) 44 cwt., 2 qrs., 11 lbs. ÷ 1 cwt., 2 qrs., 17 lbs.
(6) 272 yds., 1 qr. ÷ 7 yds., 2 qrs., 1 na.
(7) 9487 bus., 2 pks. ÷ 143 bus., 3 pks.
(8) 1416 ac., 2 ro., 16 po. ÷ 4 ac., 3 ro., 27 po.
(9) 57 lea., 1 mi., 956 yds. ÷ 7 fur., 87 yds., 1 ft., 5 in.
(10) 617 lds., 1 qr. ÷ 12 qrs., 1 pk.

128. We shall now add some examples of the Multiplication and Division of numbers, comprising different denominations, but of the same kind, by mixed numbers.

In the case of Multiplying by a mixed number, it will generally be found advantageous, first to multiply by the integral part, and then to add to the result thus obtained the result given by multiplying by the fractional part.

Thus, for example: Multiply £2. 6*s.* 8*d.* by $3\frac{7}{10}$.

$$(\pounds 2.\ 6s.\ 8d.) \times 3 = \pounds 7.$$

$$\frac{(\pounds 2.\ 6s.\ 8d.) \times 7}{10} = \frac{\pounds 16.\ 6s.\ 8d.}{10} = \pounds 1.\ 12s.\ 8d.$$

$$\text{Therefore } (\pounds 2.\ 6s.\ 8d.) \times 3\tfrac{7}{10} = \pounds 7. + \pounds 1.\ 12s.\ 8d. = \pounds 8.\ 12s.\ 8d.$$

In Division it will be found advantageous to reduce the mixed number to an improper fraction.

Thus, for example: Divide £89. 17*s.* $6\frac{3}{4}$*d.* by $19\frac{3}{4}$*d.*

$$19\tfrac{3}{4} = \tfrac{79}{4}.$$

$$\text{Now } \pounds 89.\ 17s.\ 6\tfrac{3}{4}d. \div \tfrac{79}{4} = \frac{(\pounds 89.\ 17s.\ 6\frac{3}{4}d.) \times 4}{79}.$$

$$\begin{array}{rl}
 & \pounds. \quad s. \quad d. \\
 & 89 \,.\, 17 \,.\, 6\frac{3}{4} \\
 & \qquad\qquad\quad 4 \\
\hline
79) & 359 \,.\, 10 \,.\, 3 \quad (4\pounds \\
 & \underline{316} \\
 & 43 \\
 & \underline{\;20} \\
79) & 870 \;(11s. \\
 & \underline{79} \\
 & \;80 \\
 & \underline{\;79} \\
 & \;\;1 \\
 & \underline{12} \\
79) & 15 \;(0d.
\end{array}$$

Therefore the quotient is £4. 11*s*. $0\frac{15}{79}$*d*.

Ex. XLIV.

(1) £13. 12*s*. $11\frac{1}{4}$*d*. × $2\frac{7}{9}$.
(2) £7. 0*s*. $0\frac{4}{5}$*d*. × $3\frac{4}{5}$.
(3) £40. 11*s*. $6\frac{3}{8}$*d*. × $57\frac{1}{20}$.
(4) 3 ro., 35 po., $27\frac{1}{2}$ yds. × $81\frac{1}{2}$.
(5) 4 mi., 3 fur., 37 po., $4\frac{1}{5}$ yds. × $5\frac{5}{7}$.
(6) 84 tons, 13 cwt., 3 lbs. × $23\frac{5}{9}$.
(7) £50. 10*s*. 7*d*. ÷ $\frac{1}{8}$.
(8) 597 cwt., 2 qrs., 8 lbs. ÷ $13\frac{1}{3}$.
(9) £9. 9*s*. $7\frac{5}{16}$*d*. ÷ $3\frac{3}{4}$.
(10) 6491 yrs., 8 mo. ÷ $375\frac{8}{11}$.
(11) £20. 18*s*. $2\frac{157}{310}$*d*. ÷ $12\frac{10}{12}$.
(12) 571 yds., 2 qrs., 1 na. ÷ $23\frac{5}{7}$.

Miscellaneous Examples, depending on Arts. (116—128), *worked out.*

Ex. 1. A person bought 500 yards of cloth at 15*s*. 9*d*. a-yard, and retailed it at 16*s*. 3*d*. a-yard: what was his profit?

His profit on 1 yard = 16*s*. 3*d*. − 15*s*. 9*d*.
= 6*d*.,
therefore his whole profit = (6 × 500)*d*.
= 250*s*.
= £12. 10*s*.

Ex. 2. A spring of water, which yields 75 gallons an hour, supplies 600 families: how much water may each family use daily?

The daily supply of water = (75 × 24) gallons;

therefore each family may use daily $\frac{75 \times 24}{600}$ gals., or 3 gals.

Ex. 3. How many revolutions will a wheel, which is 4 yards in circumference, make in 3 miles?

$$3 \text{ miles} = (3 \times 1760) \text{ yards} = 5280 \text{ yards},$$

and since the wheel passes over 4 yards in one revolution;

$$\frac{5280}{4} \text{ or } 1320 = \text{number of revolutions required.}$$

Ex. 4. The value of a mark being 13*s*. 4*d*., and that of a moidore 27*s*., how many half-crowns are there in 30 marks + 40 moidores?

$$\begin{aligned} 30 \text{ marks} + 40 \text{ moidores} &= (13s.\ 4d.) \times 30 + 27s. \times 40 \\ &= (160 \times 30)d. + (27 \times 12 \times 40)d. \\ &= (4800 + 12960)d. \\ &= 17760d. \end{aligned}$$

$$1 \text{ half-crown} = 30d.;$$

$$\text{therefore number of half-crowns required} = \frac{17760}{30} = 592.$$

Ex. 5. How many guineas, sovereigns, half-crowns, and shillings, and of each an equal number, are there in £1246?

$$\begin{aligned} \text{Now, } 1 \text{ guinea} + 1 \text{ sovereign} + 1 \text{ half-crown} + 1 \text{ shilling} &= (42 + 40 + 5 + 2) \text{ sixpences} \\ &= 89 \text{ sixpences}; \end{aligned}$$

$$\text{and } £1246 = (1246 \times 20 \times 2) \text{ sixpences} = 49840 \text{ sixpences};$$

the question therefore is reduced to this: How often are 89 sixpences contained in 49840 sixpences?

$$\text{Number required} = \frac{49840}{89} = 560.$$

Ex. 6. How much water must be added to a cask containing 60 gallons of spirit at 12*s*. 6*d*. a gallon, to reduce the price to 8*s*. a gallon?

$$\begin{aligned} \text{Cost of cask} &= (12s.\ 6d.) \times 60, \\ &= (150 \times 60)d. \\ 8s. &= (8 \times 12)d.; \end{aligned}$$

therefore $\frac{150 \times 60}{8 \times 12}$, or $\frac{375}{4}$, or $93\frac{3}{4}$ = the number of gallons which the cask must contain, in order that its contents may be sold at 8*s*. a gallon.

Therefore $(93\frac{3}{4} - 60)$, or $33\frac{3}{4}$ = the number of gallons of water which have to be added.

Ex. 7. How many yards of cloth, worth 3*s.* $7\frac{1}{2}$*d.* a-yard, must be given in exchange for 144 yards of cloth, worth 18*s.* $1\frac{1}{2}$*d.* a-yard?

The value of 144 yards at 18*s.* $1\frac{1}{2}$*d.* a-yard,

= (18*s.* $1\frac{1}{2}$*d.*) × 144,
= £130. 10*s.*
= 62640 half-pence;

and 3*s.* $7\frac{1}{2}$*d.* = 87 half-pence;

therefore the number of yards required $= \frac{62640}{87} = 720$.

Or thus, since 18*s.* $1\frac{1}{2}$*d.* = (3*s.* $7\frac{1}{2}$*d.*) × 5,
it is clear that the number of yards required = 144 × 5,
= 720.

Ex. 8. A traveller walks 22 miles a-day, and after he has gone 84 miles another follows him at the rate of 34 miles a-day; in what time will the second traveller overtake the first?

The second traveller has to walk over 84 miles more than the first before he can overtake him.

Each day he walks (34 − 22) or 12 miles more than the first;

therefore $\frac{84}{12}$ or 7 is the number of days required.

Ex. 9. A mixture is made of 8 gallons of spirits at 12*s.* 10*d.* a gallon, 7 gallons at 10*s.* 6*d.* a gallon, and 10 gallons at 9*s.* 1*d.* a gallon; at what price per gallon must the mixture be sold, 1st, that the seller may neither gain nor lose by his bargain; 2nd, that he may gain £1. 13*s.* by it; 3rd, that he may lose 7 guineas; and 4th, that he may reserve 10 gallons of the mixture for himself, and sell the remainder so as to realize the money he laid out?

	£.	*s.*	*d.*
8 gallons at 12*s.* 10*d.* cost	5	2	8
7 gallons at 10*s.* 6*d.* cost	3	13	6
10 gallons at 9*s.* 1*d.* cost	4	10	10
therefore 25 gallons cost	£13	7	0

1st. If he is neither to gain or lose, he must sell 1 gallon for $\frac{£13.\ 7s.}{25}$; which, worked out, gives 10*s.* $8\frac{4}{25}$*d.* as the price required.

2nd. If he is to gain £1. 13*s*.

25 gallons must be sold for £13. 7*s*. + £1. 13*s*., or £15.;

therefore, 1 gallon must be sold for $\frac{£15}{25}$; which, worked out, gives 12*s*. as the price required.

3rd. If he is to lose 7 guineas,

25 gallons must be sold for £13. 7*s*. − £7. 7*s*. or £6.;

therefore 1 gallon must be sold for $\frac{£6}{25}$; which, worked out, gives 4*s*. $9\frac{1}{2}$*d*. $\frac{2}{5}$*q*. as the price required.

4th. If he is to retain 10 gallons for his own use,

15 gallons must be sold for £13. 7*s*.;

therefore 1 gallon must be sold for $\frac{£13.\ 7s.}{15}$; which, worked out, gives 17*s*. $9\frac{1}{2}$*d*. $\frac{2}{5}$*q*. as the price required.

Ex. 10. A club, consisting of 56 persons, joined for a lottery ticket of 12 guineas value, and it came up a prize of £7000: what sum did each man contribute, and what did each man gain?

56 persons subscribe 12 guineas;

$$\text{therefore each person subscribes } \frac{12 \text{ guineas}}{56},$$

$$\text{or } 4s.\ 6d.$$

56 persons receive a prize of £7000;

$$\text{therefore each person receives } \frac{£7000}{56},$$

$$\text{or } £125;$$

$$\text{therefore each person gains } £125. - 4s.\ 6d.$$

$$= £124.\ 15s.\ 6d.$$

Ex. 11. Divide £20 among *A*, *B*, and *C*, so that *B* may have 2 guineas more than *A*, and that *C* may have 2*s*. less than *B*.

Now

$$\begin{aligned} B\text{'s share} &= A\text{'s share} + £2.\ 2s. \\ C\text{'s share} &= B\text{'s share} - 2s. \\ &= A\text{'s share} + £2.\ 2s. - 2s. \\ &= A\text{'s share} + £2. \end{aligned}$$

But, by the question,

$$A\text{'s share} + B\text{'s share} + C\text{'s share} = £20,$$

$$\text{or } A\text{'s share} + (A\text{'s share} + £2.\ 2s.) + (A\text{'s share} + £2.) = £20,$$

or 3 times *A*'s share + £4. 2*s*. = £20;
therefore evidently 3 times *A*'s share = £20. − £4. 2*s*.
= £15. 18*s*.,

$$\text{or } A\text{'s share} = \frac{£15.\ 18s.}{3} = £5.\ 6s.$$

B's share = £7. 8*s*. *C*'s share = £7. 6*s*.

Ex. 12. Divide £8. 11*s*. 6*d*. among 5 men, 6 women, and 7 boys; giving each woman twice as much as each boy, and each man thrice as much as each woman.

Since each woman's share = twice each boy's share,
therefore 6 women's shares = 12 boys' shares.
Again, since each man's share = thrice each woman's share,
therefore, 5 men's shares = 15 women's shares,
= 30 boys' shares,
but 5 men's shares + 6 women's shares + 7 boys' shares = £8. 11*s*. 6*d*.,
or 30 boys' shares + 12 boys' shares + 7 boys' shares = £8. 11*s*. 6*d*.,
or 49 boys' shares = £8. 11*s*. 6*d*.
= 343 sixpences.

$$\text{Therefore, each boy's share} = \frac{343}{49} \text{ sixpences,}$$

= 7 sixpences = 3*s*. 6*d*.
Therefore, each woman's share = 7*s*.,
each man's share = £1. 1*s*. 0*d*.

DECIMAL COINAGE.

129. It may be well to notice here some of the advantages which would result from a decimal coinage of pounds, florins, cents, and mils; the pound being of the same value as the pound sterling at present; the florin being $= \frac{1}{10}$ of £1.; the cent being $= \frac{1}{10}$th of a florin, or $= \frac{1}{100}$th of £1.; the mil (m.) being $= \frac{1}{10}$th of a cent, or $= \frac{1}{100}$th of a florin, or $= \frac{1}{1000}$th of £1. The Table would stand thus:

10 Mils make 1 cent, 1 c.
10 Cents ...... 1 florin, 1 fl.
10 Florins ...... 1 pound, £1.

130. In such a system, much of the labour of reducing superior to inferior denominations, and the converse, would be done away with; for we could at once say, £24. 3 fl. 7 c. 2 m. = 24372 m. Since by performing the operation of reduction at length, we obtain

$$\begin{array}{l}
\text{£. fl. c. m.}\\
24\ .\ 3\ .\ 7\ .\ 2\\
\underline{10}\\
240+3, \text{ or } 243 \text{ fl.}\\
\qquad\qquad \underline{10}\\
\qquad\qquad 2430+7, \text{ or } 2437 \text{ c.}\\
\qquad\qquad\qquad\qquad \underline{10}\\
\qquad\qquad\qquad\qquad 24370+2, \text{ or } 24372 \text{ m.}
\end{array}$$

or we might say £24. 3 fl. 7 c. 2 m. = £24·372;

$$\begin{aligned}
\text{for £24. 3 fl. 7 c. 2 m.} &= \text{£}\left(24+\frac{3}{10}+\frac{7}{100}+\frac{2}{1000}\right)\\
&= \text{£}\frac{24000+300+70+2}{1000}\\
&= \text{£}\frac{24372}{1000}\\
&= \text{£}24{\cdot}372.
\end{aligned}$$

Similarly, £24. 3 fl. 7 c. 2 m. = 243·72 fl., or = 2437·2 c.

Conversely 24372 mils = £24. 3 fl. 7 c. 2 m.,

for, proceeding by Rule (Art. 116), we get

$$\begin{array}{r|l}
10 & \underline{24372}\\
10 & \underline{\ \ 2437-2\text{ m.}}\\
10 & \underline{\ \ \ \ 243-7\text{ c.}}\\
 & \ \ \ \ \ \ 24-3\text{ fl.}
\end{array}$$

hence 24372 m. = £24. 3 fl. 7 c. 2 m.;

or we might say 24372 m. = £24·372;

$$\text{for } 24372 \text{ m.} = \text{£}\frac{24372}{1000} = \text{£}24{\cdot}372.$$

Similarly 24372 m. = 243·72 fl., or = 2437·2 c.

Again, £18. 3 fl. 9 m. = 18309 m.,

for, proceeding by Rule (Art. 116),

$$\begin{array}{l}
\text{£. fl. m.}\\
18\ .\ 3\ .\ 9\\
\underline{10}\\
180+3, \text{ or } 183 \text{ fl.}\\
\qquad\qquad \underline{10}\\
\qquad\qquad 1830 \text{ c.}\\
\qquad\qquad \underline{\quad 10}\\
\qquad\qquad 18300+9, \text{ or } 18309 \text{ m.}
\end{array}$$

or we might say £18. 3 fl. 9 m. = £18·309;

$$\text{for } £18.\ 3\text{ fl. } 9\text{ m.} = £\left(18 + \frac{3}{10} + \frac{0}{100} + \frac{9}{1000}\right)$$

$$= £\left(\frac{18000 + 300 + 9}{1000}\right)$$

$$= £\,\frac{18309}{1000}$$

$$= £\,18{\cdot}309.$$

Similarly £18. 3 fl. 9 m. = 183·09 fl., or = 1830·9 c.

Conversely 18309 m. = £18. 3 fl. 9 m.

for proceeding by Rule (Art. 116) we get

$$\begin{array}{r|l} & \text{m.} \\ 10 & \underline{18309} \\ 10 & \underline{1830} - 9\text{ m.} \\ 10 & \underline{183} - 0\text{ c.} \\ & 18 - 3\text{ fl.} \end{array}$$

or, 18309 m. = £18. 3 fl. 0 c. 9 m.

$$\text{Similarly } 18309\text{ m.} = £\frac{18309}{1000} = £18{\cdot}309,$$

or 18309 m. = 183·09 fl., or = 1830·9 c.

Again, £254. 5½ c. = £254. 5·5 c.

$$\begin{array}{l} \underline{\quad\ 100} \\ 25400\text{ c.} + 5{\cdot}5\text{ c.} \\ \quad = 25405{\cdot}5\text{ c.} \\ \quad = 254055\text{ m.} \end{array}$$

Also, £254. 5¼ fl. = £254. 5·25 fl.

$$\begin{array}{l} \underline{\quad\ 10} \\ 2540\text{ fl.} + 5{\cdot}25\text{ fl.} \\ = 2545{\cdot}25\text{ fl.} \\ = 25452{\cdot}5\text{ c.} \\ = 254525\text{ m.} \end{array}$$

Ex. XLV.

Reduce, expressing in each successive inferior denomination and verifying each result:

(1) £15. 6 fl. to mils, and 6 fl. 3 c. 2 m. to mils.

(2) £30. 9½ fl. to mils, and £96. 1 fl. 2 c. 9 m. to mils.

(3) £18. 6½c. to mils, and 9¼ fl. to mils.

(4) £10. 1 m. to mils, and £46. 2½ c. to mils.

131. The addition, subtraction, multiplication, and division of money would also be much simplified by the adoption of a decimal coinage, as will be evident from the following examples.

Ex. 1. Find the sum of £18. 6 fl. 3 c. 5 m.; 9 fl. 9 m.; £24. 1 m.; 3 c. 2 m.; 5¼ fl.

	m.		£.
£18. 6 fl. 3 c. 5 m. =	18635,	or =	18·635,
9 fl. 9 m. =	909,	or =	·909,
£24. 1 m. =	24001,	or =	24·001,
3 c. 2 m. =	32,	or =	·032,
5¼ fl. =	525,	or =	·525,
	44102 m.,	or =	£44·102.

each of which results = £44. 1 fl. 2 m.

Ex. 2. From £16. 3 c. 2 m., subtract £14. 4 fl. 9 m.

	m.		£.
£16. 3 c. 2 m. =	16032,	or =	16·032,
£14. 4 fl. 9 m. =	14409,	or =	14·409,
	1623 m.,	or	£1·623.

each of which results = £1. 6 fl. 2 c. 3 m.

Ex. 3. Multiply £16. 3 c. 2 m. by 23.

£16. 3 c. 2 m. = 16032 m., or = £16·032.

m.	£.
16032	16·032
23	23
48096	48096
32064	32064
368736 m.	£368·736

each of the above results = £368. 7 fl. 3 c. 6 m.

Ex. 4. Divide £368. 7 fl. 3 c. 6 m. by 23.

In other words, divide 368736 m. by 23, or £368·736 by 23.

```
        m.       m.                     £.       £.
23) 368736 (16032               23) 368·736 (16·032
    23                              23
    ---                             ---
    138                             138
    138                             138
    ----                            ----
      73                              73
      69                              69
      ---                             ---
       46                              46
       46                              46
       --                              --
```

each of the above results = £16. 0 fl. 3 c. 2 m.

Note. Similar advantages would result from the use of a decimal system in weights and measures.

Ex. XLVI.

1. Add together
   (1) £76. 8 fl. 5 c. 3 m.; £27. 9 fl. 9 m.; £84. 1 c.; £56. 3 fl. 6 c. 2 m.; £19. 1 m.
   (2) £252. 2½ fl.; £300. 2½ c.; 4½ fl.; 5⅕ c.
2. Find the difference between
   (1) £19. 5 fl., and £16. 3 fl. 9 c.
   (2) £20., and £19. 9 fl. 9 c. 9 m.
   (3) £5. 5½ fl., and £4. 4½ c.
3. Multiply
   (1) £76. 8 fl. 3 m. separately by 5 and 63.
   (2) 9 fl. 2½ c. separately by 18 and 1008.
   (3) £150. 5 m. separately by 2005 and 18576.
4. Divide
   (1) £194. 5 fl. 7 c. 5 m. by 5.
   (2) £10764. 2 fl. 4 m. by 11.
   (3) £342136. 8 fl. by 7380.

Ex. XLVII.

Miscellaneous Questions and Examples on Arts. (100—131).

Note.—Where the contrary is not expressed, a year is supposed to consist of 365 days.

I.

(1) Explain the meaning of the term 'Reduction.' Reduce 537983 half-guineas into seven-shilling pieces, and also into groats.

(2) What is the standard of the gold, and silver, and copper coinage in this kingdom? According to the present law in England for what sums respectively are copper and silver legal tenders?

(3) What is meant by 'Compound Multiplication'? Can concrete numbers of the same or different kinds be multiplied together? Give the reason. What is the cost of school accommodation for 13750 children at £1. 18*s*. 6½*d*. each?

(4) How many nobles are equivalent to £26. 14*s*.?

(5) A person bought 1763 yards of cloth at 5*s*. 3½*d*. per yard, and retailed it at 6*s*. 11*d*. per yard: what was his profit?

(6) A person's weekly income is £14, and his quarterly expenditure is £128. 10*s*.; how much will he have saved at the end of 8 years? (supposing a year to consist of 52 weeks).

(7) An equal number of guineas, pounds, half-guineas, crowns, and half-crowns amount to £398. 5*s*.: how many of each sort are there?

(8) What quantity of water must I add to a pipe of wine, which cost £90, to reduce its price to 10*s*. a gallon?

II.

(1) Explain the meaning of 'Compound Division': what different cases are there of it? If £1844. 2*s*. 8½*d*. be divided equally among 49 persons, how much will each receive?

(2) A house and its furniture are worth £6734. 5*s*. 9*d*.; but the house is worth 8 times as much as the furniture; what is the house worth?

(3) Define 'a square', 'a cube'; shew clearly by a figure how many cubic feet there are in a cubic yard. Reduce 4203239040 cub. in. to cub. yds.; and find how how many grains of wheat there are in a load, if a pint contains 7000 grains.

(4) Divide £3. 13*s*. 9*d*. between two persons, so that one shall receive half as much again as the other.

(5) A jeweller sold jewels to the value of 834 guineas, for which he received in part 1429 dollars, worth 4*s*. 6*d*. each; what sum remained unpaid?

(6) The tax on a certain property amounts to £974. 16*s*. 3½*d*. at the rate of 2*s*. 2¾*d*. in the pound. What is the value of the property?

(7) If I bottle off two-thirds of 2 pipes of wine into quarts, and the rest into pints, how many dozens of each shall I have?

(8) A servant's wages are £10. 8*s*. a year, how much ought he to receive for 7 weeks? (supposing a year to consist of 52 weeks).

III.

(1) What are the different uses to which Troy weight and Avoirdupois weight are respectively applied? Express 56 lbs. Avoirdupois in lbs., &c., Troy.

(2) A factor bought 56 pieces of stuff for £1569. 17*s.* 4*d.* at 4*s.* 10*d.* a yard: how many yards were there in each piece?

(3) How many farthings are there in 5 half-sovereigns, 5 half-crowns, 5 sixpences, and 5 half-pence?

(4) Goods are bought at 6½*d.* per lb., and the cost of carriage is 1¼*d.* per lb.; they are sold at £4. 10*s.* per cwt.: what is the gain or loss per cwt.?

(5) What is meant by a 'mean solar day'? How does the 'solar' year differ from the 'civil' year? State clearly the methods which have been adopted to correct the error arising therefrom.

(6) A gentleman laid up in the year 1851 £294. 1*s.* 6*d.*, having spent daily £1. 12*s.* 6*d.*: what was his income in that year?

(7) Divide 198 guineas among 4 persons, so that the second may have twice as much as the first, the third 3 times as much as the second, and the fourth 4 times as much as the third.

(8) A person with £5. 7 florins, 9 cents, and 1 mil. in his pocket, goes to the sea-side for 2 days: he spends in Railway fare 6 fl. 2 c. 5 m.; in cab fare 1 fl. 2 c. 5 m.; and his Hotel bill is 13 fl. 5 c. What sum does he return home with?

IV.

(1) What are the standards of weight and capacity in England, and how are they fixed?

(2) The ages of two persons amount to 51 years; the difference is 23 years: what are their respective ages?

(3) How many Rubles at 3*s.* 4½*d.* each are equal in value to 378 Napoleons, at 15*s.* 9¾*d.* to the Napoleon?

(4) A hundred sovereigns all equally light, are worth ninety-five pounds; what is the value of each in shillings?

(5) Find

1. The sum of £27. 3 c. 9 m.; £560. 2½ fl.; £30. 3 c. 7 m.
2. The quotient of £405. 5 fl. 3 c. 6 m. by 16.

(6) A person lays out £43. 9*s.* 4*d.* in spirits at 5*s.* 4*d.* a gallon; some of which leaked out in the carriage; however, he sold the remainder for £54, at the rate of 7*s.* 6*d.* a gallon: how many gallons leaked out?

(7) If a piece of ground contain 24 acres, and an inclosure of 17

acres, 3 roods be taken out of it, how many perches are there in the remainder?

(8) How many hours have elapsed since the birth of Christ to the year 1852, supposing each year to consist of 365 days, 6 hours?

V.

(1) Explain how the statute defines 'a yard,' with reference to a natural standard of length. Find the corresponding linear unit, when an acre is one hundred thousand square units.

(2) How many barley-corns will reach round the earth, supposing the circumference of it to be 25000 miles?

(3) If a single article cost 3*s.* 7*d.*, how many dozens can be bought for £86. 10*s.*?

(4) A bankrupt owes £3549, and can pay 17*s.* 6*d.* in the pound. What are his effects worth, and what loss do his creditors sustain?

(5) A piece of money is worth 16*s.* 3*d.*; how many guineas are there in 253 such pieces?

(6) How many times will a pendulum vibrate in 24 hours, which vibrates 5 times in 2 seconds?

(7) If the sum paid for 247 gallons of spirit amount, together with the duty, to £619. 11*s.* 2*d.*; and the duty on each gallon be $\frac{1}{7}$th part of its original cost; what is the duty per gallon?

(8) 12 persons on a journey each spend £23. 4 c. 6 m. in board and lodging; 6 of them agree to pay the travelling expences, the share of each amounting to £18. 1 m. Find the amount of expenditure during the journey.

VI.

(1) What is the meaning of the word 'Carat' as applied to gold, and as applied to diamonds? How many 'carats' fine is standard gold? If from 2793461 lbs. Troy of gold there be coined £130524465. 4*s.* 6*d.*, find the value of each lb.

(2) A wheel makes 514 revolutions in passing over 1 mile, 467 yards, 1 foot: what is its circumference?

(3) How much must I pay for 455 Napoleons, a Napoleon being worth 16*s.* 4½*d.*?

(4) A grocer buys a hogshead of sugar, containing half a ton, for £30, and retails it at 7½*d.* per lb.; how much money does he make?

(5) A merchant buys 10 gallons of spirit at 12*s.* a gallon: 15 gallons at 14*s.* 6*d.* a gallon; and 18 gallons at 15*s.* 9*d.* a gallon: what will be the price of a gallon of the mixture, so that he may gain £2. 5*s.* 6*d.* on his outlay?

(6) A gentleman distributed £41. 5*s.* among 12 men, 16 women, and 30 children; to every man he gave twice as much as to a woman, and to every woman three times as much as to a child: what did each receive?

(7) A chain, 11 yards long, is divided into 50 equal parts, called links; find how many square links there are in an acre.

(8) A merchant expends £1636. 5*s.* on equal quantities of wheat at £2. 2*s.* a quarter, barley at £1. 1*s.* a quarter, and oats at 14*s.* a quarter: what quantity of each will he have?

VII.

(1) How many minutes are there in the 10 years, of which the first is 1852?

(2) Divide 425 tons, 15 cwt., 2 qrs., 12 lbs., by 27: and 1361 m., 4 fur., 28 po., by 28: and find how many moidores are equivalent to 198 guineas?

(3) Two boys run a race of 1 mile, one of them gains 5 feet in every 110 yards; how far will the other be left behind at the end of the race?

(4) Light travels at the rate of 192000 miles a second: how many days will it be in coming to us from the star α Centauri, supposed to be 20 billions of miles distant?

(5) Divide £100. 2*s.* 6*d.* equally among 45 people; supposing 20 of them to have received their portions, and 10 of the remaining 25 to have given up their portions to the other 15, how much would each of the 15 receive?

(6) A father left his eldest son £5000 more than he left his second son, and the second son 1500 guineas more than the third; to the third he left 12000 guineas: what was the eldest son's portion; and what sum did the father leave to his 3 sons?

(7) A person buys 128 gallons of wine at 8*s.* 6*d.* a gallon: how many gallons of water must be added to it, in order that he may gain £5. 12*s.* on his outlay, and retail the wine at 5*s.* a gallon?

(8) A bankrupt has good debts to the amount of £456. 18*s.* 3*d.*; and the following bad debts, £360. 7*s.* 6*d.*, £120. 13*s.*, and £21. 4*s.*, for which he receives respectively 4, 5, and 10 shillings in the £; his own liabilities amount to £4558; how much can he pay in the £?

VIII.

(1) Can you attach any meaning (1) to the multiplication of 6*s.* 8*d.* by £1. 2*s.* 3*d.*, (2) to the division of 1 yard, 2 feet, 3 inches by 6 feet, 8 inches? State reasons for your answer.

(2) A carriage-load is found to weigh 1 ton, 3 cwt., 1 qr., and it consists of 315 equal packages; what is the weight of each?

(3) *A* gives to *B* 98 gallons of brandy worth 25*s.* 6*d.* a gallon, and gets in return 39 guineas and 576 yards of cloth: what is the value of the cloth per yard?

(4) A person counts on the average 7000 shillings in an hour: what sum will he count in 67 days, if he work 9 hours a day?

(5) A gentleman's average daily expenditure for the year 1852 is £2. 0*s.* 1½*d.*; and this allows him to lay by £50 at the end of the year: what is his income?

(6) Shew how to perform the following operations: (1) the addition of £896. 5 fl. 4 c. 7 m.; £391. 5 fl. 3 c. 8 m.; £23. 9 c. 6 m.: (2) the subtraction of the second sum from the first; and (3) the multiplication of the third by 248; reading off each result.

(7) A grazier left to his 5 children in equal portions 175 oxen, 2003 sheep, 563 pigs, and 87 fowls: what was the value of each of their fortunes, supposing the oxen to be worth 11 guineas each, the sheep a guinea and a half each, the pigs half-a-guinea each, and the fowls 9*d.* each?

(8) I hire a house at £90 a-year; which is assessed in the rate-book at ⅘ths of its rent; I agree to pay the rates upon it, viz., 3 poor's-rates of 9*d.*, 10*d*, and 1*s.* 2*d.* respectively in the £, a church-rate of 8*d.* in the £, and a paving-rate of 1*s.* 7*d.* in the £: what is the whole annual cost of the house?

IX.

(1) Explain the calendar as now in use. On June 21 of 1851 the Duke of Wellington had lived 30,000 days. Find the day and year of his birth.

(2) The fore-wheel of a carriage is 10 feet in circumference, and the hind-wheel is 16 feet: how many revolutions will one make more than the other in 100 miles?

(3) A loaded truck weighs 4 tons, 3 qrs., 1 lb.; the truck itself weighs a ton and a half, and it contains 758 equal packages: find the weight of each package.

(4) *A* has 35 ponies, each worth 15 guineas, and *B* has 24 horses, each worth £24. 15*s.*: should they exchange, which of them ought to give money also, and how much?

(5) Sound travels at the rate of 1142 feet a second: if a gun be discharged at the distance of 4½ miles, how long will it be, after seeing the flash, before I hear the report?

(6) How many times will a clock, which chimes the quarters, strike and chime in 1854?

(7) How long will a person be in walking from Cambridge to Ely, a distance of 16 miles, when he takes 110 steps of $2\frac{1}{2}$ feet every minute?

(8) A manufacturer employs 60 men and 45 boys, who respectively work 10 and 14 hours per day during 5 days of the week, and half the time on the remaining day; each man receives 6*d.* per hour, and each boy 2*d.* per hour: what is the amount of wages paid in the year? (a year = 52 weeks).

X.

(1) What will be the expence of forming a railway 146 miles in length, at 3 guineas a yard?

(2) A gentleman's income is 2000 guineas; he spends £18 per week upon personal expences, and his annual subscription to charities amounts to £150: what will be the state of his finances at the end of 8 years? (reckoning 52 weeks to the year).

(3) Find the value of 12 lbs., 8 oz. of copper coin, having given that 12 penny pieces weigh 8 oz.

(4) What is the price of 7 packages of cloth, each package containing 7 parcels, each parcel 27 pieces, and each piece 81 yards, at the rate of $1\frac{1}{2}$ guineas for 3 yards?

(5) A mixture is made of 6 gallons of spirits at 6 fl. 2 c. 5 m. per gallon, 4 gallons at 9 fl. per gallon, and 10 gallons at £1. 1 fl. 1 c. 5 m. per gallon; find the price of a gallon of the mixture.

(6) If 5000 people took in hand to count a billion of sovereigns, and beginning their work at the commencement of the year 1852, could each count on the average 100 sovereigns a minute (without intermission), when would they finish their task?

(7) I have a bank-note of £20, a note-of-hand for £6. 10*s.* and in several coins, as follows; in copper, 13 farthings, and 45 half-pence; in silver, 36 three-pences, 58 groats, 96 sixpences, 67 shillings, 97 half-crowns, and 126 crowns; in gold, 65 half-guineas, 77 guineas, and 34 moidores: how much have I altogether?

(8) In a manufactory there are employed 5 foremen, each at 4*s.* 6*d.* a-day, 63 workmen, each at 2*s.* 9*d.* a-day, 75 boys, each at 1*s.* 8*d.* a-day, and 47 girls, each at 1*s.* $4\frac{1}{2}$*d.* a-day; they work 6 days in the week: how much will their master have to pay in wages per week, and how much per year? (a year = 52 weeks).

XI.

(1) Divide £6842. 14*s.* 5*d.* among 3 persons, so that the first shall

have £568. 14*s*. 4*d*. more than the second, and the second £728. 18*s*. 2*d*. more than the third.

(2) If a person spend £152. 10*s*. a-week; what must be his daily income that in 15 years he may lay by £7522. 10*s*.? (a year = 52 weeks).

(3) Find how often 3 cwt., 2 qrs., 27 lbs., 15 oz. is contained in 4 tons, 13 cwt., 2 qrs., 27 lbs., 7 oz.; and verify the result.

(4) A person bought 4 bales of cloth, each bale consisting of 6 pieces, and each piece of 27 yards, at £16. 4*s*. per bale; what was the price of the whole, and what the rate per yard?

(5) Supposing 5000 persons, and 1500 carriages to pass over Waterloo Bridge, daily, during the present year, the former paying a toll of a half-penny each, the latter a toll of 2*d*. each; what will be the amount of toll raised at the year's end?

(6) If a person spend 200 guineas during the first six calendar months of the year 1853, what is his average daily expenditure?

(7) What quantity of tea at 4*s*. 5¾*d*. per lb., must be given in exchange for 5 cwt., 3 qrs. of sugar, at 7*s*. 10½*d*. per stone?

(8) A father left 5 sons; and his property consisted of £500 in cash, and 5 bills of £48. 10*s*. 6*d*. each. He ordered £20 to be bestowed on his burial, and his debts, amounting to £164, to be paid: then the residue of the property to be thus divided, viz., one-third part to go to the eldest son, and the remainder to the other four sons in equal portions: what was the share of each son?

XII.

(1) A gentleman sent a tankard to his silversmith, which weighed 100 oz., 16 dwts., and ordered him to make it into spoons, each weighing 2 oz., 16 dwts.: how many spoons did he receive?

(2) A gentleman's estate, for the 5 years ending with 1849, yielded £1227. 15*s*.: how much could he spend one day with another, so as to lay by 135 guineas?

(3) The length of a year being 365¼ days, and that of a lunar month being 29½ days, how many lunar months are there in 19 years?

(4) What is the value of a talent of silver, if silver be worth 5*s*. per oz., and a talent consists of 1000 shekels, each weighing 219 grains?

(5) Divide £17. 3*s*. 5*d*. by £14. 3*s*. 6*d*. to 4 places of decimals. Can these sums be multiplied together?

(6) A merchant bought 7 pieces of cloth, each 27 yards, for £55. 12*s*.; and sold 56 yards at 5*s*. 3½*d*. per yard; at what must he sell the remainder per yard in order to gain £3. 11*s*. on the whole?

(7) A certain number of men, twice as many women, and three times as many boys earned in 5 days £7. 15*s.*; each man earned 1*s.* 6*d.*, each woman 10*d.*, and each boy 8*d.* a-day. How many were there of each?

(8) A bankrupt owes his creditors £2963, and pays them 6*s.* $8\frac{1}{4}$*d.* in the pound. How much does he pay them altogether?

REDUCTION OF FRACTIONS.

132. *To find the value of a fractional part of a number of one denomination in terms of the same or lower denominations.*

RULE. Multiply the given number by the numerator of the fraction, and divide the product (if possible) by the denominator; if there be a remainder, multiply the numerator of the fraction which remains by the number of units connecting the given denomination with the next lower denomination, and divide the product by the denominator; if there still be a remainder, proceed with it in the same way as with the last remainder, and so on, till you come to the lowest denomination. The compound number formed of the integral parts reserved from the successive quotients, and of the result of the last reduction, will be the value required.

Note. If the given number comprise different denominations of the same kind: reduce the different denominations to the lowest denomination involved, and the above rule may be then applied; or the value may be found by the method shewn in Art. (128).

Ex. 1. Find the value of $\frac{7}{8}$ of £1.

Proceeding by the Rule given above,

$$\frac{7}{8} \text{ of } £1. = \frac{7 \times 20}{8} s. = \frac{7 \times 5}{2} s.$$

$$= \frac{35}{2} s. = 17\tfrac{1}{2} s.,$$

$$\text{and } \frac{1}{2} \text{ of } 1s. = \frac{1 \times 12}{2} d. = 6d.;$$

therefore the value required = 17*s.* 6*d.*

Reason for the above process.

$$\frac{7}{8} \text{ of } £1. \text{ is the same as } 7 \text{ times } \frac{1}{8} \text{ of } £1.,$$

$$\text{and } \frac{1}{8} \text{ of } £1. = \frac{20s.}{8} = \frac{5s.}{2};$$

$$\text{therefore 7 times } \frac{1}{8} \text{ of £1.} = 7 \text{ times } \frac{5s.}{2} = \frac{35s.}{2} = 17\tfrac{1}{2}s.$$
$$= 17s.\ 6d.$$

Ex. 2. Find the value of $\frac{2}{5}$ of 13*s*. 4*d*.

$$\frac{2}{5} \text{ of } 13s.\ 4d. = \frac{2}{5} \text{ of } 160d. = \frac{2 \times 160}{5} d.,$$
$$= (2 \times 32)d.,$$
$$= 64d.,$$
$$= 5s.\ 4d.;$$

or thus, $$\frac{2}{5} \text{ of } 13s.\ 4d. = \frac{(13s.\ 4d.) \times 2}{5} = \frac{£1.\ 6s.\ 8d.}{5}$$
$$= 5s.\ 4d.$$

Ex. 3. Find the value of $\frac{3}{8}$ of £5. 13*s*. 9*d*.

$$\frac{3}{8} \text{ of } £5.\ 13s.\ 9d. = \frac{(£5.\ 13s.\ 9d.) \times 3}{8},$$
$$= \frac{£17.\ 1s.\ 3d.}{8},$$
$$= £2.\ 2s.\ 7\tfrac{7}{8}d.$$

Ex. 4. Find the value of $\frac{3}{4}$ of a guinea + $\frac{3}{8}$ of a crown + $\frac{3}{5}$ of 7*s*. 6*d*. $- \frac{3}{4} 2d.$

$$\frac{3}{4} \text{ of a guinea} = \frac{3 \times 21}{4} s. = \frac{63}{4} s. = 15\tfrac{3}{4}s.,$$
$$\text{and } \frac{3}{4}s. = \frac{3 \times 12}{4} d. = (3 \times 3)d. = 9d.;$$

therefore $\frac{3}{4}$ of a guinea = 15*s*. 9*d*.

$$\frac{3}{8} \text{ of a crown} = \frac{3 \times 5}{8} s. = \frac{15}{8} s. = 1\tfrac{7}{8}s.,$$
$$\text{and } \frac{7}{8}s. = \frac{7 \times 12}{8} d. = \frac{7 \times 3}{2} d. = \frac{21}{2} d. = 10\tfrac{1}{2}d.;$$

therefore $\frac{3}{8}$ of a crown = 1*s*. 10½*d*.

$$\frac{3}{5} \text{ of } 7s.\ 6d. = \frac{3 \times 90}{5} d. = (3 \times 18)\,d. = 54d. = 4s.\ 6d.,$$
$$\frac{3}{4} \text{ of } 2d. = \frac{3 \times 2}{4} d. = \frac{3}{2} d. = 1\tfrac{1}{2}d.;$$

$$\text{therefore value required} = 15s.\ 9d. + 1s.\ 10\tfrac{1}{2}d. + 4s.\ 6d. - 1\tfrac{1}{2}d.$$
$$= £1.\ 2s.\ 1\tfrac{1}{2}d. - 1\tfrac{1}{2}d.$$
$$= £1.\ 2s.$$

Ex. 5. Find the value of $\frac{2}{7}$ of £15. + $3\frac{3}{7}$ of £1. + $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1. + $\frac{2}{3}$ of $\frac{3}{7}$ of 1*s*.

$$\frac{2}{7}\text{ of }£15.=£\frac{2\times 15}{7}=£\frac{30}{7}=£4\tfrac{2}{7},$$

$$£\frac{2}{7}=\frac{2\times 20}{7}\,s.=\frac{40}{7}\,s.=5\tfrac{5}{7}s.,$$

$$\frac{5}{7}s.=\frac{5\times 12}{7}\,d.=\frac{60}{7}\,d.=8\tfrac{4}{7}d.\,;$$

$$\text{therefore }\tfrac{2}{7}\text{ of }£15.=£4.\ 5s.\ 8\tfrac{4}{7}d.$$

$$£3\tfrac{3}{7}.=£3.+£\tfrac{3}{7}.$$

$$£\frac{3}{7}=\frac{3\times 20}{7}\,s.=\frac{60}{7}\,s.=8\tfrac{4}{7}s.,$$

$$\frac{4}{7}s.=\frac{4\times 12}{7}\,d.=\frac{48}{7}\,d.=6\tfrac{6}{7}d.\,;$$

$$\text{therefore }£3\tfrac{3}{7}.=£3.\ 8s.\ 6\tfrac{6}{7}d.$$

$$\begin{aligned}\tfrac{1}{3}\text{ of }\tfrac{5}{7}\text{ of }\tfrac{3}{5}\text{ of }£1.&=\tfrac{1}{7}\text{ of }£1.\\&=\frac{20}{7}\,s.\\&=2\tfrac{6}{7}s.,\end{aligned}$$

$$\frac{6}{7}s.=\frac{6\times 12}{7}\,d.=\frac{72}{7}\,d.=10\tfrac{2}{7}d.\,;$$

$$\text{therefore }\tfrac{1}{3}\text{ of }\tfrac{5}{7}\text{ of }\tfrac{3}{5}\text{ of }£1.=2s.\ 10\tfrac{2}{7}d.$$

$$\begin{aligned}\frac{2}{3}\text{ of }\frac{3}{7}\text{ of }1s.=\frac{2}{7}\text{ of }1s.&=\frac{2\times 12}{7}\,d.\\&=\frac{24}{7}\,d.=3\tfrac{3}{7}d.\,;\end{aligned}$$

$$\begin{aligned}\text{therefore required value}&=£4.\ 5s.\ 8\tfrac{4}{7}d.+£3.\ 8s.\ 6\tfrac{6}{7}d.+2s.\ 10\tfrac{2}{7}d.+3\tfrac{3}{7}d.\\&=£7.\ 17s.\ 5\tfrac{1}{7}d.\end{aligned}$$

Ex. 6. Find the value of $\frac{5}{9}$ of a bushel $-\ \frac{5}{7}$ of a peck.

$$\frac{5}{9}\text{ of a bus.}=\frac{5\times 4}{9}\text{ pks.}=\frac{20}{9}\text{ pks.}=2\tfrac{2}{9}\text{ pks.},$$

$$\frac{2}{9}\text{ pk.}=\frac{2\times 8}{9}\text{ qts.}=\frac{16}{9}\text{ qts.}=1\tfrac{7}{9}\text{ qts.}\,;$$

$$\text{therefore }\frac{5}{9}\text{ of a bus.}=2\text{ pks., }1\tfrac{7}{9}\text{ qts.},$$

$$\frac{5}{7}\text{ of a pk.}=\frac{5\times 8}{7}\text{ qts.}=\frac{40}{7}\text{ qts.}=5\tfrac{5}{7}\text{ qts.}\,;$$

$$\begin{aligned}\text{therefore required value}&=2\text{ pks., }1\tfrac{7}{9}\text{ qts.}-5\tfrac{5}{7}\text{ qts.}\\&=1\text{ pk., }4\tfrac{4}{63}\text{ qts.}\end{aligned}$$

Ex. XLVIII.

(1) Find the respective values of,

1. $\frac{3}{8}$ of £1; $\frac{2}{3}$ of £1; $\frac{5}{8}$ of £1; $\frac{3}{4}$ of a guinea; $\frac{5}{6}$ of a guinea.
2. $\frac{3}{4}$ of £1. 10*s.*; $\frac{2}{3}$ of £2; $\frac{3}{8}$ of half-a-crown; $\frac{3}{5}$ of 13*s.* 4*d.*
3. $\frac{13}{240}$ of £1; $\frac{5}{16}$ of 1*s.*; $\frac{3}{5}$ of 6*s.* 8*d.*; $\frac{2}{9}$ of 1*s.* 6*d.*; $\frac{3}{7}$ of 3*s.* 6*d.*
4. $2\frac{3}{7}$ of 7*s.* 6*d.*; $\frac{5}{7}$ of £2. 3*s.* 9*d.*; $\frac{8}{21}$ of a moidore; $\frac{2}{3}$ of £135. 16*s.* $10\frac{1}{2}$*d.*
5. $\frac{2}{5}$ of 4*s.* 7*d.*; $1\frac{3}{5}$ of £1. 2*s.* 9*d.*; $\frac{5}{14} \times \frac{11}{18}$ of 21*s.*; $\frac{1}{3}$ of $\frac{3}{5}$ of 9*s.* $10\frac{1}{2}$*d.*
6. $3\frac{1}{12}$ of 2*s.* 6*d.*; $\frac{9}{11}$ of £4. 14*s.* 5*d.*; $\frac{2}{3}$ of $\frac{4}{21}$ of 10*s.* 6*d.*; $\frac{1}{50}$ of 100 guineas.
7. $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{20}{21}$ of 5 guineas; $\frac{2}{9}$ of £16. 16*s.* $3\frac{1}{4}$*d.*; $\frac{1}{1000}$ of £441. 12*s.* 6*d.*
8. $\frac{5}{16}$ of a cwt.; $\frac{3}{4}$ of a lb. Avoird.; $\frac{4}{5}$ of a mile; $\frac{5}{8}$ of an acre.
9. $\frac{5}{11}$ of a mile; $\frac{3}{10}$ of a day; $\frac{2}{3}$ of a yard; $\frac{2}{7}$ of 3 cwt., 1 qr., 14 lb.
10. $7\frac{3}{5}$ of a lb. Avoird.; $1\frac{3}{4}$ of a lb. Troy; $2\frac{2}{5}$ of a gal.; $4\frac{5}{19}$ of an acre.
11. $3\frac{5}{12}$ of a hhd. of beer; $2\frac{3}{8}$ of a tun of wine; $6\frac{26}{29}$ of a bus.
12. $2\frac{1}{6}$ of a load; $3\frac{109}{570}$ of a cub. yd.; $9\frac{5}{12}$ guineas.
13. $\frac{7}{8}$ of $\frac{3}{5}$ of $10\frac{2}{3}$ hrs.; £$\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$; $\dfrac{7\frac{1}{8}}{8\frac{1}{7}}$ of $\dfrac{5\frac{1}{7}}{7\frac{1}{5}}$ of a moidore.
14. $\dfrac{\frac{4}{49}}{\frac{5}{14}\text{ of }1\frac{1}{3}}$ of £16. 8*s.* $1\frac{1}{2}$*d.*; $\frac{3}{7}$ of $1\frac{2}{5}$ of $12\frac{1}{2}$ of $\frac{3}{44}$ of £2 $\times \frac{3}{44}$.
15. $\frac{5}{7}$ of £1 $\times 5\frac{3}{5}$; $\frac{3}{8}$ of $\frac{2}{9}$ of £1 $\div \frac{1}{4}$.
16. $19\frac{3}{4}$ of £5. 1*s.* $1\frac{1}{4}$*d.* $\frac{45}{79}$*q.*; $2\frac{2}{5}$ of £8. 14*s.* $2\frac{7}{12}$*d.* $\div 8\frac{4}{5}$.

(2) Find the values of

1. $\frac{3}{5}$ of £1 + $\frac{2}{7}$ of a guinea + 3*s.* 2*d.*
2. $\frac{3}{8}$ of £1 + $\frac{3}{8}$ of 2*s.* 6*d.* + $\frac{3}{8}$ of 1*s.*
3. $\frac{1}{7}$ of £1 + $\frac{2}{9}$ of 1*s.* + $\frac{5}{12}$*d.*
4. $\frac{5}{23}$ of $\frac{11}{18}$ of 10*s.* 6*d.* + $\frac{5}{8}$*s.* + $\frac{1}{12}$ of 2*s.*6*d.*
5. $\frac{5}{8}$ of £1 − $\frac{5}{8}$ of 1*s.* + $\frac{5}{8}$ of a guinea − $\frac{5}{8}$ of a moidore.
6. £$3\frac{5}{8}$ + $7\frac{4}{8}$*s.* + $4\frac{3}{4}$*d.*
7. $\frac{2}{3}$ of £1 − $\frac{5}{6}$ of 2*s.* 6*d.* + $\frac{2}{9}$ of 1*s.*
8. $\frac{2}{3}$ of 10*s.* 6*d.* + $\frac{7}{8}$ of 27*s.* − $\frac{6}{11}$ of 6*s.* 8*d.*
9. $\frac{7}{64}$ of £1. 12*s.* + $\frac{2}{15}$ of £3. 5*s.* + $\frac{7}{11}$ of $1\frac{1}{2}$ guineas.
10. $\frac{3}{4}$ of $\frac{5}{8}$ of £1 + $\frac{2}{3}$ of $\frac{5}{9}$ of 2*s.* 6*d.* + $\frac{3}{4}$ of $10\frac{1}{2}$*d.*
11. $\frac{2}{9}$ of 21*s.* + $\frac{1}{2}$ of $\frac{3}{4}$ of £1 − $\frac{1}{6}$ of $\frac{3}{4}$ of 5*s.* + $\frac{1}{3}$ of $\frac{5}{8}$ of 1*s.*
12. $\frac{3}{7}$ of £15 + $\frac{5}{8}$ of $\dfrac{1}{2\frac{4}{5}}$ of £1. 12*s.* + $\frac{4}{7}$ of 3*d.*

13. $\dfrac{2}{3\frac{1}{2}}$ of $\dfrac{4}{\frac{5}{7}-\frac{1}{9}}$ of 2 guineas + $\dfrac{1\frac{7}{8}+\frac{3}{4}}{3\frac{4}{5}-\frac{2}{3}}$ of £5.

14. $\frac{3}{5}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{2}{3}$ lb.

15. $\frac{3}{8}$ lb. Troy + $\frac{5}{6}$ lb. Troy − $\frac{8}{9}$ oz. Troy.

16. $\frac{7}{10}$ of a mile − $\frac{5}{8}$ of a fur. + $\frac{4}{11}$ po.

17. $\frac{9}{111}$ cub. yds. + $2\frac{7}{8}$ cub. ft.

18. $\frac{2}{3}$ of a qr. + $\frac{3}{4}$ of a bus. − $\frac{1}{6}$ of a qr.

19. $\frac{3}{5}$ of 7 fur., 29 po., $3\frac{1}{4}$ yds. + $\frac{5}{7}$ of 5 m., 3 fur., 37 po., $4\frac{1}{5}$ yds.

20. $7\frac{2}{5}$ of $365\frac{1}{4}$ d. + $3\frac{9}{10}$ of $\frac{5}{6}$ wks. + $\frac{3}{4}$ of $5\frac{5}{9}$ hrs.

21. $\frac{1}{11}$ of 91 ac., 3 ro., 36 po., $2\frac{3}{4}$ yds. − $\frac{3}{5}$ of 6 ac., 2 ro., 17 po., $25\frac{1}{3}$ yds.

133. *To reduce a number, or a fraction, of any denomination, to a fraction of another denomination.*

Rule. "Reduce the given number, or fraction, and also the number or fraction to the fraction of which it is to be reduced, to their respective equivalent values in terms of some one and the same denomination: then the fraction of which the former is made the numerator, and the latter the denominator, will be the fraction required."

Ex. 1. Reduce 3*s*. 5*d*. to the fraction of £1.

Proceeding by the above Rule,

$$3s.\ 5d. = 41 \text{ pence},$$
$$£1 = 240 \text{ pence};$$

therefore fraction required $= \frac{41}{240}$.

Reason for the above process.

£1, or unity, is here divided into 240 equal parts; and 41 of such parts being taken, the part of unity, or £1, which they make up, is represented by $\frac{41}{240}$.

Ex. 2. Reduce $\frac{5}{8}$ of £1 to the fraction of 27*s*.

$$\tfrac{5}{8} \text{ of } £1 = 20 \text{ times } \tfrac{5}{8} \text{ of } 1s.$$
$$= \frac{5 \times 20}{8} s.$$
$$= \frac{5 \times 5}{2} s.$$
$$27s. = 27s.$$

therefore fraction required $= \dfrac{\frac{5 \times 5}{2}}{27}$,

$$= \frac{5 \times 5}{2} \times \frac{1}{27} = \frac{25}{54}.$$

For 27*s.* is divided into 27 equal parts; and $\frac{5}{8}$ of £1 is divided into $\frac{25}{2}$ of such parts; therefore the part of unity, or 27*s.*, which the latter represents, is $\frac{\frac{25}{2}}{27} = \frac{25}{54}$.

Ex. 3. Express $\frac{2}{7}$ of £1 as the fraction of a farthing.

$$\tfrac{2}{7} \text{ of } £1 = (\tfrac{2}{7} \times 20 \times 12 \times 4) \text{ farthings},$$
$$= \tfrac{1920}{7} \text{ farthings};$$
$$1 \text{ farthing} = 1 \text{ farthing};$$
$$\text{therefore fraction required} = \frac{\frac{1920}{7}}{1}.$$
$$= \tfrac{1920}{7}.$$

For the unit, or farthing, is divided into 1 part, and £$\frac{2}{7}$ contains $\frac{1920}{7}$ of such parts.

Therefore the fraction of unity, or 1 farthing, which $\frac{2}{7}$ of £1 represents, is $\frac{\frac{1920}{7}}{1} = \frac{1920}{7}$.

Ex. 4. What part of $\frac{1}{3}$ of a ton is $2\frac{2}{3}$ of $1\frac{1}{3}$ of $\frac{1}{7}$ of a cwt.?

$$2\tfrac{2}{3} \text{ of } 1\tfrac{1}{3} \text{ of } \tfrac{1}{7} \text{ of a cwt.} = \tfrac{8}{3} \text{ of } \tfrac{4}{3} \text{ of } \tfrac{1}{7} \text{ of a cwt.}$$
$$= \tfrac{8}{3} \times \tfrac{4}{3} \times \tfrac{1}{7} \text{ cwt.}$$
$$\tfrac{1}{3} \text{ of a ton} = \tfrac{20}{3} \text{ cwt.}$$
$$\text{Therefore fraction required} = \frac{\frac{8}{3} \times \frac{4}{3} \times \frac{1}{7}}{\frac{20}{3}}$$
$$= \tfrac{8}{3} \times \tfrac{4}{3} \times \tfrac{1}{7} \times \tfrac{3}{20}$$
$$= \tfrac{8}{105}.$$

Ex. XLIX.

(1) Reduce

1. 6*s.* 8*d.* to the fraction of £1.; and 3*s.* $1\frac{1}{2}$*d.* to the fraction of 1 guinea.
2. 5*d.* to the fraction of 1*s.*; and 3*s.* $4\frac{1}{2}$*d.* to the fraction of £1.
3. $7\frac{1}{2}$*d.* to the fraction of 27*s.*; and 15 sixpences to the fraction of 13*s.* 4*d.*
4. £1. 3*s.* 4*d.* to the fraction of £9. 6*s.* 8*d.*; and 2*s.* $0\frac{1}{4}$*d.* to the fraction of 10*s.* 6*d.*
5. £4. 17*s.* 6*d.* to the fraction of £5.; and 16*s.* to the fraction of £200.
6. £18. 7*s.* 6*d.* to the fraction of £2.; and 6*s.* $7\frac{6}{7}$*d.* to the fraction of 7*s.* 9*d.*

7. 1*s*. 2*d*. to the fraction of a moidore; and 3*s*. 4*d*. to the fraction of a half-guinea.
8. 3 qrs., 19 lbs. to the fraction of a ton; and $61\frac{7}{8}$ lbs. to the fraction of 4 oz.
9. 3 qrs., 4 lbs. to the fraction of 2 cwt.; and 5 oz., $2\frac{2}{3}$ drs. to the fraction of a grain.
10. 3 ro., $27\frac{1}{2}$ po. to the fraction of an acre; and $26\frac{2}{5}$ sq. yds. to the fraction of 2 acres.
11. 126 yds., 2 ft., 6 in. to the fraction of a mile; and 6 cub. ft., 100 cub. in. to the fraction of a cubic yard.
12. 2 qrs., $2\frac{2}{3}$ na. to to the fraction of an Eng. ell; and 8 h., 3 m. to the fraction of a day.
13. 1 stone, 8 lbs. to the fraction of a ton; and 1 sc., 13 grs. to the fraction of a lb.
14. 2 ac., 1 ro. to the fraction of 9 ac., 2 ro.; and 1540 yds., 2 ft., 9 in. to the fraction of 2 miles.
15. 1 ft., $\frac{7}{8}$ in. to the fraction of a sq. yd.; and 2 qts., $1\frac{1}{2}$ pt. to the fraction of a barrel.
16. 2 wks., 5 days, 7 h., 27 m. to the fraction of a day; and 1 ro., 20 po. to the fraction of an acre.
17. 4 bush., $2\frac{2}{9}$ qts. to the fraction of a load; and 3 quires, 7 sheets to the fraction of a ream.
18. $2\frac{1}{2}$ guineas to the fraction of £$2\frac{1}{2}$.; and $2\frac{1}{2}$ cwt. to the fraction of 2 tons, 12 lbs.
19. $10\frac{5}{6}$ months to the fraction of 13 months; and $100\frac{1}{2}$ guineas to the fraction of a groat.
20. 6 ft., $3\frac{5}{8}$ in. to the fraction of 13 ft., $8\frac{9}{10}$ in.; and $1\frac{1}{2}$ yds. to the fraction of $1\frac{1}{2}$ in.

(2) Reduce

1. $\frac{3}{4}$ of a crown to the fraction of £1.; and $\frac{4}{5}$ of a farthing to the fraction of 1*s*.
2. $\frac{5}{7}$ of 1*s*. to the fraction of a guinea; and $\frac{1}{8}$ of 7*s*. to the fraction of a crown.
3. $\frac{3}{5}$ of a guinea to the fraction of £1.; and $\frac{7}{9}$ of 27*s*. to the fraction of 2*s*. 6*d*.
4. $\frac{4}{5}$ of a half-guinea to the fraction of £1.; and $\frac{25}{12}$ of 1*s*. to the fraction of 2*s*. 6*d*.
5. $\frac{1}{8}$ of £74. 13s. 4*d*. to the fraction of £28.; and $\frac{1}{9}$ of a moidore to the fraction of $3\frac{1}{2}$ guineas.

6. $\frac{3}{5}$ of a dwt. to the fraction of 1 lb.; and $\frac{5}{6}$ of 2 lbs. to the fraction of $2\frac{1}{2}$ tons.
7. $\frac{3}{8}$ of a lb. to the fraction of a cwt.; and $\frac{5}{6}$ of a yd. to the fraction of a mile.
8. $\frac{1}{300}$ of £1. to the fraction of a penny; and $\frac{3}{9680}$ of a mile to the fraction of a yard.
9. $\frac{2}{5}$ of $\frac{4}{7}$ of half-a-guinea to the fraction of 2*s*. 6*d*.; and 1 oz. Troy to the fraction of 1 oz. Avoirdupois.
10. $\frac{3}{7}$ of a pole to the fraction of a league; and $3\frac{7}{8}$ furlongs to the fraction of $2\frac{7}{8}$ miles.
11. $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards to the fraction of a furlong; and $\frac{1}{7}$ of $\frac{2}{11}$ of a guinea to the fraction of 2*s*. 6*d*.
12. $\frac{7}{9}$ of 16*s*. $0\frac{1}{2}$*d*. to the fraction of 17*s*. 6*d*.; and $\frac{7}{316}$ of a lb. Troy to the fraction of a pennyweight.
13. $\frac{7}{8}$ of a lb. Avoird. to the fraction of 2 lbs. Troy; and $\frac{7}{9}$ of 2*s*. 6*d*. to the fraction of $1\frac{1}{2}$ guineas.
14. $\frac{5}{9}$ of a French ell to the fraction of a yd.; and $\frac{3}{8}$ of a crown to the fraction of $\frac{1}{3}$ of 7*s*. 6*d*.
15. $\frac{16}{23}$ of a sq. in. to the fraction of a sq. yd.; and $\frac{1}{5}$ of a yd. to the fraction of an English ell.
16. $\frac{21}{15000}$ of a year to the fraction of a day; £$\frac{\cdot036}{1875}$ to the fraction of a farthing.

(3)

1. What part of 7 guineas is $\frac{2}{3}$ of a moidore?
2. What part of £9. is $\frac{1}{2}$ of $\frac{9}{10}$ of half-a-crown?
3. What part of a second is $\frac{1}{100000}$ of a day?
4. What part of $\frac{3}{4}$ of a league is $\frac{7}{9}$ of a mile?
5. What part of $4\frac{1}{2}$ guineas is $5\frac{5}{8}$ of $\frac{4}{15}$ of £7.?
6. What part of 3 weeks, 4 days, is $\frac{1}{2}$ of $5\frac{3}{5}$ sec.?
7. What part of $\frac{1}{3}$ of an acre is $25\frac{9}{11}$ po.?
8. What part of $\frac{1}{30}$ of a min. is $\frac{7}{183}$ of a month of 28 days?
9. What part of $\frac{1}{3}$ of 4 tuns of wine is $3\frac{1}{4}$ hhds?
10. What part of 3 fathoms is $\frac{3}{14}$ of $\frac{7}{3}$ of a pole?

Examples, such as the following, are often given.

Ex. 1. Compare the values of $\frac{1}{21}$ of £1., $\frac{1}{22}$ of a guinea, and $\frac{1}{4}$ of 3*s*. $9\frac{1}{2}$*d*.

$$\frac{1}{21}\text{ £1.} = \frac{20 \times 12}{21}\,d. = \frac{80}{7}\,d.,$$

$$\tfrac{1}{22}\text{ of a guinea} = \frac{21\times 12}{22}\,d. = \tfrac{126}{11}\,d.,$$

$$\tfrac{1}{4}\text{ of }3s.\ 9\tfrac{1}{2}d. = \tfrac{1}{4}\text{ of }45\tfrac{1}{2}d. = \left(\tfrac{1}{4}\times\tfrac{91}{2}\right)d. = \tfrac{91}{8}d.$$

Therefore the equivalent fractions in one and the same denomination (namely, that of pence) are respectively

$$\tfrac{80}{7},\ \tfrac{126}{11},\ \text{and } \tfrac{91}{8}.$$

Least common multiple of the denominators $= 7\times 11\times 8$; therefore the fractions become respectively

$$\frac{80\times 11\times 8}{7\times 11\times 8} = \frac{7040}{616},$$

$$\frac{126\times 7\times 8}{11\times 7\times 8} = \frac{7056}{616},$$

$$\frac{91\times 11\times 7}{8\times 11\times 7} = \frac{7007}{616};$$

therefore $\frac{1}{22}$ of a guinea is the greatest, $\frac{1}{21}$ of £1. is the next, and $\frac{1}{4}$ of 3*s*. 9½*d*. is the least.

Ex. 2. Express £$\frac{9}{10}$ − $\frac{5}{6}$ of a guinea as the fraction of half-a-crown.

$$\pounds\tfrac{9}{10} - \tfrac{5}{6}\text{ of guinea} = \frac{9\times 20}{10}s. - \frac{5\times 21}{6}s.$$

$$= (9\times 2)s. - \frac{5\times 7}{2}s.$$

$$= \frac{36-35}{2}s. = \tfrac{1}{2}s.$$

Half-a-crown $= 2\tfrac{1}{2}s. = \tfrac{5}{2}s.$;

therefore the fraction required $= \dfrac{\frac{1}{2}}{\frac{5}{2}} = \tfrac{1}{5}$.

Ex. 3. Reduce $\dfrac{3\frac{1}{9}}{1\frac{1}{13}}$ of $\{\frac{19}{120}$ of £1. − $\frac{7}{48}$ of 1*s*.$\}$ to the fraction of a moidore.

$$\frac{3\frac{1}{9}}{1\frac{1}{13}}\text{ of }\left\{\frac{19}{120}\text{ of }\pounds1. - \frac{7}{48}\text{ of }1s.\right\}$$

$$= \frac{28}{9}\times\frac{13}{14}\text{ of }\left\{\frac{19}{120}\times 20 - \frac{7}{48}\text{ of }1\right\}s.$$

$$= \frac{26}{9}\text{ of }\left\{\frac{19}{6} - \frac{7}{48}\right\}s.$$

$$= \frac{26}{9} \text{ of } \left\{\frac{19\times 8}{6\times 8} - \frac{7}{48}\right\}s.$$

$$= \left(\frac{26}{9} \times \frac{145}{48}\right)s.$$

A moidore $=27s.$;

$$\text{therefore the fraction required} = \frac{\frac{26}{9} \times \frac{145}{48}}{27}$$

$$= \frac{13\times 145}{9\times 24\times 27}$$

$$= \frac{1885}{5832}.$$

Ex. 4. What fraction of a guinea together with 4*s.* 6*d.* is equivalent to 15*s.*?

In other words the question is, "What fraction of a guinea is equivalent to 15*s.* − 4*s.* 6*d.*, or 10*s.* 6*d.*?"

Now 10*s.* 6*d.* = 21 sixpences,
1 guinea = 42 sixpences;

therefore fraction required $= \frac{21}{42} = \frac{1}{2}$.

Ex. L.

(1) Compare the values of $\frac{1}{19}$ of £1., $\frac{1}{20}$ of a guinea, and $\frac{8}{35}$ of a crown.

(2) Compare the values of $\frac{7}{8}$ of £1., $\frac{5}{6}$ of a guinea, and $\frac{2}{3}$ of 15*s.* 7½*d.*

(3) Which is the greater, $\frac{1}{25}$ of a day, or $\frac{6}{7}$ of an hour, and by how much?

(4) Express the difference between $\frac{1}{12}$ of £1. and $\frac{1}{14}$ of a guinea as the fraction of half-a-crown.

(5) Express the difference between $\frac{2}{5}$ of a guinea and $\frac{7}{9}$ of £1. as the fraction of half-a-guinea.

(6) Reduce $\frac{3}{8}$ of a crown $+\frac{4}{5}$ of a shilling to the fraction of a guinea.

(7) Express $\frac{3}{8}$ of 2*s.* 6*d.* $+\frac{4}{5}$ of a guinea $+\frac{5}{8}$ of £1. $-\frac{1}{10}$ of a penny as the fraction of £5.

(8) Add together $\frac{3}{8}$ of £3. 7*s.* 6*d.* and $\frac{2}{3}$ of $\frac{1}{4}$ of 4½ guineas; and reduce the result to the fraction of £2.

(9) What fraction of £10. together with $3\frac{1}{2}$ guineas is equivalent to 5 guineas?

(10) What fraction of $2\frac{1}{2}$ cwt. together with 3 qrs., 14 lbs. will give a ton and a half?

REDUCTION OF DECIMALS.

134. *To reduce a decimal of any denomination to its proper value.*

Rule. "Multiply the decimal by the number of units connecting the next lower denomination with the given one, and point off for decimals as many figures in the product, beginning from the right hand, as there are figures in the given decimal. The figures on the left of the decimal point will represent the whole numbers in the next denomination. Proceed in the same way with the decimal part for that denomination, and so on."

Ex. 1. Find the value of ·0484 of £1.

Proceeding by the Rule given above,

$$\begin{array}{r} \pounds. \\ \cdot 0484 \\ \underline{20} \\ \cdot 9680s. \\ \underline{12} \\ 11 \cdot 6160d. \\ \underline{4} \\ 2 \cdot 4640q. \end{array}$$

therefore the value of ·0484 of £1. $= 11d.\ 2 \cdot 4640q.$

$= 11d.\ 2\frac{464}{1000}q.$

$= 11\frac{1}{2}\frac{29}{250}d.$

Reason for the above process.

$$\cdot 0484 \text{ of } \pounds 1. = \frac{484}{10000} \text{ of } \pounds 1.$$

$$= \frac{9680}{10000}s. = \frac{116160}{10000}d.$$

$$= 11\frac{616}{1000}d. = 11d. + \frac{616 \times 4}{1000}q.$$

$$= 11d. + \frac{2464}{1000}q.$$

$$= 11d. + 2\frac{464}{1000}q.$$

$$= 11\frac{1}{2}\frac{29}{250}d.$$

Ex. 2. Find the value of 13·3375 acres.

$$\begin{array}{r}
\text{Acres.}\\
13{\cdot}3375\\
\underline{4}\\
1{\cdot}3500\ \text{ro.}\\
\underline{40}\\
14{\cdot}0000\ \text{po.}
\end{array}$$

therefore the value is 13 ac., 1 ro., 14 po.

Ex. 3. Find the value of ·07 of £2. 10*s.*

$$\begin{array}{r}
\pounds 2.\ 10s. = 50s.\\
{\cdot}07\\
\underline{50}\\
3{\cdot}50\,s.\\
\underline{12}\\
6{\cdot}00\,d.
\end{array}$$

therefore the value of ·07 of £2. 10*s.* is 3*s.* 6*d.*

Ex. 4. Find the value of ·0474609375 of £10. 13*s.* 4*d.*

$$\begin{array}{r}
\pounds 10.\ 13s.\ 4d. = 2560d.\\
{\cdot}0474609375\\
\underline{2560}\\
28476562500\\
2373046875\\
\underline{949218750}\\
121{\cdot}5000000000d.\\
\underline{4}\\
2{\cdot}0q.
\end{array}$$

therefore the value is 121½*d.* or 10*s.* 1½*d.*

Ex. 5. Find the value of ·97291$\dot{6}$ of £1.

1st method.

$$\begin{array}{r}
\pounds.\\
{\cdot}972917\\
\underline{20}\\
19{\cdot}458340s.\\
\underline{12}\\
5{\cdot}500080d.\\
\underline{4}\\
2{\cdot}000320q.
\end{array}$$

therefore the value is 19*s.* 5½*d.* nearly.

2nd method.

$$\cdot 97291\dot{6} \text{ of } £1. = \frac{972916-97291}{900000} \text{ of } £1., \quad \text{Art. (97)},$$

$$= \frac{875625}{900000} \text{ of } £1. = \left(\frac{467}{480} \times 20\right) s.$$

$$= \frac{467}{24} s. = 19s.\ 5\tfrac{1}{2}d.$$

Note. The latter is generally the better course to adopt.

Ex. 6. Find the value of $\cdot 375$ of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s*. 3*d*. + $\cdot 0\dot{2}\dot{7}$ of £2. 15*s*.

$$\begin{array}{r}
\text{Guinea.}\\
\cdot 375\\
21\\
\hline
375\\
750\\
\hline
7\cdot 875s.\\
12\\
\hline
10\cdot 500d.\\
4\\
\hline
2\cdot 000q.
\end{array}$$

therefore ·375 of a guinea = 7*s*. 10½*d*..

$$\cdot\dot{5}\dot{4} \text{ of } 8s.\ 3d. = \frac{54}{99} \text{ of } 99d. = 54d. = 4s.\ 6d.,$$

$$\cdot 0\dot{2}\dot{7} \text{ of } £2.\ 15s. = \left(\frac{27}{990} \text{ of } 55\right) s.$$

$$= \frac{3}{110} \text{ of } 55s. = \frac{3}{2} s. = 1s.\ 6d.$$

therefore the value required = 7*s*. 10½*d*. + 4*s*. 6*d*. + 1*s*. 6*d*.,
= 13*s*. 10½*d*.

Ex. 7. Find the value of $\frac{133}{400}$ of $3\frac{3}{4}$ tons $-\cdot 3\dot{4}0\dot{5}$ of $1\frac{2}{3}$ qrs. $+\frac{\cdot 21334\dot{8}}{\cdot 326}$ of 2 cwt., 102 lbs.

$$\frac{133}{400} \text{ of } 3\tfrac{3}{4} \text{ tons} = \left(\frac{133}{400} \times \frac{15}{4}\right) \text{tons} = \frac{133 \times 3}{80 \times 4} \text{ tons},$$

$$= \left(\frac{133 \times 3}{80 \times 4} \times 20\right) \text{cwt.} = \frac{399}{16} \text{ cwt.}$$

$$= 24 \text{ cwt., } 3 \text{ qrs., } 21 \text{ lbs.}$$

$$\cdot 3\dot{4}0\dot{5} \text{ of } 1\tfrac{2}{3} \text{ qrs.} = \left(\frac{3405-3}{9990} \text{ of } \frac{5}{3}\right) \text{qrs.,}$$

$$= \left(\frac{3402}{9990} \times \frac{5}{3} \times 28\right) \text{lbs.,}$$

$$= \left(\frac{21 \times 28}{37}\right) \text{lbs.} = 15\tfrac{33}{37} \text{ lbs.}$$

$$\frac{\cdot 21334\dot{8}}{\cdot 326} \text{ of } 2 \text{ cwt., } 102 \text{ lbs.} = \left(\frac{213348-21334}{900000} \times \frac{1000}{326} \text{ of } 326\right) \text{lbs.}$$

$$= \frac{192014}{900} \text{lbs.}$$

$$= 213\frac{157}{450} \text{lbs.}$$

therefore the value of the expression

$= 24$ cwt., 3 qrs., 21 lbs. $- 15\frac{33}{37}$ lbs. $+ 213\frac{157}{450}$ lbs.
$= 24$ cwt., 3 qrs., $5\frac{4}{37}$ lbs. $+ 1$ cwt., 3 qrs., $17\frac{157}{450}$ lbs.
$= 1$ ton, 6 cwt., 2 qrs., $22\frac{7609}{16650}$ lbs.

Ex. LI.

(1) Find the respective values of

1. ·45 of £1.; ·16875 of £1.; ·87708 of £1.
2. ·28125 of £1.; ·7962 of £1.; ·359375 of £2.
3. 5·00625 of £1.; ·775625 of £5.; ·6875 of 16*s.*
4. ·0625 of a guinea; ·7635 of 10*s.*; 2·625 of 1*s.*
5. ·056713 of a guinea; 2·76543 of £1; 1·74375 of 10*s.*
6. 3·049 of £1.; ·0425 of £100; ·432 of 13*s.* 4*d.*
7. ·1875 of 5 guineas; 1·05625 of 6*s.* 8*d.*; ·875 of £3. 5*s.* 6*d.*
8. 3·10532 of 12*s.* 6*d.*; 27·5 of 2*s.* 4*d.*; 41·375 of 8*d.*
9. ·875 of a lea.; 2·5384375 of a day; ·6 of 1 lb. Troy.
10. 6·156510416 of £4.; ·046875 of 3 qrs., 12 lbs.
11. ·85076 of a cwt.; ·07325 of a cwt.; ·045 of a mile.
12. 4·16525 of a ton; 3·625 of a cwt.; ·05 of an acre.
13. 3·8343 of a lb. Troy; 2·46875 of a qr.; 4·106 of 3 cwt. 1 qr., 21 lbs.
14. 3·8375 of an acre; 3·5 of 18 gallons.
15. ·925 of a furlong; ·34375 of a lunar month.
16. 5·06325 of £100.; 3·8 of an Eng. ell.
17. 2·25 of $3\frac{1}{2}$ acres; 2·465 of 5 crowns.
18. 1·605 of £3. 2*s.* 6*d.*; 2·0396 of 1 m., 530 yds.
19. 4·751 of 2 sq. yds., 7 sq. ft.; 2·0005 of £63. 0*s.* $3\frac{1}{4}$*d.*
20. 2·009943 of 2 miles; 1·005 of 15 guineas.

(2) Find the respective values of

1. $\cdot 38\dot{3}$ of £1.; $\cdot 4708\dot{3}$ of £1.; $\cdot 469\dot{4}$ of 1 lb. Troy.
2. $\cdot 5\dot{7}4\dot{0}$ of 27*s.*; $\cdot 13\dot{8}$ of 10*s.* 6*d.*; $2\cdot\dot{6}$ of 5*s.*
3. $\cdot\dot{1}4285\dot{7}$ of 2 guineas; $3\cdot\dot{2}09532\dot{8}$ of 17*s.* 6*d.*
4. $\cdot 06\dot{3}$ of 100 guineas; $2\cdot 013\dot{8}$ of $3\cdot 5$ moidores.
5. $4\cdot 0\dot{5}$ of $1\frac{1}{2}$ sq. yds; $\cdot 16\dot{3}$ of $2\frac{1}{2}$ miles; $4\cdot\dot{9}\dot{0}$ of 4 d., 3 hrs.
6. $3\cdot 24\dot{2}$ of $2\frac{1}{2}$ acres; $\dfrac{\cdot 093\dot{1}\dot{8}}{\cdot 56\dot{8}\dot{1}}$ of $2\frac{1}{12}$ of $2\cdot 5$ days.

(3) Find the difference between $\cdot 77777$ of a pound and 8*s.* $6\cdot 6648$*d.*; and between $\cdot 70323$ of a pound and $3\cdot 5646$ of a shilling.

(4) Subtract $\frac{3}{8}$ of a crown from £$1\cdot 59375$.

(5) Find the respective values of the following expressions:

1. $\cdot 68125$ of £1. + $\cdot 375$ of 13*s.* 4*d.* + $\cdot 605$ of £3. 2*s.* 6*d.*
2. $3\frac{7}{8}$ of 6*s.* 8*d.* − $\cdot 4097\dot{2}$ of a guinea + $2\cdot 75$ of £30.
3. £$\cdot 634375$ + $\cdot 025$ of 25*s.* + $\cdot 31\dot{6}$ of 30*s.*
4. $\cdot 75$ of 6*s.* 8*d.* − $1\cdot 84375$ of 4*s.* + $3\cdot 979\dot{6}$ of 2*s.*
5. $2\cdot\dot{8}\dot{1}$ of $365\frac{1}{4}$ days + $5\cdot 75$ of a week − $\frac{3}{4}$ of $5\frac{5}{9}$ hours.
6. $\frac{7}{9}$ of $\frac{3}{14}$ of 3 acres − $2\cdot 00875$ square yards + $\cdot 02\dot{2}\dot{7}$ of $3\frac{1}{2}$ square feet.

(6) Which is the greater, $\cdot 0231$ of a guinea, or $\cdot 19$ of a half-crown.

135. *To reduce a number or fraction of any denomination, to the decimal of another denomination.*

Rule. "Reduce the given number or fraction, to a fraction of the proposed denomination; and then reduce this fraction to its equivalent decimal."

Ex. 1. Reduce $\frac{2}{5}$ of £1. to the decimal of 1 guinea.

$$\tfrac{2}{5} \text{ of } £1. = \frac{2\times 20}{5}\, s. = 8s.$$

1 guinea = 21*s.*

therefore the fraction required $= \frac{8}{21}$.

$$21\left\{\begin{matrix}7\\3\end{matrix}\right.\begin{array}{|l}8\cdot 0\\ \hline 1\cdot\dot{1}4285\dot{7}14\\ \hline \cdot\dot{3}80952\dot{3}8\end{array}$$

therefore the decimal required $= \cdot\dot{3}80952\dot{3}8$.

Ex. 2. Reduce 13*s.* 6¼*d.* to the decimal of £1.

$$13s.\ 6\tfrac{1}{4}d. = 162\tfrac{1}{4}d. = \tfrac{649}{4}d.$$

$$£1. = 240d.;$$

therefore the fraction $= \dfrac{\frac{649}{4}}{240} = \dfrac{649}{960}$.

$$\begin{array}{r}
960)\ 649{\cdot}00\ (\ {\cdot}67 \\
\underline{5760} \\
7300 \\
\underline{6720} \\
580
\end{array}$$

We may work such an example as the above more expeditiously, by first reducing ¼*d.* to the decimal of a penny, which decimal will be ·25, and then reducing 6·25*d.* to the decimal of a shilling by dividing by 12, which decimal will be $\cdot 52083\dot{3}$, and then reducing $13\cdot 52083\dot{3}s.$ to the decimal of £1. by dividing by 20, which process gives $\cdot 6760416\dot{6}$ as the required decimal of £1.

The mode of operation may be shewn thus:

$$\begin{array}{r|l}
4 & \underline{1{\cdot}00} \\
12 & \underline{6{\cdot}25} \\
2{,}0 & \underline{13{\cdot}52083\dot{3}} \\
 & {\cdot}676041\dot{6}
\end{array}$$

Ex. 3. Reduce 3 bus., 1 pk. to the decimal of a load: and verify the result.

$$\begin{array}{r|l}
4 & \underline{1{\cdot}00} \\
8 & \underline{3{\cdot}25} \\
5 & \underline{{\cdot}40625} \\
 & {\cdot}08125
\end{array}$$

therefore ·08125 is the decimal required.

$$\begin{array}{l}
{\cdot}08125 \text{ ld.} \\
\underline{\phantom{{\cdot}0812}5} \\
{\cdot}40625 \text{ qrs.} \\
\underline{\phantom{{\cdot}4062}8} \\
3{\cdot}25000 \text{ bush.} \\
\underline{\phantom{3{\cdot}250}4} \\
1{\cdot}00000 \text{ pk.}
\end{array}$$

therefore ·08125 of a load = 3 bus., 1 pk.

Ex. 4. Add together $\frac{2}{5}$ of 21*s.*, $\frac{3}{4}$ of a moidore, $\frac{5}{2}$ of 7*s.* 6*d.*, and reduce the result to the decimal of £1.

$$\tfrac{2}{5} \text{ of } 21s. = \tfrac{42}{5}s. = 8s.\ 4\tfrac{4}{5}d.$$
$$\tfrac{3}{4} \text{ of } 27s. = \tfrac{81}{4}s. = £1.\ 0s.\ 3d.$$
$$\tfrac{5}{2} \text{ of } 7s.\ 6d. = (3s.\ 9d.) \times 5 = 18s.\ 9d.$$

therefore the sum $= £2.\ 7s.\ 4\frac{4}{5}d.$

Now to reduce $£2.\ 7s.\ 4\frac{4}{5}d.$ to the decimal of £1.

$$\begin{array}{r|l} 5 & 4\cdot \\ \hline 12 & 4\cdot8 \\ \hline 2{,}0 & 7\cdot4 \\ \hline & \cdot37 \end{array}$$

therefore the decimal required $= 2\cdot37$.

Ex. 5. Express the sum of $\cdot\dot{4}2857\dot{1}$ of £15., $\frac{1}{4}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{4}{5}$ of £1. 12*s.*, and $\frac{4}{7}$ of 3*d.*, as the decimal of £10.

$$\cdot\dot{4}2857\dot{1} \text{ of } £15 = \tfrac{428571}{999999} \text{ of } £15.$$
$$= \tfrac{3}{7} \text{ of } £15 = £\tfrac{45}{7}$$
$$= £6.\ 8s.\ 6\tfrac{6}{7}d.;$$
$$\tfrac{1}{4} \text{ of } \frac{1}{2\frac{4}{5}} \text{ of } \tfrac{4}{5} \text{ of } £1.\ 12s. = \tfrac{1}{4} \text{ of } \tfrac{5}{14} \text{ of } \tfrac{4}{5} \text{ of } 32s.$$
$$= \tfrac{16}{7}s. = 2s.\ 3\tfrac{3}{7}d.;$$
$$\tfrac{4}{7} \text{ of } 3d. = \tfrac{12}{7}d. = 1\tfrac{5}{7}d.;$$
$$\text{therefore the sum} = £6.\ 8s.\ 6\tfrac{6}{7}d. + 2s.\ 3\tfrac{3}{7}d. + 1\tfrac{5}{7}d.$$
$$= £6.\ 11s.$$

$$\begin{array}{r|l} 2{,}0 & 11 \\ \hline 10 & 6\cdot55 \\ \hline & \cdot655 \end{array}$$

therefore the decimal required $= \cdot655$.

Ex. 6. Convert £17. 9*s.* 6*d.* into pounds, florins, &c.; and verify the result.

First reduce 9*s.* 6*d.* to the decimal of £1.

$$\begin{array}{r|l} 12 & 6\cdot0 \\ \hline 2{,}0 & 9\cdot5 \\ \hline & \cdot475 \end{array}$$

$$\therefore\ £17.\ 9s.\ 6d. = £17{\cdot}475$$
$$= £17.\ 4\text{ fl. } 7\text{ c. } 5\text{ m.}$$

Again, £17. 4 fl. 7 c. 5 m.

$$\begin{array}{r} = £17{\cdot}475 \\ \underline{20} \\ 9{\cdot}500\,s. \\ \underline{12} \\ 6{\cdot}000\,d. \end{array}$$

$$\therefore\ £17.\ 4\text{ fl. } 7\text{ c. } 5\text{ m.} = £17.\ 9s.\ 6d.$$

Ex. 7. Express 1 shilling and 1 half-crown in terms of the decimal coinage.

$$1s. = £\tfrac{1}{20} = £\tfrac{5}{100} = £{\cdot}05 = 5\text{ cents};$$
$$2s.\ 6d. = £\tfrac{1}{8} = £\tfrac{125}{1000} = £{\cdot}125$$
$$= 1\text{ fl. } 2\text{ c. } 5\text{ m.}$$

Ex. 8. Reduce the difference between a cent and a penny to the decimal of 3*s.* 4*d.*

$$1d. = £\tfrac{1}{240}; \qquad 1\text{ c.} = £\tfrac{1}{100};$$
$$\therefore\ \text{difference} = £(\tfrac{1}{100} - \tfrac{1}{240}) = £\tfrac{140}{24000} = £\tfrac{7}{1200}$$
$$= (\tfrac{7}{1200} \times 20 \times 12)d. = \tfrac{7}{5}d.$$
$$3s.\ 4d. = 40d.$$
$$\therefore\ \text{fraction} = \frac{\tfrac{7}{5}}{40} = \tfrac{7}{200} = \tfrac{35}{1000};$$
$$\therefore\ \text{decimal} = {\cdot}035.$$

Ex. LII.

(1) Reduce

1. 6*s.* 4*d.* to the decimal of £1; and 8*s.* $8\frac{1}{2}$*d.* to the decimal of £1.
2. 4*s.* $7\frac{1}{2}$*d.* to the decimal of £1; and 15*s.* $11\frac{1}{4}$*d.* to the decimal of £1.
3. 3*s.* $4\frac{1}{2}$*d.* to the decimal of a crown; and $\frac{3}{4}$*d.* to the decimal of £1.
4. 10*s.* $0\frac{3}{4}$*d.* to the decimal of £1; and 5*s.* $8\frac{3}{4}$*d.* to the decimal of £5.
5. 1*s.* $3\frac{1}{4}$*d.* to the decimal of 15*s.*; and 12*s.* $1\frac{1}{2}$*d.* to the decimal of a guinea.
6. 5*s.* to the decimal of 13*s.* 4*d.*; and 18*s.* 9*d.* to the decimal of 27*s.*
7. 13*s.* 6*d.* to the decimal of 10*s.*; and £1. 9*s.* $4\frac{1}{2}$*d.* to the decimal of £1.

8. £3. 11*s.* $9\frac{3}{4}$*d.* to the decimal of £1; and also to the decimal of £2. 10*s.*
9. 14*s.* $0\frac{3}{4}$*d.* to the decimal of 3 guineas; and 27*s.* to the decimal of $1\frac{1}{2}$ guineas.
10. $6\frac{1}{2}$ guineas to the decimal of £5; and $1\frac{1}{2}$*d.* to the decimal of £100.
11. £8. 0*s.* 10*d.* to the decimal of $5\frac{1}{2}$*d.*; and 7 guineas to the decimal of £5. 10*s.* 11*d.*
12. 2 oz., 13 dwts. to the decimal of 1 lb.; and 4 lbs., 2 sc. to the decimal of 1 oz.
13. 2 qrs., 21 lbs. to the decimal of 1 ton; and 3 cwt., 3 oz. to the decimal of 10 cwt.
14. 2 fur., 41 yds. to the decimal of a mile; and 1 fur., 30 po. to the decimal of a league.
15. 2 sq. ft., 73 sq. in. to the decimal of a square yard; and 3 ro., 20 po. to the decimal of an acre.
16. 14 gals., 2 qts. to the decimal of a barrel; and 3 qrs., 3 pks. to the decimal of a load.
17. 4 days, 18 hrs. to the decimal of a week; and 11 sec. to the decimal of 5 days.
18. $1\frac{1}{3}$ guineas to the decimal of £$1\frac{1}{2}$.; and 1 lb. Troy to the decimal 1 lb. Avoirdupois.
19. $2\frac{1}{8}$ inches to the decimal of $2\frac{1}{5}$ miles; and 1 st., $6\frac{1}{2}$ lbs. to the decimal of $3\frac{1}{2}$ lbs.
20. $3\frac{3}{4}$ pks. to the decimal of $3\frac{1}{2}$ qrs.; and $27\frac{1}{2}$ gals. to the decimal of $1\frac{1}{3}$ qts.
21. $5\frac{3}{4}$ yds. to the decimal of 2 Fr. ells; and 1 ton, $2\frac{1}{4}$ cwt. to the decimal of 1 cwt., $2\frac{1}{4}$ qrs.
22. 3 wks., $5\frac{1}{4}$ d. to the decimal of $5\frac{1}{2}$ hrs.; and 1 min., $2\frac{1}{4}$ sec. to the decimal of $\frac{1}{25}$ of a lunar month.
23. 3 reams to the decimal of 19 sheets; and $3\frac{1}{2}$ acres to the decimal of $3\frac{1}{4}$ sq. yards.
24. 33 yds. to the decimal of a mile; 3*s.* $5\frac{11}{200}$*d.* to the decimal of a dollar, a dollar being 4*s.* 3*d.*; and 7*s.* $8\frac{1942}{10000}$*d.* to the decimal of 10*s.* 6*d.*

(2) Reduce

1. $\frac{2}{3}$ of 13*s.* 6*d.* to the decimal of £1; and $\frac{6}{7}$ of half-a-crown to the decimal of 1*s.*
2. $\frac{5}{8}$ of a crown to the decimal of 21*s.*; and $6\frac{2}{3}$ cwt. to the decimal of a ton.

3. $\frac{15}{56}$ of a guinea to the decimal of £1; and $\frac{7}{8}$ pk. to the decimal of 2 qrs.
4. $\frac{3}{7}$ of a guinea to the decimal of £2; and $\frac{21}{15000}$ of a year to the decimal of a day.
5. $\frac{3}{7}$ of $\frac{1}{10}$ of 40 yds. to the decimal of $\frac{1}{5}$ of 2 miles; and $\frac{1}{4}$ of $3\frac{1}{2}$ sq. yds. to the decimal of 2 acres, 1 ro.
6. $\frac{3}{5}$ of $4\frac{4}{9}$ hrs. to the decimal of $365\frac{1}{4}$ days; and $9\frac{6}{11}$ of $\frac{11}{15}$ pecks to the decimal of $3\frac{1}{3}$ qrs.
7. 3 lbs., 6 oz. Troy to the decimal of 10 lbs. Avoird.; and $\frac{1}{5}$ oz. Avoird. to the decimal of $\frac{1}{3}$ oz. Troy.

(3) Express $\frac{3}{8}$ of a crown + $\frac{4}{5}$ of a shilling as a decimal of 7*s.*

(4) Express $\frac{3}{5}$ of half-a- crown + ·4 of a shilling as a decimal of £2.

(5) Add together $\frac{3}{5}$ of a day, $\frac{2}{3}$ of an hour, and $\frac{4}{5}$ of 6 hours; and express the result as the decimal of a week.

(6) Express the difference of $\frac{2}{5}$ of a guinea and $\frac{3}{4}$ of 7*s.* 6*d.* as the decimal of a moidore.

(7) Express the value of $·8\dot{3}$ of 8*s.* + $·\dot{0}\dot{5}$ of 2 guineas + 1·8 of 5*s.* as the decimal of half-a-guinea.

(8) Find the difference between $6\frac{1}{2}$ half-guineas and £3·525; and reduce the result to the decimal of a crown.

(9) Add $5\frac{1}{2}$ cwt. to 3·125 qrs.; and reduce the sum to the decimal of a ton.

(10) Convert the following sums of money into the decimal coinage of pounds, florins, &c., and verify each result:

1. 6*d.* 2. 10*d.* 3. $4\frac{1}{2}$*d.* 4. 5*s.*
5. 10*s.* 6*d.* 6. 16*s.* 7. £5. 12*s.* 6*d.*
8. £54. 7*s.* 4*d.* 9. £20. 19*s.* $7\frac{1}{2}$*d.* 10. 15*s.* $4\frac{3}{4}$*d.*
11. 14*s.* 8·16*d.* 12. £2. 15*s.* 11·088*d.* 13. £3. 0*s.* 11*d.*3·04*q.*

PRACTICE.

136. **Def.** **An Aliquot Part** of a number is such a part as, when taken a certain number of times, will exactly make up that number. Thus, 4 is an aliquot part of 12, 6 of 18, &c.

Practice is a compendious mode of finding the value of any number of articles by means of **aliquot parts**, when the value of an unit of any denomination is given.

Practice may be separated into two cases, **Simple** and **Compound**.

I. *Simple Practice.*

In this case the given number is expressed in the same denomination as the unit whose value is given: as, for instance, 26 lbs. at £2. 5*s.* per lb.; or 330 articles at 5*s.* 6½*d.* each.

The Rule for Simple Practice will be easily shewn by the following examples.

Ex. 1. Find the value of 1296 things at 16*s.* 10½*d.* each.

The method of working such an example is the following:

Supposing the cost of the things to be £1. each;

then the total cost = £1296;

therefore

	£.	*s.*	*d.*
cost at 10*s.* 0*d.* each = ½ of the above sum...... =	648	0	0
cost at 5*s.* 0*d.* each = ½ the cost at 10*s.* each =	324	0	0
cost at 1*s.* 3*d.* each = ¼ the cost at 5*s.* each ... =	81	0	0
cost at 0*s.* 7½*d.* each = ½ the cost at 1*s.* 3*d.* each =	40	10	0

therefore, by adding up the vertical columns,

	£.	*s.*	*d.*
cost at 16*s.* 10½*d.* =	£1093	10	0

The operation is usually written thus:

	£.	*s.*	*d.*	
10*s.* = ½ of £1.	1296	0	0	= cost at £1. each.
5*s.* = ½ of 10*s.*	648	0	0	= cost at 10*s.* each.
1*s.* 3*d.* = ¼ of 5*s.*	324	0	0	= cost at 5*s.* each.
7½*d.* = ½ of 1*s.* 3*d.*	81	0	0	= cost at 1*s.* 3*d.* each.
	40	10	0	= cost at 7½*d.* each.
	£1093	10	0	= cost at 16*s.* 10½*d.* each.

Note. The student must use his own judgment in selecting the most convenient 'aliquot' parts; taking care that the sum of those taken make up the *given price of the unit.*

Ex. 2. Find the value of 3825 things at £2. 17*s.* 4½*d.* each.

	£.	*s.*	*d.*	
10*s.* = ½ of £1.	3825	0	0	= value at £1. each.
			2	
	7650	0	0	= value at £2. each.
5*s.* = ½ of 10*s.*	1912	10	0	= value at 10*s.* each.
2*s.* = ⅕ of 10*s.*	956	5	0	= value at 5*s.* each.
(∴ take ⅕ of £1912. 10*s.*)	382	10	0	= value at 2*s.* each.
4*d.* = ⅙ of 2*s.*	63	15	0	= value at 4*d.* each.
½*d.* = ⅛ of 4*d.*	7	19	4½	= value at ½*d.* each.
	£10972	19	4½	= value at £2.17*s.*4½*d.* each.

Ex. 3. Find the cost of $165\frac{7}{8}$ cwt. at £2. 5*s*. 6*d*. per cwt.

The cost clearly = 165 times £2. 5*s*. 6*d*. + $\frac{7}{8}$ of £2. 5*s*. 6*d*.

	£.	*s*.	*d*.	
5*s*. = $\frac{1}{4}$ of £1.	165	0	0	= cost at £1. each.
			2	
	330	0	0	= cost at £2. each.
6*d*. = $\frac{1}{10}$ of 5*s*.	41	5	0	= cost at 5*s*. each.
	4	2	6	= cost at 6*d*. each.
	375	7	6	= cost of 165 cwt. at £2. 5*s*. 6*d*. per cwt.
$\frac{7}{8}$ of £2. 5*s*. 6*d*. =	1	19	$9\frac{3}{4}$	= cost of $\frac{7}{8}$ cwt. at £2. 5*s*. 6*d*. per cwt.
	£377	7	$3\frac{3}{4}$	= cost of $165\frac{7}{8}$ cwt. at £2. 5*s*. 6*d*. per cwt.

Ex. 4. Find the value of 6413 things at 4*s*. $10\frac{7}{16}$*d*. each.

	£.	*s*.	*d*.	
4*s*. = $\frac{1}{5}$ of £1.	6413			= value at £1. each.
6*d*. = $\frac{1}{8}$ of 4*s*.	1282	12	0	= value at 4*s*. each.
4*d*. = $\frac{1}{12}$ of 4*s*.	160	6	6	= value at 6*d*. each.
$\frac{7}{16}$*d*. = $\frac{7}{64}$ of 4*d*.	106	17	8	= value at 4*d*. each.
	11	13	$9\frac{11}{16}$	= value at $\frac{7}{16}$*d*. each.
	£1561	9	$11\frac{11}{16}$	= value at 4*s*. $10\frac{7}{16}$*d*. each.

II. *Compound Practice.*

In this case the given number is not wholly expressed in the same denomination as the unit whose value is given; as for instance, 1 cwt., 2 qrs., 14 lbs, at £2. 2*s*. per cwt.

The Rule for Compound Practice will be easily shewn by the following examples.

Ex. 1. Find the value of 84 cwt., 3 qrs., 14 lbs. of sugar at £12. 11*s*. 8*d*. per cwt.

The method of working such an example is the following:

The value of 1 cwt. of sugar being £12. 11*s*. 8*d*.,

	£.	*s*.	*d*.
the value of 84 cwt. of sugar............... =	1057	0	0
2 qrs. = $\frac{1}{2}$ (value of 1 cwt.) =	6	5	10
1 qr. = $\frac{1}{2}$ (value of 2 qrs.) =	3	2	11
14 lbs. = $\frac{1}{2}$ (value of 1 qr.) =	1	11	$5\frac{1}{2}$

therefore, by adding the vertical columns,

the value of 84 cwt., 3 qrs., 14 lbs. = £1068 . 0 . $2\frac{1}{2}$

The operation is usually written thus:

	£	s.	d.	
2 qrs. = $\frac{1}{2}$ cwt.	12	11	8	= value of 1 cwt.
			12	
	151	0	0	= value of 12 cwt.
			7	
	1057	0	0	= value of (12 × 7) or 84 cwt.
1 qr. = $\frac{1}{2}$ of 2 qrs.	6	5	10	= value of 2 qrs.
14 lbs. = $\frac{1}{2}$ of 1 qr.	3	2	11	= value of 1 qr.
	1	11	$5\frac{1}{2}$	= value of 14 lbs.
	£1068	0	$2\frac{1}{2}$	= value of 84 cwt., 3 qrs., 14 lbs.

Ex. 2. Find the value of 319 cwt., 3 qrs., 16 lbs. at £2. 12*s.* 6*d.* per cwt.

	£.	s.	d.	
2 qrs. = $\frac{1}{2}$ cwt.	2	12	6	= value of 1 cwt.
			10	
	26	5	0	= value of 10 cwt.
			4	
	105	0	0	= value of 40 cwt.
			8	
	840	0	0	= value of 320 cwt.
subtracting	2	12	6	= value of 1 cwt.
	837	7	6	= value of 319 cwt.
1 qr. = $\frac{1}{2}$ of 2 qrs.	1	6	3	= value of 2 qrs.
14 lbs. = $\frac{1}{2}$ of 1 qr.	0	13	$1\frac{1}{2}$	= value of 1 qr.
2 lbs. = $\frac{1}{7}$ of 14 lbs.	0	6	$6\frac{3}{4}$	= value of 14 lbs.
	0	0	$11\frac{1}{4}$	= value of 2 lbs.
	£839	14	$4\frac{1}{2}$	= value of 319 cwt., 3 qrs., 16 lbs.

Ex. 3. Find the value of 37 yds., 2 ft., 7 in. of silk, at 5*s.* $3\frac{1}{4}$*d.* a yard.

	£.	s.	d.	
1 ft. = $\frac{1}{3}$ of 1 yd.	0	5	$3\frac{1}{4}$	= value of 1 yd.
			4	
	1	1	1	= value of 4 yds.
			9	
	9	9	9	= value of 36 yds.
	0	5	$3\frac{1}{4}$	= value of 1 yd.
	9	15	$0\frac{1}{4}$	= value of 37 yds.
1 ft. = $\frac{1}{3}$ of 1 yd.	0	1	$9\frac{1}{12}$	= value of 1 ft.
6 in. = $\frac{1}{2}$ of 1 ft.	0	1	$9\frac{1}{12}$	= value of 1 ft.
1 in. = $\frac{1}{6}$ of 6 in.	0	0	$10\frac{13}{24}$	= value of 6 in.
	0	0	$1\frac{109}{144}$	= value of 1 in.
	£9	19	$6\frac{1}{2}$*d.* $\frac{31}{36}$*q.*	= value of 37 yds., 2 ft., 7 in.

Note. It will be found most convenient, in all examples of Practice, to work with fractions of a penny, and finally to find the value of the sum of these fractions in farthings, as in the above example.

Ex. LIII.

Find the value of

1. 645 things at 2*s*. 6*d*. each; and 69 things at 10*s*. 6*d*. each.
2. 454 things at 2*s*. 9*d*. each; and 72 things at 1*s*. 7*d*. each.
3. 52 things at 3*s*. 9*d*. each; and 1257 things at 6¾*d*. each.
4. 626 things at 8*s*. 8*d*. each; and 286 things at 12*s*. 1*d*. each.
5. 80 things at 4*s*. 4¼*d*. each; and 37 things at 5*s*. 5½*d*. each.
6. 138 things at £1. 14*s*. each; and 589 things at £1. 11*s*. 6*d*. each.
7. 95 things at £1. 2*s*. 6*d*. each; and 107 things at £24. 6*s*. 2*d*. each.
8. 457 things at £1. 8*s*. 6*d*. each; and 88 things at 1½*d*. each.
9. 111 things at £2. 5*s*. 10*d*. each; and 9251 things at 14*s*. 11*d*. each.
10. 4681 things at 8¾*d*. each; and 1209 things at 13*s*. 1*d*. each.
11. 1450 things at £1. 7*s*. 8*d*. each; and 249 things at £2. 13*s*. 9*d*. each.
12. 898 things at 18*s*. 7¾*d*. each; and 405 things at 19*s*. 8¼*d*. each.
13. 744 things at £19. 19*s*. each; and 421 things at £4. 2*s*. 6¾*d*. each.
14. 1593 things at 9*s*. 0½*d*. each: and 6602 things at 7*s*. 1¾*d*. each.
15. 7382 things at £3. 15*s*. 4½*d*. each; and 5614 things at £14. 14*s*. 5½*d*. each.
16. 6573 things at £5. 18*s*. 10¼*d*. each; and 37271 things at £6. 13*s*. 0¾*d*. each.
17. 51143 things at £4. 17*s*. 9¼*d*. each; and 6293 things at £7. 0*s*. 11¾*d*. each.
18. 3027 things at £5. 12*s*. 2½*d*. each; and 4945 things at £1. 0*s*. 1¾*d*. each.
19. 733¼ things at £1. 9*s*. 4*d*. each; and 751½ things at £2. 17*s*. 10*d*. each.
20. 1418¾ things at 7*s*. 9½*d*. each; and 1178⅜ things at 16*s*. 8*d*. each.
21. 1762⅜ things at £1. 1*s*. 10½*d*. each; and 5555$\frac{5}{9}$ things at 6*s*. 10¼*d*. each.

22. $4013\frac{5}{8}$ things at £2. 16*s.* $6\frac{1}{4}$*d.* each; and $6005\frac{2}{11}$ things at 18*s.* $3\frac{3}{4}$*d.* each.
23. $6444\frac{13}{16}$ things at £10. 14*s.* $10\frac{1}{2}$*d.* each; and $5109\frac{9}{13}$ things at £4. 16*s.* $4\frac{1}{4}$*d.* each.
24. 43·35 things at 8*s.* $11\frac{1}{4}$*d.* each; and 147·625 things at 19*s.* $7\frac{1}{4}$*d.* each.
25. 1430 things at [illegible]$\frac{2}{3}$*d.* each; and 7866 things at 2*s.* $6\frac{1}{8}$*d.* each.
26. 7400 things at 4*s.* $9\frac{7}{12}$*d.* each; and 2841 things at 5*s.* $10\frac{3}{14}$*d.* each.
27. 6352 things at £1. 13*s.* $7\frac{9}{10}$*d.* each; and 2731 things at £4. 8*s.* $9\frac{2}{3}$*d.* each.
28. Find the value of 5 cwt., 2 qrs., 14 lbs. at £2. 5*s.* 6*d.* per cwt.
29. Find the value of 33 cwt., 3 qrs., 7 lbs. at £6. 7*s.* 8*d.* per cwt.
30. Find the value of 72 cwt., 3 qrs., 17 lbs. of sugar at £1. 4*s.* 6*d.* per cwt.
31. Find the value of 60 cwt., 3 qrs. 12 lbs. at £7. 13*s.* 6*d.* per cwt.
32. Find the value of 3 cwt., 2 qrs., 16 lbs. at £3. 7. 6*d.* per cwt.
33. Find the value of 9 yds., 2 ft., 10 in. at 5*s.* $7\frac{1}{2}$*d.* per yd.
34. Find the value of 39 cwt., 10 lbs. at £3. 15*s.* $7\frac{3}{4}$*d.* per cwt.
35. Find the cost of 30 cwt., 2 qrs., 14 lbs. at £1. 17*s.* $8\frac{1}{4}$*d.* per qr.
36. Find the value of 15 oz., 6 dwts., 17 grs. at 5*s.* 10*d.* per oz.
37. What is the rent of 23 ac., 3 ro., 5 po. at $2\frac{1}{2}$ guineas an acre?
38. What is the cost of 7 cwt., 1 qr., $15\frac{1}{2}$ lbs. at £2. 0*s.* 7*d.* per cwt.?
39. What is the rent of 225 ac., 1 ro., 19 po. at 13*s.* $2\frac{1}{2}$*d.* a rood?
40. Find the cost of 22 qrs., 4 bus., 3 pks. of wheat at 5*s.* $8\frac{1}{4}$*d.* per bus.
41. Find the cost of 2 tons, 15 cwt., 27 lbs. at £5. 11s. $7\frac{3}{5}$*d.* per cwt.
42. Find the cost of 48 sq. yds., 8 sq. ft., 114 sq. in. at 13*s.* $7\frac{3}{4}$*d.* per sq. yd.
43. Find the cost of 2 hhds., 1 bar. 5 qts. of beer at £2. 0*s.* 6*d.* a bar.

SQUARE AND CUBIC MEASURE.

CROSS MULTIPLICATION, DUODECIMALS.

137. **Definitions:**

(1) A **Parallelogram** is a four-sided figure, of which the opposite sides are parallel.

(2) A **Rectangle** is a right-angled parallelogram.

(3) The **Area** of a figure is the quantity of surface contained in it; and is estimated numerically by the number of times or parts of a time it contains a certain fixed area, which is assumed for its measuring unit.

(4) A **Solid** is that which hath length, breadth, and thickness.

(5) The **Capacity**, or **Volume** of a solid, is the quantity of space, comprehending length, breadth, and thickness, which it contains or takes up.

(6) The word *Content* is also frequently used to denote length, area, and capacity or volume; the length of a line being called its *linear content;* the area of a figure, its *superficial content;* and the capacity or volume of a body, or of a portion of space, comprehending length, breadth, and thickness, its *solid content.*

(7) A **Parallelopiped** is a solid contained by six quadrilateral figures, whereof every opposite two are parallel.

(8) A **Rectangular Parallelopiped** is one in which the several angles of the quadrilateral figures, which contain it, are right angles.

138. By reference to the Tables, Arts. (106, 107), and the observations upon them, we see that, in the sense there indicated, length multiplied by length produces area, and area multiplied by length produces capacity; the units in the products in these cases differing in kind from the units in the factors; thus, a rectangular area, whose adjacent sides are 4 and 3 feet respectively, is divisible into (4 × 3) or 12 equal squares, as shewn by the accompanying figure, the length of a side of each square being one linear foot. The rectangular area in this case is said to be the product of the two adjacent sides, represented respectively by numbers, the units in the numerical product being no longer linear feet, but square feet. Similarly, if the adjacent edges of a rectangular parallelopiped be 3, 4, and 2 feet, respectively, the capacity of the solid is equivalent to 24 cubes, each containing one cubic foot; and thus the capacity of the parallelopiped is correctly expressed by the product of the three adjacent edges represented respectively by numbers, the units in the numerical product being no longer linear feet, as in the factors, but cubic feet.

	1	2	3	4
1	1	2	3	4
2	5	6	7	8
3	9	10	11	12

Perhaps the readiest way of working examples in square and cubic measure is that of reducing all the different denominations to the same denomination; and proceeding as in the examples subjoined.

Ex. 1. Find the area of a rectangular court-yard 17 ft. 6 in. long, and 13 ft. 4 in. broad.

$$\begin{aligned}\text{The area} &= (17\text{ ft. }6\text{ in.}) \times (13\text{ ft. }4\text{ in.})\\ &= 17\tfrac{1}{2}\text{ ft.} \times 13\tfrac{1}{3}\text{ ft.}\\ &= \left(\frac{35}{2} \times \frac{40}{3}\right)\text{sq. ft.}\\ &= \frac{700}{3}\text{ sq. ft.}\\ &= 233\tfrac{1}{3}\text{ sq. ft.}\\ &= 25\text{ sq. yds., }8\text{ sq. ft., }48\text{ sq. in.}\end{aligned}$$

Ex. 2. Find the expense of paving a floor, whose length is 33 ft. 2 in. and breadth 18 ft., at 6*s*. per square yard.

$$\begin{aligned}\text{Area of floor} &= (33\text{ ft. }2\text{ in.}) \times 18\text{ ft.}\\ &= 33\tfrac{1}{6}\text{ ft.} \times 18\text{ ft.}\\ &= \left(\frac{199}{6} \times 18\right)\text{sq. ft.}\\ &= \left(\frac{199}{6} \times \frac{18}{9}\right)\text{sq. yds.}\\ &= \frac{199 \times 2}{6}\text{ sq. yds.}\end{aligned}$$

Therefore cost, which = cost per yard × number of yards

$$\begin{aligned}\text{is} &= \left(6 \times \frac{199 \times 2}{6}\right)s.\\ &= 398s.\\ &= £19.\ 18s.\end{aligned}$$

Ex. 3. How many square yards, feet, and inches will remain out of 400 square feet of carpeting, after covering the floor of a room 21 ft. 9 in. long and 16 ft. 11 in. broad?

$$\begin{aligned}\text{Area of the floor} &= (21\tfrac{3}{4} \times 16\tfrac{11}{12})\text{ sq. ft.}\\ &= \left(\frac{87}{4} \times \frac{203}{12}\right)\text{sq. ft.}\\ &= \frac{29 \times 203}{4 \times 4}\text{ sq. ft.} = \frac{5887}{16}\text{ sq. ft.}\end{aligned}$$

Therefore the number of square feet of carpet remaining after covering the floor $= 400 - \frac{5887}{16}$

$$=\frac{6400-5887}{16}$$
$$=\frac{513}{16}$$
$$=32 \text{ sq. ft., } 9 \text{ sq. in.}$$
$$=3 \text{ sq. yds., } 5 \text{ sq. ft., } 9 \text{ sq. in.}$$

Ex. 4. Find the capacity of a cube, of which each edge is 1 ft. 18 in.

$$\text{Capacity} = \text{length} \times \text{breadth} \times \text{height,}$$
$$=(1\tfrac{2}{3} \times 1\tfrac{2}{3} \times 1\tfrac{2}{3}) \text{ cub. ft.}$$
$$=\left(\frac{5}{3} \times \frac{5}{3} \times \frac{5}{3}\right) \text{cub. ft.}$$
$$=\frac{125}{27} \text{ cub. ft.}$$
$$=4 \text{ cub. ft., } 1088 \text{ cub. in.}$$

Ex. 5. A block of stone is 2 yds. 1 ft. 3 in. long, 1 ft. 7 in. broad, and 2 ft. thick; find its solid content, and its value at 2*s.* 3*d.* per cub. ft.

$$\text{Its content} = (2 \text{ yds. } 1 \text{ ft. } 3 \text{ in.}) \times (1 \text{ ft. } 7 \text{ in.}) \times 2 \text{ ft.}$$
$$=7\tfrac{1}{4} \text{ ft.} \times 1\tfrac{7}{12} \text{ ft.} \times 2 \text{ ft.}$$
$$=(7\tfrac{1}{4} \times 1\tfrac{7}{12} \times 2) \text{ cub. ft.}$$
$$=\left(\frac{29}{4} \times \frac{19}{12} \times 2\right) \text{cub. ft.}$$
$$=\frac{29 \times 19}{2 \times 12} \text{ cub. ft.} = 22 \text{ cub. ft., } 1656 \text{ cub. in.}$$
$$\text{Its value} = \left(2\tfrac{1}{4} \times \frac{29 \times 19}{2 \times 12}\right) s. = \frac{1653}{32} s. = \pounds 2.\, 11s.\, 7\tfrac{7}{8}d.$$

Note 1. Since linear feet multiplied by linear feet give square feet, it follows that square feet divided by linear feet give linear feet. Similarly, square yards divided by linear yards give linear yards, and so on. Again, since linear feet multiplied by square feet give cubic feet, it follows that cubic feet divided by linear feet give square feet, and cubic feet divided by square feet give linear feet.

Note 2. When surface alone is concerned, length and breadth, or length and height, or breadth and height, have to be multiplied together; but never length and breadth and height to be multiplied together; the latter only takes place where solid content is required.

Ex. 6. Find the expense of carpeting a room 15 ft. 9 in. long, and 12 ft. 5 in. broad, with carpet $\frac{3}{4}$ yd. wide, at 4*s.* per yard.

$$\text{Area of floor} = (15\tfrac{3}{4} \times 12\tfrac{5}{12})\text{ sq. ft.}$$
$$= \left(\frac{63}{4} \times \frac{149}{12} \times \frac{1}{9}\right)\text{sq. yds.}$$
$$= \left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}$$

It is clear that the required length of carpet in yards × given width of carpet in yards must give the whole area of floor in square yards;

$$\therefore\ \text{req}^{\text{d}}.\text{ length of carpet in linear yds.} \times \frac{3}{4}\text{ linear yd.} = \left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}$$

$$\therefore\ \frac{\text{req}^{\text{d}}.\text{ length of carpet in linear yds.} \times \frac{3}{4}\text{ linear yd.}}{\frac{3}{4}\text{ linear yd.}} = \frac{\left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}}{\frac{3}{4}\text{ linear yd.}},$$

$$\text{or, req}^{\text{d}}.\text{ length of carpet in linear yds.} = \frac{7}{4} \times \frac{149}{12} \times \frac{4}{3} = \frac{7 \times 149}{12 \times 3}.$$

$$\therefore\ \text{cost of carpet} = \left(4 \times \frac{7 \times 149}{12 \times 3}\right)s.$$
$$= £5.\ 15s.\ 10\tfrac{2}{3}d.$$

Ex. 7. What must be the length of a beam, the end of which is 18 sq. in., in order that its solid content may be the same as that of another beam, whose width, depth, and length are respectively 4 ft. 6 in., 3 ft. 9 in., and 12 ft. 10 in.?

Content of 1st beam = (length of beam in linear ft. × 1½ × 1½) cub. ft.

……………2nd…… = (4½ × 3¾ × 12⅚) cub. ft.

By the question,

$$\therefore\ (\text{length of beam in linear ft.} \times 1\tfrac{1}{2} \times 1\tfrac{1}{2})\text{ cub. ft.} = (4\tfrac{1}{2} \times 3\tfrac{3}{4} \times 12\tfrac{5}{6})\text{ cub. ft.};$$

$$\therefore\ \frac{\left(\text{length of beam in linear ft.} \times \frac{3}{2} \times \frac{3}{2}\right)\text{cub. ft.}}{\left(\frac{3}{2} \times \frac{3}{2}\right)\text{sq. ft.}} = \frac{\left(\frac{9}{2} \times \frac{15}{4} \times \frac{77}{6}\right)\text{cub. ft.}}{\left(\frac{3}{2} \times \frac{3}{2}\right)\text{sq. ft.}},$$

$$\text{or length of beam in linear ft.} = \frac{9}{2} \times \frac{15}{4} \times \frac{77}{6} \times \frac{2}{3} \times \frac{2}{3}$$
$$= \frac{385}{4} = 96\tfrac{1}{4} = 96\text{ ft. } 3\text{ in.}$$
$$= 32\text{ yds. } 0\text{ ft. } 3\text{ in.}$$

Ex. 8. Find the expense of painting the walls and ceiling of a room, whose height, length, and breadth are 17 ft. 6 in., 35 ft. 4 in., and 20 ft. respectively, at $7\frac{1}{2}d.$ per sq. yd.

The area of 2 of the walls = 2 length × height,
.... of the other 2 ... = 2 breadth × height,
.... of the ceiling = length × breadth.

Therefore whole area to be painted

$$= 2 \text{ height} \times \text{length} + 2 \text{ height} \times \text{breadth} + \text{length} \times \text{breadth}$$

$$= 2 \text{ height} \times (\text{length} + \text{breadth}) + \text{length} \times \text{breadth}$$

$$= 2 \times 17\tfrac{1}{2} \text{ ft.} \times (35\tfrac{1}{3} + 20) \text{ ft.} + (35\tfrac{1}{3} \times 20) \text{ sq. ft.}$$

$$= \left(2 \times \frac{35}{2} \times 55\tfrac{1}{3} + \frac{106}{3} \times 20\right) \text{sq. ft.}$$

$$= \frac{7930}{3} \text{ sq. ft.} = \left(\frac{7930}{3} \times \frac{1}{9}\right) \text{sq. yds.}$$

$$\text{therefore cost} = \left(\frac{15}{2} \times \frac{7930}{3} \times \frac{1}{9}\right) d.$$

$$= £9.\ 3s.\ 6\tfrac{7}{9}d.$$

Ex. 9. Let it be required to find the expense of papering the walls of the above room with paper $\frac{5}{8}$ yd. wide, at $13\frac{1}{2}d.$ per yd., there being three doorways in it, which each measure 7 ft. by $4\frac{1}{2}$ ft.

Now area of walls to be papered

$$= \left(2 \times 17\tfrac{1}{2} \times 55\tfrac{1}{3} - 3 \times 7 \times \frac{9}{2}\right) \text{sq. ft.}$$

$$= \frac{11053}{6} \text{ sq. ft.} = \left(\frac{11053}{6} \times \frac{1}{9}\right) \text{sq. yds.}$$

$$\therefore \text{no. of linear yds. of paper req}^{d}. \times \tfrac{5}{8} \text{ linear yd.} = \left(\frac{11053}{6} \times \frac{1}{9}\right) \text{sq. yds.}$$

$$\therefore \text{no. of linear yds. of paper req}^{d}. = \frac{11053}{6} \times \frac{1}{9} \times \frac{8}{5}.$$

$$\text{Therefore cost} = \left(\frac{27}{2} \times \frac{11053}{6} \times \frac{1}{9} \times \frac{8}{5}\right) d.$$

$$= £18.\ 8s.\ 5\tfrac{1}{5}d.$$

139. It may be well to note that each of the foregoing Examples in square and cubic measure might have been worked by bringing all the denominations into the *lowest* denomination, or by reducing the lower denominations into decimals of the highest involved.

There is however a method of working examples in square and cubic measure without reducing the different denominations to the same denomination. This method is styled CROSS MULTIPLICATION OR DUODECIMALS, and it is generally employed by painters, bricklayers, &c., in measuring work. They take the dimensions of their work in feet, inches, parts, &c., decreasing from the left to the right in a twelve-fold proportion; thus, 12 inches = 1 foot, 12 parts = 1 inch, &c.: the inches, parts, &c., are termed primes, seconds, thirds, &c., and are distinguished by the accents $'$, $''$, $'''$, &c. placed a little to the right above the numbers to which they belong.

The Rule for performing Cross Multiplication is the following:

Write the terms of the multiplier under the corresponding terms of the multiplicand. Multiply every term in the multiplicand, beginning at the lowest, by each term of the multiplier successively, beginning with the highest; divide each product which is not of the denomination of feet by 12, add the quotient to the next product, and place the remainder under the term of the multiplicand just used, when the denomination of the multiplier is feet, one place removed to the right when it is primes, two places when it is seconds, three when it is thirds, &c. Add the products together, carrying 1 for every 12, and the sum will be the answer.

Ex. 1. Multiply 4 ft. 7 in. by 9 ft. 6 in.

Proceeding by the Rule given above,

$$\begin{array}{rcccc}
\text{ft.} & & & & \\
4 & . & 7' & & \\
9 & . & 6' & & \\
\hline
41 & . & 3 & & \\
2 & . & 3 & . & 6'' \\
\hline
43 & . & 6 & . & 6''
\end{array}$$

which is the required product, and is = 43 square feet + $\frac{6}{12}$ths of a square foot (or 6 *superficial primes*, as they are called) + $\frac{6}{12}$ths of a superficial prime, *i.e.* $\frac{6}{144}$ths of a square foot (or 6 *superficial* seconds as they are called).

We may express this product in square feet and inches, thus:

$$\left(43+\frac{6}{12}+\frac{6}{144}\right)\text{ sq. ft.} = 43\text{ sq. ft.} + \left(\frac{6\times 12+6}{144}\right)\text{ sq. ft.}$$

$$= 43\text{ sq. ft.} + \frac{78}{144}\text{ sq. ft.}$$

$$= 43\text{ sq. ft. } 78\text{ sq. in.}$$

Reason for the above process.

$$9 \text{ ft.} \times 4 \text{ ft.} = 36 \text{ sq. ft.};$$

$$9 \text{ ft.} \times 7' = \left(9 \times \frac{7}{12}\right) \text{ sq. ft.} = \frac{63}{12} \text{ sq. ft.}$$

$$= \left(\frac{60+3}{12}\right) \text{ sq. ft.}$$

$$= 5 \text{ sq. ft.} + \frac{3}{12} \text{ sq. ft.}$$

$$= 5 \text{ sq. ft.} + 3 \text{ superficial primes.}$$

$$6' \times 4 \text{ ft.} = \left(\frac{6}{12} \times 4\right) \text{ sq. ft.} = \frac{24}{12} \text{ sq. ft.} = 2 \text{ sq. ft.};$$

$$6' \times 7' = \left(\frac{6}{12} \times \frac{7}{12}\right) \text{ sq. ft.} = \frac{42}{144} \text{ sq. ft.} = \left(\frac{36+6}{144}\right) \text{ sq. ft.} = \left(\frac{3}{12} + \frac{6}{144}\right) \text{ sq. ft.}$$

$$= 3 \text{ superficial primes} + 6 \text{ superficial seconds.}$$

Now 36 sq. ft. + 5 sq. ft. + 3 superficial primes + 2 sq. ft. + 3 superficial primes + 6 superficial seconds

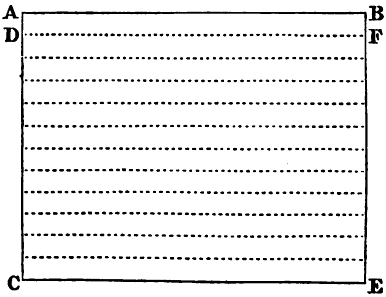
$$= 43 \text{ sq. ft.} + 6 \text{ sup. primes} + 6 \text{ sup. seconds}$$

$$= \left(43 + \frac{6}{12} + \frac{6}{144}\right) \text{ sq. ft.}$$

$$= \left(43 + \frac{12 \times 6 + 6}{144}\right) \text{ sq. ft.}$$

$$= 43 \text{ sq. ft. } 78 \text{ sq. in.}$$

Note. Attention to the accompanying geometrical figure may perhaps explain more clearly the result obtained by multiplying 9 ft. by 7 primes.



Take $AB = 9$ ft.,
$AC = 7$ ft.,
$AD = 7$ in.

Then 9 ft. × 7 ft., or rect. *AB*, *AC* = rectangular figure *ACEB*, which contains 63 sq. ft., and 9 ft. × 7 primes, or rect. *AB*, *AD* = rectangular figure *ADFB*, which is $\frac{1}{12}$th part of 63 sq. ft.

For since there are 12 lines in *AC*, each = *AD*, it follows that there are 12 rectangular figures, each = *ADFB*, in rectangular figure *ACEB*.

Ex. 2. Multiply 17 ft. 3 in. 6 pts. by 12 ft. 6 in. 3 pts.

$$\begin{array}{rrrrr}
\text{ft.} & & & & \\
17\,. & 3'\,. & 6'' & & \\
12\,. & 6\,. & 3 & & \\
\hline
207\,. & 6\,. & 0 & & \\
8\,. & 7\,. & 9''\,. & 0''' & \\
 & 4\,. & 3\,. & 10'''\,. & 6'''' \\
\hline
216\,. & 6\,. & 0\,. & 10'''\,. & 6''''
\end{array}$$

$$=216 \text{ sq. ft.}+\left(\frac{6}{12}+\frac{0}{144}+\frac{10}{12\times144}+\frac{6}{12\times12\times144}\right)\text{ sq. ft.}$$

$$=216 \text{ sq. ft.}+\left(\frac{72}{144}+\frac{10}{12\times144}+\frac{6}{12\times12\times144}\right)\text{ sq. ft.}$$

$$=216 \text{ sq. ft.}+72 \text{ sq. in.}+\frac{10}{12}\text{ sq. in.}+\frac{6}{144}\text{ sq. in.}$$

Ex. 3. Find by cross multiplication the capacity of a cube whose edge is 2 ft. 8 in.; and prove the truth of the result by vulgar fractions.

$$\begin{array}{rrrr}
\text{ft.} & & & \\
2\,. & 8' & & \\
2\,. & 8 & & \\
\hline
5\,. & 4 & & \\
1\,. & 9\,. & 4'' & \\
\hline
7\,. & 1\,. & 4'' & \\
2\,. & 8 & & \\
\hline
14\,. & 2\,. & 8 & \\
4\,. & 8\,. & 10\,. & 8''' \\
\hline
18\,. & 11\,. & 6\,. & 8
\end{array}$$

$$=18 \text{ cub. ft.}+\left(\frac{11}{12}+\frac{6}{144}+\frac{8}{1728}\right)\text{cub. ft.}$$

$$=18 \text{ cub. ft.}+\frac{1584+72+8}{1728}\text{ cub. ft.}$$

$$=18\tfrac{1664}{1728}\text{ cub. ft.}$$

$$=18 \text{ cub. ft. } 1664 \text{ cub. in.}$$

Proof by Vulgar Fractions.

$$\text{Content}=(2\tfrac{2}{3}\times2\tfrac{2}{3}\times2\tfrac{2}{3})\text{ cub. ft.}$$

$$=\left(\frac{8}{3}\times\frac{8}{3}\times\frac{8}{3}\right)\text{ cub. ft.}=\frac{512}{27}\text{ cub. ft.}$$

$$=18 \text{ cub. ft. } 1664 \text{ cub. in.}$$

140. In the examples of Cross Multiplication we see that a mixed decimal and duodecimal scale of notation is employed, the figures of the feet being expressed and multiplied in the ordinary way; whereas in other places the number 12 is always used instead of 10: Cross Multiplication is not, therefore, properly termed Duodecimal Multiplication or Duodecimals; because, although the different denominations are connected with each other by the number 12, still the different digits of those denominations are connected with each other by the number 10.

Ex. LIV.

1. Find the area of a rectangular board, whose sides are 2 ft. 9 in. and 10 ft. 4 in. respectively.
2. A room is 17 ft. 3 in. long, and 13ft. 10 in. broad; find the area of the floor in feet and inches.
3. Find the number of square feet and inches in a rectangular piece of ground 9 ft. 3 in., by 3 ft. 5 in.
4. The floor of a room, which is 15½ ft. wide, contains 91 sq. yards; find the length of the room.
5. A rectangular plot of ground 26 ft. broad contains 92 sq. yds. 4 sq. ft.; find its length.
6. Find the breadth of a room, whose length is 22½ ft. and whose area is 397½ ft.
7. How many planks 12 ft. 6 in. long and 8½ in. wide will floor a room 50 ft. by 16 ft.?
8. Find the area of a square building, whose side is 26 yd. 5 in.
9. An area, measuring 30 ft. 6 in. by 8 ft. 9 in., is to be paved; what will it cost at the rate of 4*s.* 8*d.* per sq. ft.?
10. Find the cost of a slab 5 ft. 7 in. long and 3 ft. 8 in. broad at 3*s.* per square foot.
11. Find the area of a floor which measures 18 ft. 6 in. by 12 ft. 3 in., and the expense of carpeting it at 3*s.* per square yard.
12. What will be the expense of painting the surfaces, which measure respectively as follows?
    (1) 23 ft. 6 in. by 20 ft., at 4*s.* 6*d.* per sq. yd.
    (2) 14 ft. 3 in. by 11 ft. 11 in., at 1*s.* 4*d.* per sq. ft.
    (3) 13 ft. 6 in. by 8 ft. 9 in., at 7*s.* 8*d.* per sq. yd.
13. Work by Cross Multiplication each of the following examples, and prove the truth of each result by Vulgar Factions.
    (1) 18 ft. 9 in. × by 14 ft. 7 in.

(2) 23 ft. 8 in. × by 16 ft. 9 in.
(3) 27 ft. 6′. 9″ × by 5 ft. 3′.
(4) 22 ft. 8¾ in. × by 16 ft. 7½ in.
(5) 4 ft. 6′. 5″ × by 9 ft. 4′. 7″.
(6) 75 ft. 7⅓ in. × by 38 ft. 3¼ in.
(7) 5 yds. 2 ft. 2 in. 3 pts. × by 5 yds. 11 in. 7 pts.

14. How many yards of carpet ½ yd. wide will cover a room 40 ft. 8 in. by 24 ft. 6 in.?

15. What length of paper ¾ of a yard wide will be required to cover a wall 15 ft. 8 in. long by 11 ft. 3 in. high?

16. Find the cost of a carpet ¾ yard wide at 3*s*. 9*d*. a yard for a room 20 feet by 18.

17. Find the expense of carpeting the following rooms:

(1) 12 ft. 4 in. long, and 12 ft. 6 in. broad, with carpet ¾ yd. wide, at 4*s*. 6*d*. a yard.
(2) 29½ ft. long, and 14¼ ft. broad, with carpet ⅝ yd. wide, at 3*s*. 6*d*. a yard.
(3) 15 ft. 6 in. long, and 12 ft. 9 in. broad, with carpet 24 in. wide at 7*s*. 8*d*. a yard.
(4) 26½ ft. long, and 18 ft. broad, with carpet ¾ yd. broad, at 3*s*. 4*d*. a yard.
(5) 19 ft. 7 in. long, and 18 ft. 11 in. broad, with carpet 25 in. broad, at 4*s*. 8*d*. a yard.

18. Find the content, and (when required) the cost, of the following:

(1) A piece of timber, whose length, breadth, and thickness are respectively 54½ ft.. 5 ft., and 2 ft. 5 in., at 9*d*. a solid foot.
(2) A cube, whose edge is 1 ft. 8 in., at 6*d*. a solid inch.
(3) Digging a cubical cellar, whose length is 12 ft., at 9*d*. a solid yard.
(4) A cistern 6 ft. deep, having a square bottom of which each side is 2½ ft.
(5) A wall 1000 ft. long, 10½ ft. high, and 2 ft. 1½ in. thick.
(6) A cube, whose edge is 13 ft. 7′. 7″.

19. Find the number of feet and inches in the floor, and the number of cubic feet and inches in the volume of a room 23 ft. 10 in. long, 18 ft. 4 in. broad, and 11 ft. 3 in. high.

20. Find the length of paper, ⅞ths of a yard wide, required to cover the walls of a room, whose length is 27 ft. 5 in., breadth 14 ft. 7 in., and height 12 ft. 10 in.

21. What would be the cost of painting the four walls of a room whose length is 24 ft. 3 in., breadth 15 ft. 8 in., and height 11 ft. 6 in., at 4*s.* a square foot?

22. Find the expense of painting the walls and ceilings of each of the first two, and the walls of each of the last two of the following rooms.

    (1) A room whose length is 16 ft. 8 in., breadth 15 ft. 9 in., and height 14 ft, at 1*s.* a sq. yd.

    (2) One whose length is 15 ft., breadth 10 ft., and height 9 ft. 9 in., at 1*s.* 4*d.* a sq. yd.

    (3) One whose circuit is 41½ ft., and height 8 ft. 5 in., at 11*d.* a sq. yd.

    (4) One whose circuit is 72 ft., and height 10¼ feet, at 10½*d.* a sq. yd.

    And find also the expense of papering the walls of the first two of the above rooms with paper 1 ft. 9 in. wide, at the following prices—the first at 3*s.* 6*d.* a yard, and the second at 1*s.* 2*d.* a yard.

23. The length, breadth, and height of a room are 7 yds. 1 ft. 3 in., 5 yd. 2 ft. 9 in., and 4 yds. 6 in., respectively. What length of paper two feet broad will be required to cover the walls, and what will it cost at 9*d.* per yard?

24. Supposing the cost of a carpet in a room 25 feet long, at 5*s.* a square yard, to be £6. 5*s.*, determine the breadth of the room.

25. In a rectangular court, which measures 96 ft. by 84 ft., there are four rectangular grass-plots, measuring each 22½ ft. by 18 ft.; find the cost of paving the remaining part of the court at 8½*d.* per square yard.

26. If a piece of cloth be 94½ yds. long, and 1⅛ yds. broad, how broad is a piece of the same content, whose length is 74⅔ yds.?

27. How many sq. ft. and sq. in. remain out of 313 sq. ft. of carpeting, after covering a room 16 ft. 9 in. by 12 ft. 11 in.? What is the price of the requisite carpeting at 3*s.* 6*d.* a yard?

28. On laying down a bowling-green with sods 2 ft. 6 in. long by 9 in. wide, it is found that it requires 75 sods to form one strip extending the whole length of the green, and that a man can lay down one strip and a quarter each day: find the space laid down in 8 days.

29. A piece of land, whose length is 151 yds. 1¼ ft., and breadth

35 yds., is to be exchanged for part of a strip of land of the same quality, whose breadth is 15 yds. $2\frac{1}{2}$ ft. Find the length of the equivalent strip.

30. Find the difference between the content of a floor 80 ft. 9 in. long and 65 ft. 6 in. broad, and the sum of the contents of three others, the dimensions of each of which are exactly one-third of those of the other.

31. A reservoir is 24 ft. 8 in. long, by 12 ft. 9 in. wide; how many cubic feet of water must be drawn off to make the surface sink 1 foot?

32. Divide 1532 ft. $9\frac{9}{12}$ in. by 81 ft. 9 in.: and find the breadth of a room, the length of which is $17\frac{1}{2}$ ft., and the area $250\frac{5}{6}$ ft.

33. How many sq. ft. of glazing are contained in the windows of a house of 4 stories, each story containing 12 windows, the breadth of each window being 3 ft. 6 in.; the height of the windows on the ground and first floors being $7\frac{1}{2}$ ft., on the second floor 6 ft. 10 in., and on the third floor 6 ft.? What will the cost be at 10*d.* a sq. ft.?

34. How many bricks will be required to build a wall 20 yds. long, $7\frac{1}{2}$ ft. high, and 14 in. deep; supposing a brick to be 9 in. long, $3\frac{1}{2}$ in. broad, and $2\frac{1}{2}$ in. deep?

35. How many tons of water are there in a cistern 18 ft. 8 in. long, 18 ft. 4 in. broad, and 6 ft. 9 in. deep, supposing a cubic foot of water to weigh 1000 oz.?

36. How many rods of brickwork are there in a wall 77 ft. long, 16 ft. high, and 1 ft. $10\frac{1}{2}$ in. thick?

37. Find the expense of painting the outside of a cubical iron chest, whose edge is 2 ft. 5 in., at 1*s.* 3*d.* per sq. yd.

38. What will the painting of a room cost which is $20\frac{1}{4}$ ft. long, $18\frac{1}{2}$ ft. broad, and 10 ft. high, containing 2 windows whose dimensions are 7 ft. by 4 ft. each, at the rate of 2*s.* 6*d.* per sq. yd.?

39. A piece of cloth 5 times as long as broad cost £19; supposing the price of cloth to be 4*s.* 9*d.* a square yard, find the dimensions of the piece.

40. What length must be cut off a straight plank $1\frac{1}{2}$ ft. broad, and $\frac{3}{4}$ ft. deep, in order that it may contain $11\frac{1}{4}$ cubic ft.?

41. A Turkey carpet, measuring 11 ft. 6 in. by 9 ft. 8 in., is laid down on the floor of a room measuring 14 ft. by 12 ft. 6 in.; determine the quantity of Brussels carpet, $\frac{3}{4}$ yd. wide, which will be

required to complete the covering of the area; what will be the cost of it at 3*s*. 9*d*. a yd.?

42. Shew by Cross Multiplication and by Vulgar Fractions how many cubic feet are contained in a beam 20 ft. 4 in. long, 1 ft. 5 in. broad, and 10 in. thick.

43. If 69 yds. of carpet. ¾ yd. wide, cover a room which is 10½ yds. long, find the width of the room.

44. If a postage stamp be an inch long and ⅘ths of an inch broad, how many stamps will be required for papering a room 16 ft. 10 in. long. 15 ft. 9 in. broad, and 12 ft. 6 in. high?

45. The length, width, and height of a room are respectively 36 ft., 24 ft., and 20 ft.; how many yards of painting are there in the walls of it, deducting for a fire-place 6 ft. by 5½ ft., and two windows, each 7½ ft. by 3¾ ft.?

What would it cost to paper the above room with paper 2¼ ft. wide, at 11*d*. a yard?

46. How many bricks, each 9 in. long, 4½ in. wide, and 3 in. thick, will be required for a wall 100 yds. long, 15 ft. high, and 1ft. 10½ in. thick?

47. A gentleman has a garden 200 ft. long and 180 ft. broad, and a gravel walk is to be made to run lengthways across it; how wide must the path be so as to take up ⅛th of the garden?

48. A wall is to be built 15 yds. long, 7 ft. high, and 13 in. thick, containing a doorway 6 ft. high, and 4 ft. wide. How many bricks will it require, the solid content of a brick being 108 cubic inches?

49. What would be the cost of paving a road of a uniform breadth of 4 yards extending round a rectangular piece of ground, the length of which is 85 yds. and breadth 56 yds., the cost of paving a square yard being 1*s*. 2*d*.?

50. How many paving-stones, each of them a foot long and $\frac{4}{15}$ of a foot wide, will be required for paving a street 45 ft. wide, surrounding a square, the side of which is 225 ft.?

51. What will be the expense of paving a rectangular court-yard, whose length is 126 ft. and breadth 98 ft., with pebbles, at 9*d*. per sq. yd.; and by how much will the expense be increased if a granite path, 5½ ft. wide, at 10*s*. 6*d*. per sq. yd., be laid down all round between the outside walls and the pebbles?

52 A gentleman wishes to raise his lawn (which is 1902 ft. long and 1020 ft. broad) 2 ft., and for that purpose digs a moat round it

17 yds. broad in every part; supposing the depth of the moat to be uniform, how deep must it be in order that he may have soil sufficient for his purpose?

53. Find the expense of lining a cistern, 10 ft. 3 in. long, 6 ft. 6 in. broad, and 5 ft. $4\frac{1}{2}$ in. deep, with lead, at £2. 2*s*. a cwt., which weighs 8 lbs. per sq. ft.

54. How many imperial gallons will a cistern contain whose length, depth, and breadth are 7 ft. 3 in., 3 ft. 8 in., and 2 ft. 10 in. respectively?

141. Examples which are usually classed under particular Rules, such as the Rule of Three, &c., can nevertheless be readily solved independently by means of the foregoing principles.

The following examples, which are worked out, are intended to exemplify various methods of reasoning. In the examples for practice which follow them, questions will be found the solution of which may be easily arrived at in a similar way: the number of such questions in this place must necessarily be very limited, and therefore the student is strongly recommended to apply to all questions which are hereafter classed under particular Rules, an independent method of solution, as well as the one denoted by the Rule to which they are respectively affixed.

Ex. 1. Express a degree in metres, 32 metres being = 35 yards.

$$35 \text{ yards} = 32 \text{ metres},$$

$$\therefore 1 \text{ yard} = \frac{32}{35} \text{ metres};$$

$$\therefore 1 \text{ degree} = (69\tfrac{1}{2} \times 1760) \text{ yards} = (139 \times 880) \text{ yards},$$

$$= \left(\frac{139 \times 880 \times 32}{35}\right) \text{ metres} = 111835\tfrac{3}{7} \text{ metres}.$$

Ex. 2. If $\frac{2}{3}$rds of a lottery ticket be worth £220, what is the value of $\frac{3}{11}$ths of the same?

$$\therefore \tfrac{2}{3}\text{rds of the ticket} = £220.$$

$$\therefore \tfrac{1}{3}\text{rd of the ticket} = £110.$$

$$\therefore \text{whole ticket} = £(110 \times 3) = £330.$$

$$\therefore \tfrac{3}{11}\text{ths of the ticket} = \tfrac{3}{11} \text{ of } £330 = £\frac{330 \times 3}{11} = £90.$$

Ex. 3. A person has $\frac{3}{7}$ths of an estate of 4000 acres left him; he sells $\frac{2}{3}$rds of his share: how many acres has he remaining, and what fraction of the whole estate will they be?

$$\text{He sells } \frac{2}{3} \text{ of } \frac{3}{7} \text{ of 4000 acres, or } \frac{2}{7} \text{ of 4000 acres,}$$

$$\therefore \text{ he has remaining } \left(\frac{3}{7} \text{ of } 4000 - \frac{2}{7} \text{ of } 4000\right) \text{ acres}$$

$$= \frac{1}{7} \text{ of 4000 acres} = 571\tfrac{3}{7} \text{ acres.}$$

Ex. 4. The sum of £463. 16*s*. is to be raised in a parish, the assessment of which is £6184.; what is the rate in the £.?

$$\text{£6184 produce £}463\tfrac{4}{5} \text{ or £}\frac{2319}{5},$$

$$\therefore \text{ £1. produces £}\left(\frac{2319}{5} \times \frac{1}{6184}\right), \text{ or } \left(\frac{2319}{5} \times \frac{1}{6184} \times 20\right)s.$$

$$\text{or } \frac{2319}{1546}s., \text{ or } 1s.\ 6d.$$

Ex. 5. After taking from my purse ⅔ of my money, I find that ¼ of what is then left amounts to 7*s*. 6*d*.; what money had I in my purse at first?

Let unity, or 1, denote the sum in the purse at first. After taking away ¼, ¾ remains. Now by the question

$$\frac{2}{3} \text{ of } \frac{3}{4} \text{ of unity, or } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of the sum in the purse at first} = 7s.\ 6d.$$

$$\text{or } \frac{1}{2} \text{ of the sum in the purse at first} = 7s.\ 6d.,$$

$$\therefore \text{ sum in the purse at first} = 15s.$$

Ex. 6. *A* met two beggars, *B* and *C*; and having $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of a moidore in his pocket, gave *B* $\frac{1}{7}$ of $\frac{3}{4}$ of that sum, and *C* $\frac{3}{5}$ of the remainder; what did each receive?

$$A \text{ had at first } \frac{\frac{40}{11}}{\frac{30}{7}} \text{ of } \frac{\frac{75}{7}}{\frac{15}{2}} \text{ of } \frac{77}{540} \text{ of } 27s., \text{ or } \frac{14}{3}s.$$

$$B \text{ received } \frac{1}{7} \text{ of } \frac{3}{4} \text{ of } \frac{14}{3}s., \text{ or } \frac{1}{2}s., \text{ or } 6d.$$

$$A \text{ had left afterwards } \left(\frac{14}{3} - \frac{1}{2}\right)s. = \frac{25}{6}s.,$$

$$\therefore C \text{ received } \frac{3}{5} \text{ of } \frac{25}{6}s., \text{ or } \frac{5}{2}s., \text{ or } 2s.\ 6d.$$

Ex. 7. A farmer pays a corn-rent of 5 quarters of wheat and 3 quarters of barley, Winchester measure: what is the money value of his rent, when wheat is at 60*s.*, and barley at 54*s.* per quarter, imperial measure; 32 imperial gallons being = 33 Winchester gallons?

Rent is 5 qrs. of wheat Win. mea. + 3 qrs. of barley Win. mea.

$$\text{But } 1 \text{ Win. gal.} = \frac{32}{33} \text{ imp. gal.},$$

$$\therefore\ 1 \text{ Win. qr.} = \frac{32}{33} \text{ imp. qr.}$$

$$\therefore\ \text{rent is } 5 \times \frac{32}{33} \text{ imp. qrs. of wheat} + 3 \times \frac{32}{33} \text{ imp. qrs. of barley},$$

$$\therefore\ \text{money value of rent} = \left(5 \times \frac{32}{33} \times 60 + 3 \times \frac{32}{33} \times 54\right) s. = £22.\ 8s.$$

Ex. 8. If £1. sterling be worth 25 francs, 60 centimes; and also worth 6 thalers, 20 silber groschen; how many francs and centimes is a thaler worth? (One thaler = 30 silber groschen, 1 franc = 100 centimes.)

$$6 \text{ thalers, } 20 \text{ silber groschen} = 25 \text{ francs, } 60 \text{ centimes},$$

$$\text{or } 6\tfrac{20}{30} \text{ thalers} = 25\tfrac{60}{100} \text{ francs},$$

$$\therefore\ 1 \text{ thaler} = (25\tfrac{3}{5} \div 6\tfrac{2}{3}) \text{ francs}$$

$$= \frac{384}{100} \text{ francs} = 3 \text{ francs, } 84 \text{ centimes.}$$

Ex. 9. Standard gold contains 12 parts of pure gold to one part of copper, and 20 lbs. Troy are coined into 934 sovereigns and a half-sovereign; find the weight of pure gold in a sovereign.

Number of parts $= 12 + 1 = 13$, of which $\frac{12}{13}$ is pure gold.

By the question,

$$934\tfrac{1}{2} \text{ sovereigns weigh } 20 \text{ lbs. Troy},$$

$$\therefore\ 1 \text{ sov. weighs } \frac{20 \times 2}{1869} \text{ lbs. Troy},$$

$$\therefore\ \text{weight of pure gold in a sov.} = \left(\frac{12}{13} \times \frac{20 \times 2}{1869}\right) \text{lb. Troy}$$

$$= 113\tfrac{6413}{8099} \text{ grs.}$$

Ex. 10. If a person, travelling 13¾ hours a day, perform a journey in 27⅝ days, in what length of time will he perform the same if he travel $10\frac{2}{7}$ hours a day?

If he travel $13\frac{3}{4}$ hrs. a day, he does the journey in $27\frac{5}{8}$ days,
.... 1 hr. ...., ... $(27\frac{5}{8}\times 13\frac{3}{4})$ days,
.... $10\frac{2}{7}$ hrs. ......, ... $\dfrac{27\frac{5}{8}\times 13\frac{3}{4}}{10\frac{2}{7}}$ days,
which, worked out, gives $36\frac{2141}{2304}$ days.

Ex. 11. If 858 men in 6 months consume 234 quarters of wheat, how many quarters will be required for the consumption of 979 men for three months and a half?

858 men in 6 months consume 234 quarters,

$\therefore$ 1 man in 1 month consumes $\dfrac{234}{858\times 6}$ qrs.,

$\therefore$ 979 men in 1 month consume $\dfrac{979\times 234}{858\times 6}$ qrs.,

$\therefore$ 979 men in $3\frac{1}{2}$ months consume $\left(\dfrac{979\times 234}{858\times 6}\times\dfrac{7}{2}\right)$ qrs., or $155\frac{3}{4}$ qrs.

Ex. 12. If 5 men or 7 women can do a piece of work in 37 days; in what time will 7 men and 5 women do a piece of work twice as great?

$$5 \text{ men} = 7 \text{ women},$$

$$\therefore 1 \text{ man} = \frac{7}{5} \text{ woman},$$

$$\therefore 7 \text{ men} = \frac{49}{5} \text{ women},$$

$$\therefore 7 \text{ men and } 5 \text{ women} = \left(\frac{49}{5}+5\right) \text{women} = \frac{74}{5} \text{ women}.$$

Now by the question,

7 women in 37 days do the piece of work,

$\therefore$ 1 woman in (37×7) days does ....

$\therefore$ 74 women in $\dfrac{37\times 7}{74}$ days do.....................

$\therefore$ $\dfrac{74}{5}$ women in $\dfrac{37\times 7\times 5}{74}$ days do ....

$\therefore$ $\dfrac{74}{5}$ women in $\dfrac{37\times 7\times 5\times 2}{74}$ or in 35 days do twice as much.

Ex. 13. A bankrupt owes three creditors *A*, *B*, and *C* £250, £330, and 400 guineas respectively, and his property is worth £125; how much will each creditor receive, and how many shillings in the pound?

Debts amount to £(250 + 330 + 420), or £1000.

If the bankrupt has £1, he pays $\frac{1}{1000}$ part of debt,

.... £125......... $\frac{125}{1000}$ part of debt,

.... $\frac{1}{8}$ part of debt.

∴ *A* gets £31. 5*s*., *B* gets £41. 5*s*., and *C* gets £52. 10*s*. He pays $\frac{1}{8}$ of £1, or 2*s*. 6*d*., in the £.

Ex. 14. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required for 16 cwt. of powder.

The whole number of parts $= (15 + 3 + 2) = 20$.

Of every 20 parts,

$\frac{15}{20}$ or $\frac{3}{4}$ is nitre, $\frac{3}{20}$ is charcoal, $\frac{2}{20}$ or $\frac{1}{10}$ is sulphur.

∴ $\frac{3}{4}$ of 16 cwt., or 12 cwt. = quantity of nitre required.

$\frac{3}{20}$ of 16 cwt., or $2\frac{2}{5}$ cwt. = ...charcoal ....

$\frac{1}{10}$ of 16 cwt., or $1\frac{3}{5}$ cwt. = ... sulphur ....

Ex. 15. The price of a work which comes out in parts is £2. 16*s*. 8*d*. But if the price of each part were 13*d*. more than it is, the price of the work would be £3. 7*s*. 6*d*. How many parts are there?

£2. 16*s*. 8*d*. + (number of parts × 13)*d*. = £3. 7*s*. 6*d*.

∴ (number of parts × 13)*d*. = 10*s*. 10*d*.

= 130*d*.

∴ number of parts $= \frac{130}{13} = 10$.

Ex. 16. Divide 1860 guineas between *A*, *B*, and *C*, so that as often as *A* gets £5., *B* shall get £4., and as often as *B* gets £3., *C* shall get £1.

It is clear that *B*'s share = 3 times *C*'s share,

4 times *A*'s share = 5 times *B*'s share,

or, *A*'s share $= \frac{5}{4}$ times *B*'s share,

$= \left(\frac{5}{4} \times 3\right)$ times *C*'s share,

but *A*'s share + *B*'s share + *C*'s share = 1860 guineas;

∴ $\frac{15}{4}$ *C*'s share + 3 *C*'s share + *C*'s share = 1860 guineas,

$$\text{or } \left(\frac{15}{4}+4\right) C\text{'s share} = 1860 \text{ guineas,}$$

$$\text{or } \frac{31}{4}\, C\text{'s share} = 1860 \text{ guineas;}$$

$$\therefore\ C\text{'s share} = \left(\frac{1860}{1} \times \frac{4}{31}\right) \text{guineas} = 240 \text{ guineas,}$$

B's share = 720 guineas, and *A*'s share $= \left(240 \times \frac{15}{4}\right)$ guineas = 900 guineas.

Ex. 17. Of a certain dynasty, $\frac{1}{3}$ of the kings are of the same name, $\frac{1}{4}$ of another, $\frac{1}{8}$ of a third, and $\frac{1}{12}$ of a fourth, and there are 5 besides: how many are there of each name?

Representing the whole dynasty by unity, or 1.

$\frac{1}{3}$ = number of kings of one name,

$\frac{1}{4}$ = ...of a second,

$\frac{1}{8}$ = ...of a third,

$\frac{1}{12}$ = ...of a fourth.

Now $\frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{12}=\frac{19}{24}$,

∴ whole dynasty $-\frac{19}{24}$, or $1-\frac{19}{24}$, or $\frac{5}{24}$ = no. of remaining kings in it.

But by the question,

$$\frac{5}{24} \text{ of unity, or } \frac{5}{24} \text{ of the whole dynasty} = 5;$$

$$\therefore 1, \text{ or the whole dynasty, } = 5 \times \frac{24}{5} = 24;$$

∴ there are 8 kings of the 1st name, 6 of the 2nd, 3 of the 3rd, and 2 of the 4th.

Ex. 18. *A* can do a piece of work in 5 days, *B* can do it in 6 days, and *C* can do it in 7 days; in what time will *A*, *B*, and *C*, all working at it, finish the work? Find also in what time *A* and *B* working together, *A* and *C* together, and *B* and *C* together, could respectively finish it.

Representing the work by unity, or 1.

In one day *A* does $\frac{1}{5}$ part of the work, In one day *B* does $\frac{1}{6}$ ....,

...............*C* does $\frac{1}{7}$ ....;

$\therefore$ ....$A+B+C$ do $\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)$, or $\frac{107}{210}$ part;

$\therefore$ time in which $A+B+C$ would finish the work

$$= \frac{1}{\frac{107}{210}} \text{ days} = \frac{210}{107} \text{ days} = 1\tfrac{103}{107} \text{ days.}$$

Again, in one day $A+B$ do $\left(\frac{1}{5}+\frac{1}{6}\right)$, or $\frac{11}{30}$, of the work; therefore time in which they would finish it $=\frac{1}{\frac{11}{30}}$ or $2\tfrac{8}{11}$ days.

In like manner, it may be shewn that *A* and *C* would finish the work in $2\tfrac{11}{12}$ days; and *B* and *C* in $3\tfrac{3}{13}$ days.

Ex. 19. It being given that *A* and *B* can do a piece of work in $2\tfrac{8}{11}$ days; that *A* and *C* can do the same in $2\tfrac{11}{12}$ days; and that *B* and *C* can do it in $3\tfrac{3}{13}$ days: find the time in which *A*, *B*, and *C* would do the work: working, first, all together, secondly, separately.

In one day *A* and *B* do $\frac{11}{30}$ of the work,

.... *A* and *C* do $\frac{12}{35}$ ....,

.... *B* and *C* do $\frac{13}{42}$ ....

$\therefore$ by addition,

In one day $2A+2B+2C$ would do $\left(\frac{11}{30}+\frac{12}{35}+\frac{13}{42}\right)$, or $\frac{214}{210}$, of the work,

$\therefore$ in one day $A+B+C$ do $\frac{107}{210}$ ....

$$\therefore \text{ time required} = \frac{1}{\frac{107}{210}} = \frac{210}{107} \text{ days} = 1\tfrac{103}{107} \text{ days.}$$

Again,

work done by $A+B+C$ in one day $-$ work done by $B+C$ in one day,

or, work done by *A* in one day $=\frac{107}{210}-\frac{13}{42}=\frac{1}{5}$;

therefore time required, in which *A* would do the work, $=5$ days.

In like manner it may be shewn, that *B* would do the work in 6 days, and that *C* would do it in 7 days.

Ex. 20. A cistern is fed by a spout which can fill it in 2 hours, how long would it take to fill it if the cistern has a leak which would empty it in 10 hours?

In one hour spout fills $\frac{1}{2}$ of the cistern,

…………leak empties $\frac{1}{10}$ ……………

Therefore in one hour, when the spout and leak are both open, the part of the cistern filled by what runs in − what runs out,

$$= \left(\frac{1}{2} - \frac{1}{10}\right) = \frac{2}{5},$$

$$\therefore \text{ time required for filling the cistern} = \frac{\frac{1}{2}}{\frac{2}{5}} \text{ hrs.} = \frac{5}{2} \text{ hrs.} = 2\tfrac{1}{2} \text{ hrs.}$$

Ex. 21. *A* can perform a certain quantity of work in 5 days, *B* twice as much in 6 days, and *C* 4 times as much in 9 days; in what time can *A*, *B*, and *C* working together, perform a piece of work 11 times as great?

In one day *A* does $\frac{1}{5}$ of the work,

…………… *B* does $\frac{2}{6}$ or $\frac{1}{3}$ ………………

…………… *C* does $\frac{4}{9}$ ………………;

$\therefore$ in one day $A+B+C$ do $\left(\frac{1}{5} + \frac{1}{3} + \frac{4}{9}\right)$ or $\frac{44}{45}$ of the work,

$\therefore$ they would finish this piece of work in $\frac{45}{44}$ days,

$\therefore$ they would finish required piece of work in $\left(\frac{45}{44} \times 11\right)$ or $11\frac{1}{4}$ days.

Ex. 22. *A* and *B* can do a piece of work in 15 and 18 days respectively; they work together at it for 3 days, when *B* leaves, but *A* continues, and after 3 days is joined by *C*, and they finish it together in 4 days; in what time would *C* do the piece of work by himself?

Representing the work by unity, or 1.

$$\text{In one day } A+B \text{ do } \left(\frac{1}{15}+\frac{1}{18}\right) \text{ of the work,}$$

$$\text{in 3 days they do } \left(\frac{1}{15}+\frac{1}{18}\right)\times 3 \text{ ....}$$

$$\text{or } \frac{11}{30}\text{..............}$$

$$\therefore \frac{19}{30} \text{ of the work remains to be done.}$$

In 3 days more A does $\frac{3}{15}$ or $\frac{1}{5}$ of the work;

$\therefore$ when A is joined by C,

$$\frac{19}{30}-\frac{1}{5}, \text{ or } \frac{13}{30} \text{ of the work remains to be done.}$$

In 4 days more A does $\frac{4}{15}$ of the work;

$\therefore$ work which has to be done by C in 4 days

$$=\frac{13}{30}-\frac{4}{15}=\frac{5}{30}=\frac{1}{6};$$

$$\therefore \text{ part of work to be done by } C \text{ in one day} = \frac{1}{24},$$

$\therefore$ time in which C would do the whole work $=24$ days.

Ex. LV.

Miscellaneous Questions and Examples on preceding Arts.

I.

1. State the rules for the multiplication and division of decimals, and divide 34·17 by $3\frac{1}{4}$.

2. What is the value in English money of 1556·85 francs, when the exchange is at 24·25 francs per £?

3. Reduce $\frac{1}{3}+\frac{1}{6}+\frac{1}{14}+\frac{3}{56}$ to a decimal fraction. What decimal of a cwt. is 1 qr. 7 lbs.?

4. Explain the principle of the Rule of Practice. Find (by Practice) the cost of $365\frac{1}{2}$ tons of coals at 13*s.* $5\frac{3}{4}$*d.* a ton; and the rent of 315 ac., 3 ro., 7 po. at £1. 16*s.* 8*d.* an acre.

5. If $\frac{5}{7}$ of an estate be worth £1003. 17*s.* 1*d.*, what is the value of $\frac{2}{5}$ of it?

6. If a bankrupt pay 3*s.* 4*d.* in the pound, what will be received on a debt of £3678. 16*s.*?

7. A person possessing $\frac{3}{14}$ of an estate, sold $\frac{2}{7}$ of $\frac{1}{3\frac{1}{5}}$ of his share for £120$\frac{5}{8}$; what would $\frac{1}{5}$ of $\frac{3}{16}$ of the estate sell for at the same rate?

8. A man, his wife, and 3 children earn £1. 7*s.* 6*d.* a week; the wife earns twice as much as each child, and the man three times as much as his wife; required the man's weekly earnings.

9. If £1. sterling be worth 12 florins, and also worth 25 francs, 56 centimes; how many francs and centimes is one florin worth? (100 centimes = 1 franc.)

10. The wages of 5 men for 6 weeks being £14. 5*s.*, how many weeks will 4 men work for £19?

II.

1. What is meant by saying that one sum is a certain fraction (for example $\frac{2}{5}$) of another? If 26 francs are equivalent to a pound, what fraction of a shilling is a franc? Give the reasons for the process which you adopt in answering the question.

2. Express $\frac{2}{3}$ of 1$\frac{5}{9}$ of a mile in terms of a metre, supposing 32 metres = 35 yards.

3. *A, B,* and *C* rent a pasture for £40. *A* puts in 8 cattle, *B*, 9, and *C*, 11: how much should each pay for his share?

4. Reduce 3$\frac{3}{4}$*d.* to the decimal of 10*s.*, and divide the result by 12·5. Explain the process employed.

5. Find the value of 45 ac., 3 ro., 20 po. at £111. 11*s.* 4*d.* per acre, by Practice.

6. If the property in a town be assessed at £60000, what must be the rate in the £ in order that £2500 may be raised?

7. If the circumference of a circle = Diameter × 3·14159; find the number of revolutions passed over by a carriage-wheel 5 ft. in diameter in 10 miles.

8. A farmer has to pay yearly to his landlord the price of 7$\frac{1}{6}$ bushels of wheat at 4*s.* 9*d.* per bushel, and 9$\frac{1}{4}$ of malt at 5*s.* 3*d.*, and 6$\frac{2}{9}$ of oats at 2*s.* 4*d.* What is the whole amount of his rent?

If there were a decimal coinage of pounds, florins, &c., how many of them would he have to pay?

9. *A* alone can do a piece of work in 10 hours, and *B* can do it in 12 hours, find the time in which both working together can do it.

10. Ten excavators dig 12 loads of earth in 16 hours, whilst 12 others

can dig only 9 loads in 15 hours; in what time will they jointly dig 100 loads?

III.

1. Divide 28 tons, 4 cwt., 3 qrs. into 36 equal portions; and find the value of one of them at £7. 10*s.* 8*d.* per cwt.

2. Reduce 186 yds., 2 ft., 8$\frac{1}{25}$ in. to the decimal of a chain. If one chain = 10 chainlets = 100 links = 1000 linklets; express the above in chains, chainlets, links, linklets.

3. If £1. sterling be worth 45 Pauls, 9 Baiocchi (Roman), and be worth 25½ francs (French;) shew that a Napoleon of 20 francs = 36 Pauls. (10 Baiocchi = 1 Paul.)

4. If the rents of a parish amount to £2514. 7*s.* 6*d.* and a rate is granted of £83. 16*s.* 3*d.*, how much is that in the £? and how much must be paid by an estate whose rental is £115. 12*s.* 6*d.*?

5. If a tradesman with a capital of £1000. gains £90. in 7 months, in what time will he gain £20. 5*s.* with a capital of £315?

6. What is the difference between simple and compound Practice?

Required the price of 3 cwt., 3 qrs., 16 lbs. at £4. 6*s.* 0½*d.* per quarter, by Practice.

7. Determine the expence of papering a room 12 ft. high, measuring 20 ft. by 15 ft., at the rate of 2½*d.* per square yard.

8. In the civil year 97 days are intercalated in 400 years; what is the average length of the year?

9. If 15 horses and 148 sheep can be kept 9 days for £75. 15*s.*, what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses to eat as much as 84 sheep?

10. *A*, *B*, and *C* are three workmen: *A* can do half a piece of work in 3 hours, being twice as much as *B* can do; and *A*, *B*, and *C* can together do the whole in 2½ hours. Shew that *C* can do in 5 hours as much as *B* can do in 9 hours.

IV.

1. Explain how whole numbers are represented in the decimal or common system of notation. Multiply 729 by 37, and explain the process.

2. Add together the fifth of a shilling, two-sevenths of a crown, and four-ninths of a guinea; and reduce the result to the decimal of £25.

3. Two persons gained in trade £375; one having put in £500 and the other £850; what part of the profit ought each person to receive?

4. Taking the circumference of a circle at $3\frac{1}{7}$ times its diameter; find the cost of a marble column of two feet breadth, and five yards height, marble being at 15*s*. 6*d*. per cub. ft. (Area of circle $= \frac{1}{2}$ circumference × semi-diameter.)

5. If a certain number of men can throw up an entrenchment in 12 days, when the day is 6 hours long, in what time will they do it when the day is 8 hours long?

6. Find the entire cost of 10 lbs. of tea at 4*s*. 3*d*. per lb., 18 lbs. of coffee at 1*s*. $3\frac{1}{2}$*d*. per lb., 23 lbs of sugar at $4\frac{1}{4}$*d*. per lb., and 16lbs. of candles at $7\frac{3}{4}$*d*. per lb., and divide the amount equally among 14 persons.

7. Reduce $2375\frac{3}{4}$ Spanish dollars to English money, the exchange being at 3*s*. 4*d*. per dollar. And find the value of 1,000,000 rupees at 2*s*. $3\frac{7}{8}$*d*. each.

8. The roller used for rolling a bowling-green, being 6 ft. 6 in. in circumference, by 2 ft. 3 in. wide, is observed to make 12 revolutions as it rolls from one extremity of the green to the other; find the area rolled when the roller has passed 10 times the whole length of it.

9. Divide £1400. among *A*, *B*, and *C*, in such a manner that as often as *A* gets £5, *B* shall get £4, and as often as *B* gets £3, *C* shall get £2.

10. A fraudulent wine-merchant sells, as brandy, a mixture of brandy and rum at £2. 5*s*. a gallon, which is the proper price of his brandy, that of his rum being a guinea a gallon. Supposing one-third of the whole mixture to be rum, ascertain how much a gallon he gains by his dishonesty.

V.

1. Divide 550974 by 1472; find the quotient and remainder. Explain the operation, and prove the result.

2. Shew that the value of a fraction is not altered by multiplying the numerator and denominator by the same number.

Express the fractions $\frac{6}{45}$, $\frac{5}{7}$, and $\frac{9}{35}$ by corresponding fractions having the same denominator, and find the sum.

3. If 1lb. Avoirdupois be equivalent to 7000 grains Troy, and 1869 sovereigns weigh 40 lbs. Troy, how many sovereigns will weigh 1 Avoirdupois ounce?

4. A quarter of wheat is consumed annually by each person in England; if wheat be at 45*s*. a quarter, and the population 27,500,000, what is the value of a quarter of a year's consumption?

5. A certain number of men mow 4 acres in 3 hours; and a certain number of others mow 8 acres in 5 hours; how long will they be mowing 11 acres, if all work together?

6. The depth of water in a cistern whose base contains 1344 sq. in. is 3 ft. 9 in. Find (1) the number of cub. ft. of water in it, and (2) the depth of the same quantity of water in another cistern whose base contains 1088 square inches.

7. If a man can do a piece of work in 8½ days by working 6 hours a day, how many hours a day must he work to finish it in 5 days?

8. If 7 men or 11 women can finish a piece of work in 17 days, how many days will it take 11 men and 7 women to finish it?

9. A room is 20 ft. 6 in. long by 15 ft. 6 in. wide, and 16 ft. high; it has two doors, each 8 ft. high by 3 ft. 9 in. wide, and 3 windows, one 5 ft. by 7 ft., the other two 5 ft. by 4 ft. each. What will it cost to paper the room with paper a yard wide at 10*d.* a-yard?

10. A bankrupt owes *A* £515. 12*s.* 6*d.*, *B* £407, and *C* £293. 6*s.* 8*d.*; his estate is worth £911. 19*s.* 4½*d.*; how much can be paid in the £, and what will *A*, *B* and *C* each receive?

VI.

1. Multiply £10. 17*s.* 6¾*d.* by 8784; and find by Practice the value of 8764 things at £10. 17*s.* 6¾*d.* each.

2. A bankrupt's assets amounted to £542. 6*s.* and his creditors received 11*s.* in the pound: find the amount of his debts.

3. A piece of cloth, when measured with a yard measure which is two-thirds of an inch too short, appears to be 10½ yards long, what is its true length?

4. How many francs must be transmitted from Paris to Berlin to discharge a debt of 420 thalers? a thaler being equivalent to 3 shillings, and 24 francs to one pound sterling.

5. Estimate the cost of a dish of almonds and raisins consisting of six ounces of almonds and three quarters of a pound of raisins: supposing almonds to be ten pence, and raisins eleven pence a pound.

6. If 5 cwt., 3 qrs., 14 lbs. cost £6. per cwt., what will be the cost per pound when the cost of the whole has been reduced by £7. 16*s.* 8*d.*?

7. A grocer buys 10 cwt. 3 qrs. 21 lbs. of sugar for £30., and pays 12*s.* 6*d.* for expenses; at what rate must he sell it per pound to clear £15. 6*s.* 3*d.* by his bargain?

8. Explain the difference between Cross Multiplication and Duodecimals.

Find the cost of papering a room 20 ft. long, 16½ ft. broad, and 12 ft. high, the price of a piece of paper 12 yds. long and 3 qrs. broad being 4*s.* 6*d.*

9. If a snail, on the average, creep 2 ft. 7 in. up a pole during

12 hrs. in the night, and slip down 16 in. during the 12 hrs. in the day, how many hours will he be in getting to the top of a pole 35 ft. high?

10. The profits of a tradesman average £54. 6*s*. 5*d*. per week, out of which he pays 3 foremen, 10 shopmen, and 5 assistants, at the rate of 2 guineas, 1 guinea, and 17*s*. 6*d*. per week respectively: His yearly outgoings for rent, &c. amount to £723. 11*s*. 8*d*. Find his net annual profit.

VII.

1. What is meant by a fraction? Find the value of $\frac{2}{3}$ of $\frac{1}{4}$ of 13 guineas; and then express the result as the fraction and decimal of £237. 10*s*.

2. By what number must £5. 6*s*. 3¼*d*. be multiplied in order to give as product £85. 0*s*. 4*d*.? Divide £34. 13*s*. into 3 parts one of which shall be twice and the other 4 times as great as the third.

3. If a year consist of 365·242264 days, in how many years will its defect from the civil year of 365¼ days amount to 1 day?

4. If 15 men take 17 days to mow 300 acres of grass, how long will 27 men take to mow 167 acres?

5. If 20 men can perform a piece of work in 12 days, how many men will accomplish another piece of work, which is six times as great, in a tenth part of the time?

6. I am owner of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{1}{3}$ of a ship worth £5161. 3*s*. 9*d*., and sell $\frac{1}{6}$th of the ship; what part of her will then belong to me, and what will it be worth?

7. A bankrupt owes £900. to his three creditors; and his whole property amounts to £675; the claims of two of his creditors are £125. and £375. respectively; what sum will the remaining creditor receive for his dividend?

8. Shew that a cistern 13 ft. 4 in. long, 8 ft. broad, and 5 ft. 3 in. deep holds just twice as much water as another which is 7 ft. long, 6 ft. 8 in. broad, and 6 ft. deep.

9. There are in a manufactory a certain number of workmen who receive 60*s*. a-week, twice as many who receive 31*s*. 6*d*. a-week, and eleven times as many who receive 14*s*. a week, and the total amount of the workmen's wages for one week is £96. 19*s*.; find the number of workmen.

10. Reduce £405. 6*s*. 8*d*. to francs and centimes, at the rate of 25¼ francs to £1, and 100 centimes to a franc.

VIII.

1. Find the value at £3. 5*s*. per oz. of 13 lbs., 9 oz., 3 dwts. of gold *dust*.

2. If a florin be made the unit of money, what number will represent £1. 11*s*. $6\frac{1}{4}$*d.*?

3. If £1. be worth 12 guldens, and one penny 3 kreutzers, what fraction of one gulden is 5 kreutzers?

4. A creditor receives on a debt of £296. a dividend of 12*s*. 4*d*. in the pound, and he receives a further dividend, upon the deficiency, of 3*s*. 9*d*. in the pound; what does the creditor receive in the whole?

5. Reduce 12 ft. $4\frac{1}{2}$ in. to the fraction of a mile, and find the corresponding decimal.

6. If 29,040 copies of a paper be printed, each copy consisting of 3 sheets, and each sheet being $3\frac{1}{2}$ feet long, by 2 feet broad; how many acres will one edition cover?

7. A man has an income of £200 a year; an income-tax is established of 7*d*. in the pound, while a duty of $1\frac{1}{2}$*d*. per lb. is taken off sugar; what must be his yearly consumption of sugar that he may just save his income-tax?

8. If A can do as much work in 5 hours as B can do in 6 hours, or as C can do in 9 hours, how long will it take C to complete a piece of work, one-half of which has been done by A working 12 hours and B working 24 hours?

9. Find the number of shillings and pence which are equivalent to the recurring decimal ·3333...... of a pound.

10. The gross earnings of an undertaking average £3000, and the expenses £775. 14*s*. 2*d*. per week, one tenth of the remainder is put aside for wear and tear, and the annual charges amount to £24141. 13*s*. 8*d*. What is the net annual profit? (1 year = 52 weeks.)

IX.

1. Explain the process of Long Division.

Reduce $\frac{9275}{371} + \frac{1522842}{719}$ to its equivalent whole number.

2. Shew how to convert any proper fraction into a decimal.

Reduce $\frac{2}{5}$ and $\frac{75}{1875}$ to the decimal form.

3. State what kind of vulgar fractions can be expressed in finite decimals. Can the quantity $\frac{1}{3} - \frac{1}{6} - \frac{1}{24}$ be so expressed?

How many shillings should be given in exchange for $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$ of a pound?

4. If two-thirds of an academic term exceed one-half of it by $13\frac{1}{3}$ days, how many days are there in the whole term?

5. In a decimal coinage of pounds, florins, &c., how many of these may be obtained for £19. 17*s*. $6\frac{1}{2}$*d*.? How much is lost by the exchange?

6. The length of a rectangular field which contains 7 acres, 1 rood, 15 poles, is 453 yds., 2 ft., 3 in.; find its breadth.

7. A butler concocts a bowl of punch, of which the following are the ingredients: milk $2\frac{1}{6}$ quarts, the rind of one lemon, 2 eggs, 1 pint of rum, and half-a-pint of brandy. Compute the value of the punch, reckoning milk at *3d.* a quart, lemons at *2s.* a dozen, eggs at 16 a shilling, rum at 13*s.* per gallon, and brandy at £1. 4*s.* 8*d.* per gallon.

8. A Cochin China hen eats a pint of barley and lays a dozen eggs, while an English hen eats half-a-pint of barley and lays five eggs. Supposing the eggs of the English hen to be half as large again as those of the Cochin China, which is the more economical layer?

9. If 72 men dig a trench 20 yds. long, 1 ft. 6 in. broad, 4 feet deep, in 3 days of 10 hours each, how many men would be required to dig a trench 30 yards long, 2 ft. 3 in. broad, and 5 feet deep, in 15 days of 9 hours each?

10 A crew consists of 420 men, and a certain number of boys; the men receive each £3. per month; and the amount of wages of the whole crew is £1500. per month; find the number of boys supposing each to receive £1. 10*s.* per month.

X.

1. Explain the rule for the addition of decimals; add together $\frac{3}{5}$ and ·061; subtract ·003 from ·02; and divide ·0672 by ·006.

2. Subtract $\frac{1}{2}$ of $\frac{5}{6}$ from $\frac{2}{3}$ of $\frac{7}{11}$, and multiply the result by $\frac{3}{7}$ of $\frac{5}{8}$.

3. If £1 sterling = 10 florins = 100 cents = 1000 mils, shew that £25 10*s.* $7\frac{1}{2}$*d.* = 255 florins, 3 cents, $1\frac{1}{4}$ mils.

4. If 6 men earn £7. 6*s.* 3*d.* in $7\frac{1}{2}$ days, how much will 10 men earn in $11\frac{3}{4}$ days?

5. A person expends £72 in the purchase of cloth, how much can he buy at the rate of 2*s.* 2*d.* a yard?

6. What is the cost per hour of lighting a room with ten burners, each consuming 4 cub. in. of gas per second; the price of gas being 6*s.* for a thousand cubic feet.

7. What is the value of 8 qrs., 5 bushels, 3 pecks of wheat at 7*s.* 8*d.* a bushel?

If 8 qrs., 6 bushels, 2 pecks of malt cost £21. 3*s.*, what is the price per bushel?

8. What length of paper $\frac{3}{4}$ of a yard wide will be required to cover a wall 15 ft. 8 in. long by 11 ft. 3 in. high?

9. "Define a *Rectangular* Parallelopiped."

A block of wood, in the form of a rectangular parallelopiped, measures along its edges 18½ feet, 5¼ feet, and 3 feet, respectively; determine its value on the supposition that a cubical block, measuring 11 inches along the edge, is worth 3*s.* 6*d.*

10. If 36 men, working 8 hours a day for 16 days, can dig a trench 72 yards long, 18 wide, and 12 deep, in how many days will 32 men, working 12 hours a day, dig a trench 64 yards long, 27 wide, and 18 deep?

XI.

1. Express $\frac{11}{133}$ as a decimal; and thence find its value when unity represents £300.

2. A parish containing 2456 acres is rated on a rental of £3070; a rate of 8*d.* in the pound being levied, what on the average is the charge per acre?

3. Find the price of 2 tons, 16 cwt., 17 lbs of sugar at 10*d.* for 2½ lbs.

4. If 1 cwt. of an article cost £7, at what price per lb. must it be sold to gain $\frac{1}{10}$ of the outlay?

5. Find in inches and fractions of an inch the value of $\cdot 000035511\dot{3}\dot{6}$ of a mile. Explain the process employed.

6. Express each silver coin now current in England by a decimal of 2⅖*d.* If $\frac{1}{10}$th of 2⅖*d.* be the unit of money, what decimal will express a halfpenny?

7. An American dollar is 4*s.* 3⅗*d.*, and is 5·42 francs; find the number of francs in £1 sterling, and express both a dollar and a franc in terms of the unit of money mentioned in the last question.

8. *A* and *B* can do a piece of work in 6 days, *B* and *C* in 7 days, and *A*, *B*, and *C* can do it in 4 days; how long would *A* and *C* take to do it?

9. If a sheet of paper 5½ feet long by 2½ feet broad be cut into strips an inch broad; how many sheets would be required to form a strip that would reach round the earth, (25,000 miles)?

10. A bag contains a certain number of sovereigns, three times as many shillings, and four times as many pence, and the whole sum in the bag is £280; find how many sovereigns, shillings, and pence it contains respectively.

RULE OF THREE.

142. We may compare one number with another, or ascertain the relation which one bears to the other in respect of magnitude, in two different ways; either by considering how much one is greater or less than the other; or by considering what multiple, part, or parts, one is of the other, that is, how many times or parts of a time, or both, one number is contained in the other. Thus if we compare the number 12 with the number 3, we observe, adopting the first mode of comparison, that 12 is greater than 3 by the number 9; or, adopting the second mode of comparison, that 12 contains 3 four times, and is thus $\frac{12}{3}$ or four times as great as 3. Again if we compare the number 7 with the number 13, we observe, according to the first mode of comparison, that 7 is less than 13 by the number 6; and, according to the second, that as 1 is one thirteenth part of 13, so 7 is seven thirteenth parts of 13, or $\frac{7}{13}$ths of 13.

143. The relation of one number to another in respect of magnitude is called Ratio; and when the relation is considered in the first of the above methods, that is, when it is estimated by the difference between the two numbers, it is called Arithmetical Ratio; but when it is considered according to the second method, that is, when it is estimated by considering what multiple, part, or parts, one number is of the other, or, which is seen from above to be the same thing, by the fraction which the first number is of the second, it is called Geometrical Ratio. Thus for instance, the arithmetical ratio of the numbers 12 and 3 is 9; while their geometrical ratio is $\frac{12}{3}$ or 4. In like manner the arithmetical ratio of 7 and 13 is 6, while their geometrical ratio is $\frac{7}{13}$.

144. It is more common, however, in comparing one number with another to estimate their relation to one another in respect of magnitude according to the second method, and to call that relation so estimated by the name of Ratio. According to this mode of treatment, which we shall adopt in what follows, "Ratio is the relation which one number has to another in respect of magnitude, the comparison being made by considering what multiple, part, or parts, the first number is of the second, or how many times or parts of a time, or both, the second is contained in the first."

145. It is plain that, for any two numbers, the fraction in which the *first is numerator* and the second denominator, will correctly express the

multiple or part, or both, which the first number is of the second, or the number of times or parts, or both, of a time the second is contained in the first. Thus if we take the numbers 12 and 3, the fraction $\frac{12}{3}$, which is equivalent to the whole number 4, shows the multiple which 12 is of 3, or the number of times 3 is contained in 12. And again, if we take the numbers 7 and 13, the fraction $\frac{7}{13}$ will express the part or parts which the number 7 is of 13, or will express the part or parts of a time that 13 is contained in 7: for 1 is one thirteenth part of 13, so that 7 must be seven thirteenth parts of 13, that is, $\frac{7}{13}$ths of it; and 1 is contained 7 times in 7, so that 13 must be contained only $\frac{7}{13}$ths of a time in 7. We conclude therefore that the ratio of one number to another may be estimated and expressed by the fraction in which the former number is the numerator and the latter the denominator.

146. The ratio of one number to another is often denoted by placing a colon between them. Thus the ratio of 7 to 13 is denoted by 7 : 13. As we have shown that the ratio of one number to another may be expressed by the fraction in which the former is the numerator and the latter the denominator, we see that $7 : 13$ is $= \frac{7}{13}$. The two numbers which form a ratio are called its *terms;* the first number, or the number compared, being called the first term, or THE ANTECEDENT, and the second number or that with which the former is compared, the second term, or THE CONSEQUENT, of the ratio.

147. If the two numbers to be compared together be concrete, they must be of the *same kind.* We cannot compare together 7 days and 13 miles in respect of magnitude; but we can compare 7 days with 13 days; and it is clear that 7 days will have the same relation to 13 days in respect of magnitude, which the number 7 has to the number 13, so that the ratio of 7 days to 13 days will be the same as the ratio of the abstract number 7 to the abstract number 13, and may be expressed by the fraction $\frac{7}{13}$. If however the concrete numbers, though of the same kind, be not in the same denomination of that kind, it will be convenient to reduce them to one and the same denomination in order to find their ratio. Thus, if one of the numbers be 7 days and the other be 13 hours, the ratio of the former to the latter will not be that of 7 to 13, but that of 7 *days* to 13 *hours,* that is, 168 hours to 13 hours, which will clearly be the same as that of the abstract number 168 to the abstract number 13, and so will be expressed not by $\frac{7}{13}$, but by $\frac{168}{13}$. We see, then, that 7 days : 13 hours is the same as 168 : 13, and that each is $= \frac{168}{13}$. Thus it is plain that when the num-

bers are concrete, we may reduce them to one and the same denomination, and then, in considering their ratio, treat them as abstract numbers.

148. Proportion is the equality of two ratios; so that, when the ratio of one number to a second is equal to the ratio of a third number to a fourth, proportion is said to exist among the numbers, and the numbers are called Proportionals. Thus, the ratio of 8 to 9 is equal to that of 24 to 27, for the former ratio is $\frac{8}{9}$, and the latter ratio is $\frac{24}{27}$, which is also equal to $\frac{8}{9}$. The ratios being equal, proportion exists among the numbers 8, 9, 24, 27; and thus those numbers are proportionals.

149. When proportion exists among four numbers, that is, when the ratio of the first to the second is equal to that of the third to the fourth, this proportion or equality is often denoted by writing down the two ratios in the manner mentioned in (Art. 146) in one line, and placing a double colon (::) between them. Thus the existence of proportion among the numbers 3, 4, 9, 12, is indicated as follows,

$$3 : 4 :: 9 : 12$$

which is commonly read thus, "three are to four as nine to twelve," or "as three to four so nine to twelve." It will appear from what has preceded, that by the expression 3 : 4 :: 9 ; 12, it is meant in fact that $\frac{3}{4}=\frac{9}{12}$.

150. In order to form a proportion four numbers are required. It may indeed happen that the second and third are the same, in which particular case it might be said that only three numbers are required; thus 9 : 6 :: 6 : 4; but even in such a case it is better to consider the second and third as distinct numbers, and to regard the proportion as consisting of four numbers, of which indeed two are equal. The four numbers required to form a proportion are called its *terms*. In the proportion 3 : 4 :: 9 : 12, we have 3 for the first term, 4 for the second, 9 for the third, and 12 for the fourth term, of the proportion.

151. It has been stated that proportion is the equality of two ratios, and we have explained that the two numbers constituting a ratio must either be both abstract, or (if concrete) both of the same kind. In a proportion if one of the ratios be formed by two abstract numbers, the other may arise from two concrete numbers. For it has been explained (Art. 147) that if a ratio consist of two concrete numbers, we may reduce them both to the same denomination, and then treat the resulting numbers as abstract, the ratio of those abstract numbers being the same as that of the two concrete numbers from which they have arisen. For the

same reason, one of the two ratios constituting a proportion may be formed from concrete numbers of one kind, while the other is formed from concrete numbers of a different kind; for 7 days : 13 days :: 7 miles : 13 miles, each ratio being in fact that of 7 to 13. Indeed it appears by (Art. 147) that the ratio of two concrete numbers may always be expressed by a ratio of two abstract numbers. If both or either of the ratios in a proportion be formed from concrete numbers, we may thus replace each such ratio by one arising from abstract numbers, and in this way every term of the proportion will become an abstract number; so that, notwithstanding the remark in note (Art. 26), any one of the terms may then be multiplied or divided by any other.

152. It is readily seen that if proportion exist among four numbers taken in a certain order, it will exist also among the same numbers taken in the contrary order. Thus the numbers 8, 9, 24, 27, being proportionals in the order in which they stand, the numbers 27, 24, 9, 8, will also be proportionals. For,

$$\frac{8}{9}=\frac{24}{27};$$

$$\therefore 1\div\frac{8}{9}=1\div\frac{24}{27},$$

$$\text{or } 1\times\frac{9}{8}=1\times\frac{27}{24},$$

$$\text{or } \frac{9}{8}=\frac{27}{24};$$

$$\therefore \frac{27}{24}=\frac{9}{8},$$

$$\text{or } 27 : 24 :: 9 : 8.$$

It is apparent also from (Art. 66) that $\frac{27}{24}=\frac{9}{8}$.

153. If only three of the numbers in a proportion be given, we can by means of them find the fourth, and the method or Rule by which it may be found is one of great importance in Arithmetic. We have seen that proportion exists among the numbers 8, 9, 24, 27. If the first three numbers only were given, and we were required, by means of these, to find the fourth, the method or Rule to be adopted ought to determine a number to which 24 would have the same ratio, as 8 to 9; or, which is seen from the last article to be the same thing, it ought to determine a number which will have the same ratio to 24, which 9 has to 8; this number being of course 27. Almost all questions which arise in the

common concerns of life, so far as they require calculation by numbers, might be brought within the scope of the Rule of Three, which enables us to find the fourth term in a proportion, and which, on account of its great use and extensive application, is often called the Golden Rule.

154. The Rule of Three, then, is a method by which we are enabled, from three numbers which are given, to find a fourth which shall bear the same ratio to the third as the second to the first, that is, shall be the same multiple, part, or parts of the third, as the second is of the first; in other words, it is a Rule by which, when three terms of a proportion are given, we can determine the fourth.

As most of the practical cases in which this Rule is made use of relate to concrete numbers, we shall express the Rule with especial reference to such cases, adding however a short direction for cases in which abstract numbers only are concerned.

155. Rule. "Leaving out of consideration superfluous quantities, find, out of the three quantities which are given, that which is of the same kind as the fourth or required quantity; or that which is distinguished from the other terms by the nature of the question: place this quantity as the third term of the proportion.

"Now consider whether, from the nature of the question, the fourth term will be greater or less than the third; if it be greater, then put the larger of the other two quantities in the second term, and the smaller in the first term; but if less, put the larger in the first term and the smaller in the second term.

"Take care to reduce the first and second terms to one and the same denomination, and also to reduce the third so that it may be wholly in one denomination; remembering, however, that if the quantities involved be all of the same kind, it is unnecessary to reduce all the three terms to the same denomination, but only the first and second terms to one and the same denomination, and the third to a single denomination, which will not necessarily be the same as the former. When the terms have been properly reduced, multiply the second and third together, and divide by the first, treating all three as abstract numbers. The quotient will be the answer to the question, in the denomination to which the third term was reduced."

If the case be one in which abstract numbers only are concerned, the question itself will show at once which of the numbers will form the third term of the proportion: the second and first will be determined as above explained; and then the answer to the question will be found by such multiplication and division as are directed in the Rule*.

156. It may be proper to observe that the Rule of Three is applicable in two different kinds of cases, according to which it is called the Rule of Three Direct or the Rule of Three Inverse. The method just stated (Art. 155) is applicable to both kinds of cases: but as the distinction between the two is commonly noticed by writers on Arithmetic, it will be right to show in what it consists.

The Rule of Three Direct is that in which more requires more, or less requires less; or, in other words, in which a greater number requires a greater answer, or a less number a less answer. Thus in the question, "If 4 acres of land cost £250, find the cost of 15 acres, after the same rate." The 15 acres being more than the four acres, will require a larger sum than £250 for their purchase, and so, in this case, more requires more. Again in the question, "If 15 acres of land cost £937. 10*s.* find the cost of 4 acres, after the same rate," the 4 acres being less than the 15 acres, will require a less sum than £937. 10*s.* for their purchase, and therefore, in this case, less requires less. Such cases belong to the Rule of Three Direct.

The Rule of Three Inverse is that in which more requires less, or less requires more: or, in other words, in which a greater number requires a less answer, or a less number a greater answer. Thus in the question, "If 4 men can mow a certain meadow in 3 days, find the time in which 6 men ought to mow it," the six men being more than the four, should perform the work in less time, and so, in this case, more requires less. Again, in the question, "If 6 men can mow a certain meadow in 2 days, find the time in which 4 men ought to mow it," the 4 men, being fewer than the 6, will require a longer time for performing the work, and therefore, in this case, less requires more. Such cases belong to the Rule of Three Inverse.

Rule of Three Direct.

Ex. 1. Find the value of 37 yards of silk, when 25 yards cost £4. 7*s.* 6*d.*

There are here three given quantities, 25 yards, 37 yards, and

---

* The arrangement of the given terms in the manner mentioned at the beginning of the Rule, is commonly called *stating the question.* Sometimes a word or two, or a letter, or other symbol, will be added to represent the fourth or required term.

£4. 7*s*. 6*d*., and we have to find a fourth which will be the price of 37 yards. It is manifest that the three given quantities, 25 yards, 37 yards, £4. 7*s*. 6*d*., and the required sum, must form a proportion, because the 25 yards must have the same relation in respect of magnitude to the 37 yards, which the £4. 7*s*. 6*d*. (cost of 25 yards) has to the required sum (cost of 37 yards). Proceeding then by Rule (Art. 155) we observe that the £4. 7*s*. 6*d*. is of the same kind as the required term, viz. money; we make that the third term of the proportion; and since the required sum (cost of 37 yards) must necessarily be greater than £4. 7*s*. 6*d*. (cost of 25 yards), we make 37 the second term, and 25 the first. We have thus the first three terms arranged as follows:

25 yds. : 37 yds. :: £4. 7*s*. 6*d*.

And the entire proportion will be as follows:

25 yds. : 37 yds. :: £4. 7*s*. 6*d*. : required cost.

The first and second terms are in one and the same denomination, and require no reduction. The third and fourth must be reduced to the lowest denomination in either of them, namely pence. Then since £4. 7*s*. 6*d*.=1050 pence, the proportion becomes

25 yds. : 37 yds. : 1050 pence : no. of pence in required sum.

And by our rule we must now treat the numbers as abstract, multiply the second and third together, and divide by the first.

$$\begin{array}{r}1050\\ \underline{37}\\ 7350\\ \underline{3150}\end{array}$$

$$25\begin{cases}5\\5\end{cases}\begin{array}{|r}\underline{38850}\\ \underline{7770}\\ 1554\end{array}$$

The quotient 1554 gives the number of *pence* in the required sum of money, that being the denomination to which the third term was reduced. We must now then reduce the 1554 pence to pounds, shillings and pence.

$$\begin{array}{r|l}12 & \underline{1554d.}\\ 2,0 & \underline{12,9-6d.}\\ & \pounds 6.\ 9s.\ 6d.\end{array}$$

therefore the required answer is £6. 9*s*. 6*d*.

The above process would in common use be more compendiously written down as follows:

$$\begin{array}{rr|l}
 & & \text{yds.}\quad \text{yds.}\quad \pounds\quad s.\quad d.\\
 & & 25 : 37 :: 4 \,.\, 7 \,.\, 6\\
 & & \underline{20}\\
 & & 87\\
 & & \underline{12}\\
 & & 1050\\
 & & \underline{37}\\
 & & 7350\\
 & & \underline{3150}\\
25 \left\{ \right. & 5 & \underline{38850}\\
 & 5 & \underline{7770}\\
 & 12 & \underline{1554d.}\\
 & 20 & \underline{129s.\ 6d.}\\
 & & \pounds 6.\ 9s.\ 6d.
\end{array}$$

Reason for the above process.

We have the cost of 25 yards given, viz. £4. 7*s.* 6*d.*, in order to enable us to find the cost of 37 yards.

It is manifest that the required sum must have the same relation in respect of magnitude to £4. 7*s.* 6*d.*, which 37 yards have to 25 yards; that is, the ratio of the required sum to £4. 7*s.* 6*d.*, or of the number of pence in the required sum to 1050 pence, must be equal to that of 37 yards to 25 yards.

Now the ratio of the number of pence in the required sum to 1050 pence, is the same as that of the *abstract* number which indicates how many pence the required sum contains to the abstract number 1050, and may (if the former number be called the *required number*) be expressed by the fraction $\frac{\text{required number}}{1050}$.

And the ratio of 37 yards to 25 yards is the same as that of the abstract number 37 to the abstract number 25, and may therefore, in like manner, be expressed by the fraction $\frac{37}{25}$.

$$\therefore \frac{\text{required number}}{1050} = \frac{37}{25};$$

$$\therefore \frac{\text{required number}}{1050} \times 1050 = \frac{37}{25} \times 1050,$$

$$\text{or } \frac{\text{required number} \times 1050}{1050} = \frac{37 \times 1050}{25},$$

$$\text{or required number} = \frac{37 \times 1050}{25}, \text{(Art. 66)},$$

$$\text{or} = \frac{1050 \times 37}{25}.$$

This result shows that if we arrange the three given terms, 25 yards, 37 yards, and £4. 7*s.* 6*d.* in the following manner,

$$\overset{\text{yds.}}{25} : \overset{\text{yds.}}{37} :: \overset{\pounds}{4} . \overset{s.}{7} . \overset{d.}{6},$$

and then reduce the £4. 7*s.* 6*d.* to pence, the lowest denomination contained in it, so that the arrangement becomes

$$\overset{\text{yds.}}{25} : \overset{\text{yds.}}{37} :: \overset{d.}{1050},$$

and then consider the numbers to be abstract, as if they had been written

$$25 : 37 :: 1050,$$

we shall obtain the abstract number which will show us how many pence there are in the required sum by multiplying the second and third terms together and dividing the product by the first; and then by treating this number as concrete, that is, as so many pence, we have the required answer in pence.

The reason for the process may also be shown as follows:

The cost of 25 yards is £4. 7*s.* 6*d.* or 1050 pence;

$$\therefore \text{ the cost of 1 yard is } \frac{1050}{25} \text{ pence;}$$

$$\therefore \text{ the cost of 37 yards is } \left(\frac{1050}{25} \times 37\right) \text{ pence} = \frac{1050 \times 37}{25} \text{ pence;}$$

or if we arrange the numbers, after the reduction of the £4. 7*s.* 6*d.* to pence, in the form

$$\overset{\text{yds.}}{25} : \overset{\text{yds.}}{37} :: \overset{d.}{1050},$$

and then treat them as abstract numbers, multiply the second and third together, and divide the product by the first, the quotient will give the number of the pence in the required sum of money.

Ex. 2. If a workman earn £17. 6*s.* in 102½ days, how long will he be in earning 50 guineas?

Here the required quantity is *time*, and as the given quantity of that kind is 102½ days, we must place that as the third term in the proportion. The earning of 50 guineas will require a longer time than the earning of £17. 6*s.*: we must therefore place the 50 guineas as the second term, and the £17. 6*s.* as the first. Then reduce, according to the Rule; observing, that as there is half-a-day in the third term, we may, if we please, reduce that term to half-days, in which case the answer will be obtained in half-days, and must be reduced to days.

£17. 6*s*. : 50*g*. :: $102\frac{1}{2}$ days : required time,

$$\begin{array}{rrr} 20 & 21 & 2 \\ \hline 346 & 1050 & 205 \\ & 205 & \\ \hline & 5250 & \\ & 2100 & \\ \hline \end{array}$$

$$\begin{array}{r} 346)\,215250\,(622\frac{38}{346} \text{ half-days} \\ 2076 \\ \hline 765 \\ 692 \\ \hline 730 \\ 692 \\ \hline 38 \end{array}$$

∴ the required time $= 622\frac{38}{346}$ half-days,
$= 311\frac{19}{346}$ days.

The process is briefly represented thus:

$$\text{Required time} = \frac{1050 \times 205}{346} \text{ half-days} = 311\frac{19}{346} \text{ days.}$$

Ex. 3. If the tax on £195. be £14. 8*s*., what will be the tax on £874?

The £14. 8*s*. being of the same nature with the sum required, must be placed as the third term in the proportion; and as the required tax must clearly be greater than £14. 8*s*. we must place £874 as the second, and £195 as the first term.

£195 : £874 :: £14. 8*s*. : the required tax,

$$\begin{array}{r} 20 \\ \hline 288 \\ 874 \\ \hline 1152 \\ 2016 \\ 2304 \\ \hline \end{array}$$

$$\begin{array}{r} 195)\,251712\,(1290\frac{162}{195} \\ 195 \\ \hline 567 \\ 390 \\ \hline 1771 \\ 1755 \\ \hline 162 \end{array}$$

∴ the required tax is $1290\frac{162}{195}$ shillings $=$ £64. $10\frac{162}{195}$*s*.

But $\frac{162}{195}s. = (\frac{162}{195} \times 12)d. = \frac{1944}{195}d. = 9\frac{189}{195}d.$

And $\frac{189}{195}d. = (\frac{189}{195} \times 4)q. = \frac{756}{195}q. = 3\frac{171}{195}q. = \frac{3}{4}d.\ \frac{171}{195}q.$

∴ the required tax is £64. 10*s*. $3\frac{3}{4}d.\ \frac{171}{195}q.$

$$
\begin{array}{rrl}
 & & \quad\ 2,0 \\
\text{Or thus,}\ 195) & 251712 & (\overline{129,0s.} \\
 & \underline{195} & \ \ \pounds 64.\ 10s. \\
 & 567 & \\
 & \underline{390} & \\
 & 1771 & \\
 & \underline{1755} & \\
 & 162 & \\
 & \underline{12} & \\
195) & 1944 & (9d. \\
 & \underline{1755} & \\
 & 189 & \\
 & \underline{4} & \\
195) & 756 & (3q.\ \text{or}\ \tfrac{3}{4}d. \\
 & \underline{585} & \\
 & \frac{171}{195}q. &
\end{array}
$$

The answer is therefore £64. 10*s*. $9\frac{3}{4}d.\ \frac{171}{195}q.$

The operation, as before, is briefly represented thus:

$$\text{Required tax} = \frac{874 \times 288}{195}s. = \pounds 64.\ 10s.\ 9\tfrac{3}{4}d.\ \tfrac{171}{195}q.$$

Note 1. The student who is expert in the use and reduction of fractions will very often find it convenient, after reducing the terms of his proportion in the manner mentioned in the Rule, to defer the *actual* multiplication and division, and express the required result in a fractional form; to reduce the fraction as much as possible by the method indicated in Art. 77, note 3; and to effect the requisite multiplication or division or both, after the fraction has been so simplified.

Ex. 4. If I can travel 198 miles by railway for £2. 9*s*. 6*d*., how far at the same rate of charge ought I to be carried for £8. 0*s*. 10½*d*.?

$$
\begin{array}{rrl}
\pounds 2.\ 9s.\ 6d. & :\ \pounds 8.\ 0s.\ 10\tfrac{1}{2}d. & ::\ 198\,\text{m.}\ :\ \text{required distance.} \\
\underline{20} & \underline{20} & \\
49 & 160 & \\
\underline{12} & \underline{12} & \\
594 & 1930 & \\
\underline{4} & \underline{4} & \\
2376 & 7722 &
\end{array}
$$

$$\therefore \text{ Required distance} = \frac{7722 \times 198}{2376} \text{ miles} = \frac{3861 \times 198}{1188} \text{ miles}$$

$$= \frac{3861}{6} \text{ miles} = 643\tfrac{3}{6} \text{miles} = 643\tfrac{1}{2} \text{ miles.}$$

Ex. 5. The annual poor's rates on a net rental of £365. 7*s.* 3*d.* amount to £36. 8*s.* 9*d.*; what should be the net rental of an estate for which the poor's rates amount to £24. 5*s.* 10*d.* per annum?

$$\begin{array}{llll}
\pounds 36.\ 8s.\ 9d. & : \pounds 24.\ 5s.\ 10d. & :: \pounds 365.\ 7s.\ 3d. & : \text{required rental.} \\
\underline{\quad 20} & \underline{\quad 20} & \underline{\quad 20} & \\
728 & 485 & 7307 & \\
\underline{\quad 12} & \underline{\quad 12} & \underline{\quad 12} & \\
8745 & 5830 & 87687 &
\end{array}$$

$$\therefore \text{ Required rental} = \frac{87687 \times 5830}{8745} d. = \frac{87687 \times 1166}{1749} d.$$

$$= \frac{29229 \times 1166}{583} d. = (29229 \times 2) d.$$

$$= 58458 d. = \pounds 243.\ 11s.\ 6d.$$

Note 2. There are certain examples in which at first sight more than three terms appear to be given, but they nevertheless in certain cases come under this rule, as in the following instance.

Ex. 6. If the carriage of 5 cwt., 7 lb. for 84 miles cost me £3. 18*s.* 4*d.*, what will it cost me to have 21 cwt., 1 qr., 14 lbs. carried the same distance?

The 84 miles may evidently be left out of consideration, since the distance in both cases is the same.

Proceeding then according to our Rule,

5 cwt., 7 lbs. : 21 cwt., 1 qr., 14 lbs. :: £3. 18*s.* 4*d.* : required cost;

whence it will be found that

Required cost = £16. 10*s.* 8$\frac{3}{4}$*d.* $\frac{5}{9}$*q.*

Rule of Three Inverse.

Ex. 7. If a piece of cloth is 20 yards in length and $\frac{3}{4}$ yard in breadth, how broad is another piece which is 12 yards long, and which contains as much cloth as the other?

As the length of the second piece is less than that of the first, its breadth must necessarily be greater, in order that the content may be the same. Therefore in this case a less length requires a greater breadth, and so the example belongs to Rule of Three Inverse.

We have the *breadth* of the second piece to find. That of the first piece is $\frac{3}{4}$ yard: place this therefore as the third term. Now the required breadth is to be greater than this. Therefore place the 20 yards as the second term, and the 12 yards as the first.

12 yds. : 20 yds. :: 3 qrs. of a yd. : required breadth in qrs. of a yd.

$$\begin{array}{r|l} & 3 \\ \hline 12 & 60 \\ \hline \end{array}$$

5 qrs. of a yard $=1\frac{1}{4}$ yard.

Or thus;

12 yds. : 20 yds. :: $\frac{3}{4}$ yd : required breadth in yds.

$$\therefore \text{ required breadth} = \frac{20 \times \frac{3}{4}}{12} \text{ yds.}$$

$$= \frac{5 \times 3}{12} \text{ yds.} = \frac{5}{4} \text{yds.} = 1\frac{1}{4} \text{yds.}$$

The required breadth is therefore a yard and a quarter.

Ex. 8. If 12 men can reap a field in 4 days, in what time can the same work be performed by 32 men?

It is clear that 32 men can perform the work in a less time than 12 men, and so the time required will be less than 4 days, the third term in our proportion. We must therefore place the 12 as the second term and the 32 as the first.

32 : 12 :: 4 days : required time in days.

$$\begin{array}{l} \quad\;\; 4 \\ 32\overline{)48}\,(1\frac{16}{32} \text{ days.} \\ \quad\;\; 32 \\ \quad\;\; \overline{16} \end{array}$$

$\therefore$ the required time is $1\frac{16}{32}$ days $= 1\frac{1}{2}$ days.

Or thus:

$$\text{Required time} = \frac{12 \times 4}{32} \text{ days} = \frac{12}{8} \text{ days} = \frac{3}{2} \text{ days} = 1\frac{1}{2} \text{ days.}$$

Ex. 9. What was the price of wheat per bushel when the penny loaf weighed 8 ounces; the statute being that it must weigh 10 oz. when wheat is at 12*s.* a bushel?

Here are two numbers viz. 1 bushel and 1 penny, which can evidently have no effect on the answer, for if any other measure had been named in place of the bushel, and any other loaf in place of the penny loaf, the answer would be the same.

Now as wheat is dearer, or as the price is more, the weight of any given loaf is less, and conversely, as the weight of a given loaf is less, the price of wheat is greater; so that the price required must clearly be greater than 12*s.*, which according to our Rule must be the third term of the proportion. Therefore the 10 oz. must be the second term, and the 8 oz. the first.

$$8 \text{ oz.} : 10 \text{ oz.} :: 12s.$$

$$\begin{array}{r|l} & 12 \\ \hline 8 & 120 \\ \hline & 15s. \text{ the required price per bushel.} \end{array}$$

Or thus;

$$\text{Required price} = \frac{10 \times 12}{8} s. = \frac{10 \times 3}{2} s. = 15s.$$

Note 3. Examples, such as the following, are easily worked out by the Rule of Three.

Ex. 1. A watch, which is 4 min. $8\frac{9}{32}$ sec. too fast at half-past nine A.M. on Tuesday, loses 2 min. 45 sec. daily; what will be the time indicated by the watch at a quarter-past five P.M. on the following Friday?

From $9\frac{1}{2}$ A.M. on Tuesday, till $5\frac{1}{4}$ P.M. on Friday, there are $79\frac{3}{4}$ hours.

$$\therefore 24 \text{ hrs.} : 79\tfrac{3}{4} \text{ hrs.} :: 2'.45'' : \text{time lost by clock},$$

$$\text{whence, time lost by clock} = 9'.8\tfrac{9}{32}'';$$

$\therefore$ time by the clock at $5\frac{1}{4}$ P.M. on Friday

$$= 4'.8\tfrac{9}{32}'' + 5 \text{ hrs. } 15' - 9'.8\tfrac{9}{32}'' = 5 \text{ hrs. } 10 \text{ min.}$$

Ex. 2. A hare, pursued by a greyhound, was 130 yards before him at starting; whilst the hare ran 5 yards the dog ran 7 yards: how far had the hare gone when she was caught by the greyhound?

For every 5 yards the hare runs, the dog gains 2 yards, and when he has gained 130 yards he will have caught her.

$$\therefore 2 \text{ yds.} : 130 \text{ yds.} :: 5 \text{ yds.} : \text{required number of yards};$$

$$\text{whence, required number of yards} = 325.$$

Ex. 3. A gentleman spends on the average 30 guineas a fortnight; what must be his daily income in order that with his savings at the end of $3\frac{1}{2}$ years he may buy an estate worth £1719. 18*s.*? (supposing a year to consist of 52 weeks).

His expenditure in $3\frac{1}{2}$ years is $(30 \times 26 \times 3\frac{1}{2})$ guineas,
∴ his income in $3\frac{1}{2}$ years must be $(30 \times 26 \times 3\frac{1}{2})$ guineas + £1719. 18*s.* = £4586. 8*s.*

∴ $(3\frac{1}{2} \times 364)$ days : 1 day :: £4586. 8*s.* : daily income,
whence, daily income = £3. 12*s.*

Ex. 4. Two places, *A* and *B*, are distant from each other 324 miles by railway. A train leaves *A* for *B* at the same time that a train leaves *B* for *A*; the trains meet at the end of 6 hours, the train from *A* to *B* having travelled 16 miles an hour more than the other. How many miles did each travel an hour?

Each train is supposed to run with uniform speed: when the trains meet, the whole distance must have been passed over by them.
∴ 6 hrs. : 1 hr. :: 324 miles : miles passed over by both trains in 1 hr.,
whence, miles passed over by both trains in 1 hr. = 54.
therefore by question, $(54-16)\div 2$, or 19 = miles travelled per hour by one train, and therefore $54-19$, or 35 = miles travelled per hour by the other.

Ex. 5. A gentleman, after paying an income-tax of 7*d.* in the £, has £248. 10*s.* 8*d.* left, what was his gross annual income?

For every 19*s.* 5*d.* which he now has, he had £1. before he paid his income-tax;

∴ 19*s.* 5*d.* : £248. 10*s.* 8*d.* :: £1. : required income,
whence, required income = £256.

Ex. LVI.

1. If 4 yards of cloth cost 12*s.*, what will 96 yards of the same cloth cost?

2. If 9 yards of cloth cost £5. 12*s.*, how many yards can be bought for £44. 16*s.*?

3. If 7 bushels of wheat be worth £1. 16*s.* 9*d.*, what will be the value of 3 bushels of the same quality?

4. The rent of 42 acres of land is £63, how many acres of the same quality of land ought to be rented for £273?

5. If the cost of 72 tons of coals be £55. 16*s.*, what will be the cost of 54 tons?

6. How much must be given for 13 articles at the rate of £3. 16*s.* 6*d.* for 6 articles?

7. How long will a person be saving £3, if he put by 1*s.* 6*d.* per *week?*

8. Find a number which shall bear the same ratio to 9, which 20 does to 15.

9. If 2 cwt., 3 qrs., 14 lbs. of sugar cost £6. 14*s.* 2*d.*, what quantity of the same quality of sugar can be bought for £29. 15*s.*?

10. If 3 cwt., 3 qrs. cost £6. 16*s.*, what will be the price of 2 cwt., 2 qrs.?

11. Find the value of 23 yds., 1 ft. of cloth, supposing 4 yds., 31 in. of the same quality to cost £3. 15*s.*

12. What will be the income-tax, at 7*d.* in the pound, on £257. 10*s.*?

13. If an income of £185. 10*s.* pay an income-tax of £5. 8*s.* 2½*d.*, what ought an income of 1000 guineas to pay?

14. What is the tax upon £302. 3*s.* 7*d.*, when £429. 8*s.* 3*d.* is rated at 13*s.* 6*d.*?

15. If one bushel of malt cost 5*s.* 10*d.*, how much can I buy for £27. 5*s.* 5*d.*?

16. Find the price of 2 tons, 3 cwt., 14 lbs. at 8*s.* 9½*d.* per quarter.

17. *A* pays half yearly an income-tax of £10. 1*s.* 3*d.*; find his income, the tax being 7*d.* in the £.

18. Find the amount of a servant's wages for 215 days at 2*s.* 4*d.* a day?

19. A bankrupt's debts amount to £204. 16*s.* and his assets to £179. 4*s.*; how much in the pound can he pay?

20. Find the cost of a stone of sugar at the rate of £98. 1fl. 2c. 5m. for 1 ton, 8 cwt., 4 lbs.

21. A bankrupt pays 12*s* 8*d.* in the pound and his assets amount to £950.; find the amount of his debts.

22. Find the cost of 1 ton, 4 cwt., 8 lbs. of an article, 3½ stone of which are worth 6 fl. 1¼ c.

23. If 26 yards of butter cost 48*s.*, what must it be sold at per foot, in order to gain 4*s.* on the purchase?

24. If a farm containing 400 ac., 2 ro., 20 po. be let at £841. 3 fl. 1 c. 2½ m. for the year, what is the rent per acre?

25. Find a fourth proportional to the numbers 3, 3·75, and 40.

26. If 10 men can mow a field in 12 days, in how many days will 15 men mow it?

27. If a man walk 62 miles in 3 days, in how many days will he walk 80 miles?

28. How many yards worth 3*s.* 7½*d.* a yard must be given in exchange for 935½ yards worth 18*s.* 1½*d.* per yard?

29. A bankrupt pays 5fl. 7c. 5m. in the pound; what sum will be lost on a debt of £11793. 5fl.?

30. Find the price of 2 tons, 16 cwt., 17 lbs. of sugar at 10*d.* for $2\frac{1}{2}$ lbs.

31. If a person travelling 12 hours a day perform a journey in 24 days, in what length of time will he perform the same journey if he travel 16 hours a day?

32. If $3\frac{5}{9}$ oz. Avoir. cost 7*s.*, what will $30\frac{3}{7}$ lbs. cost?

33. How many men must be employed to finish a piece of work in 15 days, which 5 men can do in 24 days?

34. If 356 ac., 3 ro, $39\frac{1}{2}$ po. be rented at £951. 19*s.* 10*d.*, what is the rent of 2 acres?

35. If the rents of a parish amount to £2514. 7*s.* 6*d.* and a rate be granted of £83. 16*s.* 3*d.*, how much is this in the pound? And how much must be paid by an estate whose rental is £115. 12*s.* 6*d.*?

36. If 27 bus., 2 pks. cost £10. 7*s.* $2\frac{1}{4}$*d.*, what is the price of $16\frac{1}{2}$ bus.?

37. How many yards of drugget an ell wide will cover 40 yards of carpet $\frac{3}{4}$ yd. wide?

38. *A* borrowed of *B* 400 guineas for $6\frac{1}{2}$ months, afterwards *A* would requite *B*'s kindness by lending him £910; how long should he lend it?

39. A field is 121 yds. long, and 86 yds. broad; what will be its value at £80. an acre?

40. If the price of 1 lb. of sugar be ·5625 fl., what is the value of ·75 of a cwt.?

41. If $3\frac{1}{2}$ shares in a mine cost £11. 5*s.*, what will $28\frac{3}{4}$ shares cost?

42. If $34\frac{1}{2}$ yards of cloth cost £12. 7*s.* $11\frac{5}{8}$*d*, how many yards can be bought for £3. 19*s.* $0\frac{3}{4}$*d.*?

43. Find the rent at 30*s.* an acre of a rectangular field whose sides are respectively 50 chains 40 links, and 56 chains 25 links.

44. In what time will 25 men do a piece of work which 12 men can do in 3 days?

45. If $\cdot\dot{3}$ of 4·5 cwt. cost £11. 5 fl. 5c., what is the price per lb.?

46. A piece of gold at £3. 17*s.* $10\frac{1}{2}$*d.* per oz. is worth £150; what will be the worth of a piece of silver of equal weight at 54*s.* 6*d.* per lb.?

47. If a piece of building land 375 ft. 6 in. by 75 ft. 6 in. cost £118. 2*s.* $6\frac{1}{4}$*d.*, what will be the price of a piece of similar land 278 ft. 9 in. by 151 feet?

48. A servant enters on a situation at 12 o'clock at noon on Jan. 1, 1854, at a yearly salary of 35 guineas, he leaves it at noon on the 27th of May following; what ought he to receive for his services?

49. *A* was owner of $\frac{4}{17}$ of a vessel, and sold $\frac{3}{11}$ of $\frac{2}{3}$ of his share for £$\frac{400}{33}$; what was the value of $\frac{1\frac{2}{3}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of the vessel?

50. *A* exchanged with *B* 60 yards of silk worth 7*s*. 3*d*. a yard for 48 yards of velvet; what was the price of the velvet a yard?

51. A person, after paying 7*d*. in the £. for income-tax on his income, has £1632. 18*s*. 10*d*. remaining; what had he at first?

52. If a person's estate be worth 3000 guineas a year, and the land-tax be assessed at 2*s*. 9½*d*. in the £., what is his annual income?

53. A watch is 10 minutes too fast at 12 o'clock (noon) on Monday, and it gains 3′. 10″ a day; what will be the time by the watch at a quarter past 10 o'clock A.M. on the following Saturday?

54. The circumference of a circle is to its diameter as 3·1416 : 1; find (in feet and inches) the circumference of a circle whose diameter is 22½ feet.

55. A bankrupt's estate amounts to £455. 1*s*. 6¾*d*., and his debts to £937. 10*s*. What can he pay in the £? and what will a creditor lose on a debt of £114?

56. If the carriage of 3 cwt. cost 10*s*. for 40 miles, how much ought to be carried for the same price for 25¾ miles?

57. If I spend 20 guineas in a fortnight, what must my income be that I may lay by £200. in the year 1855?

58. The house-tax upon a house rated at 175 guineas is £6. 17*s*. 9¾*d*., what will be the tax upon one rated at £120?

59. A silver tankard, which weighs 1 lb., 10 oz., 10 dwts. cost £6. 3*s*. 9*d*., what is the value of the silver per ounce?

60. A man, working 7½ hours a day, does a piece of work in 9 days; how many hours a day must he work to finish it in 4½ days?

61. If a pound of silver costs £3. 6*s*., what is the price of a salver which weighs 7 lbs., 7 oz., 10 dwts., subject to a duty of 1*s*. 6*d*. per ounce, and an additional charge of 1*s*. 10*d*. per ounce for the workmanship?

62. How much did a person spend in 63 days, who with an annual income of £818. is 90 guineas in debt at the end of a year?

63. If 15 men, 12 women, and 9 boys, can complete a piece of work in 50 days, what time would 9 men, 15 women, and 18 boys take to do four times as much, the parts done by each in the same time being as the number 3, 2, and 1?

64. A person possesses £800. a-year, how much may he spend per day in order to save £48. 2 fl. 5 c. after paying a tax of £5. on every £100. of income?

65. If 3 cows or 7 horses can eat the produce of a field in 29 days, in how many days will 7 cows and 3 horses eat it up?

66. How many yards of carpet ¾ yard wide will cover a room whose width is 16 feet, and length 27½ feet?

67. A person buys 100 eggs at the rate of 2 a penny, and 100 more at the rate of 3 a penny: what does he gain or lose by selling them at the rate of 5 for 2*d.*?

68. A church-clock is set at 12 o'clock on Saturday night; at noon on Tuesday it is 3 minutes too fast: supposing its rate regular, what will be the true time when the clock strikes four on Thursday afternoon?

69. A person after paying a poors' rate of 10*d.* in the pound has £728. 6*s.* 8*d.* remaining, what had he at first?

70. If a piece of work can be done in 50 days by 35 men working at it together, and if, after working together for 12 days, 16 of the men were to leave the work; find the number of days in which the remaining men could finish the work.

71. A regiment of 1000 men are to have new coats; each coat is to contain 2½ yards of cloth 1¼ yards wide; and it is to be lined with shalloon of ¾ yard wide; how many yards of shalloon will be required?

72. If 5 ounces of silk can be spun into a thread two furlongs and a half long, what weight of silk would supply a thread sufficient to reach to the Moon, a distance of 240,000 miles?

73. How many revolutions will a carriage-wheel, whose diameter is 3 feet, make in 4 miles? (See Ex. 54).

74. If 8 oz. of sugar be worth ·5625*s.*, what is the value of ·75 of a ton?

75. The price of ·0625 lbs. of tea is ·458$\dot{3}$*s.*, what quantity can be bought for £61. 12*s.*?

76. Two watches, one of which gains as much as the other loses, viz. 2′. 5″ daily, are set right at 9 o'clock A. M. on Monday; when will there be a difference of one hour in the times denoted by them?

77. How many yards of matting, 2·5 feet broad, will cover a room 9 yards long, and 20 feet broad?

78. A person bought 1008 gallons of spirits for £640.; 48 gallons leaked out: at what rate must he sell the remainder per gallon so as not to lose by his bargain?

79. If a soldier be allowed 12 lbs. of bread in 8 days, how much will serve a regiment of 850 men for the year 1856?

80. If 2000 men have provisions for 95 days, and if after 15 days 400 men go away; find how long the remaining provisions will serve the number left.

81. A gentleman has 10000 acres; what is his yearly rental, if his weekly rental for 20 square poles be $1\frac{1}{2}$*d.*? (1 year = 52 weeks).

82. If an ounce of gold be worth £$4·18958\dot{3}$, what is the value of $·36822916\dot{6}$ lbs.?

83. If 1000 men have provisions for 85 days, and if after 17 days 150 of the men go away; find how long the remaining provisions will serve the number left.

84. What is the quarter's rent of $182·\dot{3}$ acres of land, at £4·65 per acre for a year?

85. A grocer bought 2 tons, 3 cwt., 3 qrs. of goods for £120, and paid 50*s.* for expences; what must he sell the goods at per cwt. in order to clear £61. 5*s.* on the outlay?

86. What must be the breadth of a piece of ground whose length is $40\frac{1}{2}$ yards, in order that it may be twice as great as another piece of ground whose length is $14\frac{5}{8}$ yards, and whose breadth is $13\frac{2}{13}$ yards?

87. If 3·75 yards of cloth cost £3·825, what will 38 yds., 2 qrs., 3 nails cost?

88. Four horses and 6 cows together find sufficient grass on a certain field; and 7 cows eat as much as 9 horses; what must be the size of a field relatively to the former, which will support 18 horses and 9 cows?

89. *A* alone can reap a field in 5 days, and *B* in 6 days, working 11 hours a-day, find in what time *A* and *B* can reap it together, working 10 hours a day.

DOUBLE RULE OF THREE.

157. There are many questions, which are of the same nature with those belonging to the Rule of Three, but which if worked out by means of that Rule as before given, would require two or more distinct applications of it. Every such question, in fact, may be considered to contain two or more distinct questions belonging to the Rule of Three, and when each of those questions has been worked out by means of the Rule, the answer obtained for the last of them will be the answer to the original question.

158. The following example may serve to illustrate the preceding observations. "If the carriage of 15 cwt. for 17 miles cost me £4. 5*s.*, what would the carriage of 21 cwt. for 16 miles cost me?"

We observe that this question, though of a like nature with those which engaged our attention under the Rule of Three, is nevertheless of a more complicated description; and the student, without further explanation, would find some difficulty in obtaining an answer to it by means of a single application of the Rule. For we observe, that instead of three given quantities, we have five, every one of which must necessarily have a bearing on the answer, so that none of them can be superfluous. If however the question be divided into two distinct questions, each of these, when superfluous terms are rejected, will be found to comprise only three given terms of a proportion, from which three terms the fourth is to be ascertained; and the student would have no difficulty in working out each of these two questions by means of a single application of the Rule, so that in this way he will obtain the correct answer by applying the Rule of Three twice over.

The first question may be this; "If the carriage of 15 cwt. for 17 miles cost me £4. 5*s.*, what would the carriage of 21 cwt. for 17 miles cost me?" In this question the 17 miles would have no effect upon the answer, because the distance is the same in both parts of the question, and the answer would clearly remain unaltered, if any other number of miles, or if the words "a certain distance," had been used instead of the 17 miles. This number may therefore be neglected as superfluous, and we have then three terms of a proportion remaining, and the fourth is to be found. Solving the question by the Rule of Three, we find that the answer will be £5. 19*s.*

The second question may be this: "If the carriage of 21 cwt. for 17 miles cost me £5. 19*s.*, what will the carriage of 21 cwt. for 16 miles cost me?" In this question, for reasons similar to those before given, the 21 cwt. will be a superfluous quantity. Applying the Rule of Three to the question, we find the answer to be £5. 12*s.*

From the connection of the two questions with that originally proposed, we observe that £5. 12*s.*, thus obtained through two distinct applications of the Rule of Three, must be the answer to the original question.

159. We might give still more complicated instances, in which more than two distinct applications of the Rule of Three would be needed, in order to obtain the required answer; but the practical questions which

most commonly occur, of the kind we have been treating of, would require only a double application of the Rule of Three, and, like the question which has been used by way of illustration, would comprise only five given quantities for the determination of a sixth which is not given.

160. The Double Rule of Three is a shorter or more compendious method of working out such questions as would require two or more applications of the Rule of Three; and it is sometimes called the Rule of Five, from the circumstance, that in the practical questions to which it is applied, there are commonly five quantities given to find a sixth.

161. The method of working out examples which belong to the class under consideration, may be expressed as follows:

Rule. "Set down the quantities which express the *conditions* of the question in one line. Under each of these quantities set its corresponding one in another line, leaving a blank in that place in which the required term would have fallen. Where two quantities stand one above the other reduce them both to one and the same denomination, if they be not already in that state; and as to the remaining quantity under which the blank lies, reduce it so as to be wholly in one denomination, if not already so; then the resulting numbers may be treated as abstract. Multiply together the numbers which arose from the producing quantities of one line and those which arose from the produced quantities of the other, or, in other words, those numbers in one line which represent *causes*, and those in the other which represent *effects*, for a dividend; and then the remaining numbers, whether arising from *causes or effects*, or both, for a divisor. The quotient of the division will be the answer to the question, in that denomination to which the quantity standing over the blank was reduced."

The rule above laid down, though expressed at greater length, is substantially the same as that which is stated by Dr. Olinthus Gregory, in his edition of Hutton's Course of Mathematics, to have been given in 1706, by W. Jones, Esq. F.R.S., the father of the celebrated Sir William Jones.

Ex. 1. If a tradesman with a capital of £2000 gain £50 in 3 months, how long will it take him with a capital of £3000 to gain £175?

The quantities which express the conditions of the question are £2000, £50, and 3 months. Our arrangement may therefore be the following:

£2000.	3 m.	£50.
£3000.		£175.

The producing quantities or the causes in the first line are £2000, and 3 months; for these together, that is, £2000 employed for 3 months, produce the £50, which is the produced quantity or the effect in the first line. The produced quantity or the effect in the second line is £175. Hence, according to our Rule,

$$\text{Dividend} = 2000 \times 3 \times 175 = 1050000$$
$$\text{Divisor} = 3000 \times 50 = 150000.$$

Effecting the division, we have

$$15{,}0000)\ 105{,}0000\ (7$$
$$\underline{105.}$$

The quotient is 7, which therefore, according to our Rule, is the answer to the question, in the same denomination as the term under which the blank was left, that is, in months; so that the required answer is 7 months.

Or thus;

$$\text{Required time} = \left(\frac{2000 \times 3 \times 175}{3000 \times 50}\right)\text{months} = \left(\frac{2 \times 175}{50}\right)\text{months}$$
$$= \frac{350}{50}\text{ months} = 7\text{ months.}$$

Reason for the above process.

The tradesman, with a capital of £2000 gains £50 in 3 months. Let us first find, by the Rule of Three, how long he would be in gaining £175 with the *same* capital. Thus

$$£50 : £175 :: 3\text{m.} : \text{required time}$$

$$\text{Required time} = \left(\frac{175 \times 3}{50}\right)\text{months.}$$

Since then the tradesman with a capital of £2000 would gain £175 in $\left(\frac{175 \times 3}{50}\right)$ months, let us next find, by the Rule of Three, how long it would take him to gain the same sum with a capital of £3000, and we shall have the answer to the original question. Thus

$$£3000 : £2000 :: \frac{175 \times 3}{50}\text{ months} : \text{required time.}$$

$$\text{Required time in months} = \left(\frac{175 \times 3}{50} \times 2000\right) \div 3000.$$

$$=\frac{175\times3\times2000}{50}\div3000$$
$$=\frac{175\times3\times2000}{50}\times\frac{1}{3000}$$
$$=\frac{175\times3\times2000}{50\times3000}$$
$$=\frac{2000\times3\times175}{3000\times50};$$

that is, if we arrange the given quantities as follows,

£2000. 3 m. £50
£3000. £175,

we obtain the answer to our original question by multiplying together the producing numbers or causes of one line and the produced number or effect of the other, for a dividend, and the two remaining numbers for a divisor, and then carrying out the division; and we obtain the answer in the denomination of that term which stands over the blank.

Or thus:

A capital of £2000 gains £50 in 3 months,
.... £1 .... £50 in (3 × 2000) months,
.... £1 .... £1 in $\left(\frac{3\times2000}{50}\right)$ months,
.... £3000 ...... £1 in $\left(\frac{3\times2000}{50\times3000}\right)$ months,
.... £3000 ...... £175 in $\left(\frac{3\times2000\times175}{50\times3000}\right)$ months,
or $\left(\frac{2000\times3\times175}{3000\times50}\right)$ months;

that is, if we arrange the given quantities as follows,

£2000 3m. £50
£3000 £175,

we obtain the required time in months by multiplying together the producing numbers or causes of one line and the produced number or effect of the other for a dividend, and the remaining numbers for a divisor, and finding the quotient.

Ex. 2. If a tradesman with a capital of £2000 gain £50 in 3 months, what sum will he gain with a capital of £3000 in 7 months?

The terms which express the conditions of the question being £2000, £50, and 3 months, one arrangement may be as follows;

£2000.	3 m.	£50.
£3000.	7m.	

In the first line the two producing terms are £2000 and 3 months, and the produced term £50. In the second line there are two producing terms, £3000 and 7 months, but no produced term appears. We must therefore in this case, in applying our Rule, multiply together the two producing terms of the second line and the produced term of the first, for a dividend, and the two remaining terms for a divisor.

$$\text{Dividend} = 3000 \times 7 \times 50 = 1050000$$
$$\text{Divisor} = 2000 \times 3 = 6000.$$

Dividing the former by the latter, we have

$$6{,}000)\ \underline{1050{,}000}$$
$$175$$

The answer will therefore be 175 units of that denomination which belongs to the term under which the blank lies, namely, pounds; so that the answer is £175.

Or thus:

$$\text{Answer} = £\left(\frac{3000 \times 7 \times 50}{2000 \times 3}\right) = £\left(\frac{7 \times 50}{2}\right) = £(7 \times 25) = £175.$$

Ex. 3. If 7 horses be kept 20 days for £14, how many will be kept 7 days for £28?

The arrangement may be:

h.	d.	£.
7	20	14
	7	28

$$\text{Dividend} = 7 \times 20 \times 28 = 3920$$
$$\text{Divisor} = 7 \times 14 = 98$$

$$98)\ 3920\ (40$$
$$\underline{392}$$

The answer is therefore 40 horses.

Ex. 4. If I get 8 oz. weight of bread for 6*d.* when wheat is 15*s.* a bushel, what ought a bushel of wheat to be when I get 12 oz. of bread for 4*d.*?

The arrangement may be:

oz.	*s.*	*d.*
8	15	6
12		4

Here the 15*s.* per bushel and the 8 oz. must be considered as causing or producing the charge of 6*d.* For the 8 oz. cost that sum, or may be said to produce it, and yet would not produce it unless wheat were 15*s.* a bushel. It is clear therefore that, wheat being 15*s.* a bushel, and 8 oz. being taken, a charge of 6*d.* is produced by those two circumstances conjoined. Hence,

$$\text{Dividend} = 8 \times 15 \times 4 = 480$$

$$\text{Divisor} \; = 12 \times 6 \quad = 72$$

$$72)\,480\,(6$$
$$\underline{432}$$
$$\frac{48}{72} = \frac{2}{3}$$

Hence the required price will be $6\frac{2}{3}$*s.* = 6*s.* 8*d.*

Ex. 5. If 20 men can perform a piece of work in 12 days, find the number of men who could perform another piece of work 3 times as great in $\frac{1}{5}$th of the time.

The first piece of work being reckoned as 1, the second must be reckoned as 3. Our arrangement may then be

men	days	work
20	12	1
	$\frac{12}{5}$	3

$$\text{Dividend} = 20 \times 12 \times 3 = 720$$

$$\text{Divisor} \; = \frac{12}{5} \times 1 = \frac{12}{5}$$

$$\text{Required number of men} = 720 \div \frac{12}{5} = 720 \times \frac{5}{12}$$

$$= \frac{720 \times 5}{12} = 60 \times 5 = 300.$$

Ex. 6. If 252 men can dig a trench 210 yds. long, 3 wide, and 2 deep, in 5 days of 11 hours each; in how many days of 9 hours each will 22 men dig a trench 420 yds. long, 5 wide, and 3 deep?

The first trench contains (210 × 3 × 2) cubic yards.

= 1260 cub. yds.

The second........................ (420 × 5 × 3) cub. yds.

= 6300 cub. yds.

On the supposition therefore that 252 men can remove 1260 cub. yds. of earth in 55 hours, we have to find in how many hours 22 men can remove 6300 cub. yds. Our arrangement will be

men	hrs.	cub. yds.
252	55	1260
22		6300

Dividend = 252 × 55 × 6300 = 87318000

Divisor = 22 × 1260 = 27720

```
27720) 87318000 (3150 hours
       83160
       ------
        41580
        27720              9 | 3150
        ------               -------
        138600                350 days
        138600                of 9 hours each.
        ------
             0
```

The 22 men will therefore do their work in 3150 working hours, or 350 days of 9 hours each.

Or thus:

$$\text{Required time} = \frac{252 \times 55 \times 6300}{22 \times 1260} \text{ working hours,}$$

$$= \frac{252 \times 55 \times 5}{22} \text{ working hours,}$$

$$= 126 \times 5 \times 5 \text{ working hours,}$$

$$= 630 \times 5 \text{ working hours,}$$

$$= 3150 \text{ working hours,}$$

$$= \frac{3150}{9} \text{ days of 9 hours each.}$$

$$= 350 \text{ such days.}$$

Ex. 7. If 4 men earn £15 in 20 days, how many men will earn 10 guineas in 7 days?

men	days	£
4	20	15
		guin.
	7	10

The £15 and the 10 guineas, being in different denominations, must, in accordance with our Rule, be reduced to one and the same denomination.

Thus, £15. being = 300*s.*, and 10 guineas being = 210*s.*, the arrangement is:

men	days	*s.*
4	20	300
	7	210

$$\text{Dividend} = 4 \times 20 \times 210 = 16800$$

$$\text{Divisor} \quad = 7 \times 300 = 2100$$

$$\begin{array}{l} 21{,}00)\ 168{,}00\ (8 \\ \qquad\ \ \underline{168} \end{array}$$

The answer therefore is 8 men.

Ex. 8. If 560 flag-stones, each $1\frac{1}{2}$ feet square, will pave a court-yard, how many will be required for a yard twice the size, each flag-stone being 14 in. by 9 in.?

Superficial content of each of former flag-stones

$$= (1\tfrac{1}{2} \times 1\tfrac{1}{2})\ \text{sq. ft.} = (\tfrac{3}{2} \times \tfrac{3}{2})\ \text{sq. ft.} = \tfrac{9}{4}\ \text{sq. ft.}$$

Superficial content of each of the latter flag-stones

$$= (\tfrac{14}{12} \times \tfrac{9}{12})\ \text{sq. ft.} = (\tfrac{7}{6} \times \tfrac{3}{4})\ \text{sq. ft.} = \tfrac{7}{8}\ \text{sq. ft.}$$

Considering the first court-yard as 1, and therefore the second as 2, our arrangement of the given quantities will be

flag-stones.	sq. ft.	court-yd.
560	$\frac{9}{4}$	1
	$\frac{7}{8}$	2

$$\text{Dividend} = 560 \times \tfrac{9}{4} \times 2 = 140 \times 18,$$

$$\text{Divisor} \quad = \tfrac{7}{8} \times 1 = \tfrac{7}{8}.$$

$$\begin{aligned} \text{Required number of flag-stones} &= (140 \times 18) \div \tfrac{7}{8} \\ &= (140 \times 18) \times \tfrac{8}{7} \\ &= \frac{140 \times 18 \times 8}{7} \\ &= 20 \times 18 \times 8 \\ &= 360 \times 8 \\ &= 2880. \end{aligned}$$

Ex. 9. If 10 cannon, which fire 3 rounds in 5 minutes, kill 270 men in an hour and a half, how many cannon, which fire 5 rounds in 6 minutes, will kill 500 men in one hour?

The first 10 cannon, firing $\frac{3}{5}$ of a round in a minute, kill 270 men in $\frac{3}{2}$ hours. It is required to find how many cannon, firing $\frac{5}{6}$ of a round in a minute will kill 500 men in 1 hour. Our arrangement may be:

cannon	round	hours	men
10	$\frac{3}{5}$	$\frac{3}{2}$	270
	$\frac{5}{6}$	1	500

The producing quantities in the first line are the 10 cannon, the rate of firing, namely, $\frac{3}{5}$ of round per minute, and the duration of the firing, namely, $\frac{3}{2}$ hours; these causes conjoined produce 270 dead men.

$$\text{Dividend} = 10 \times \tfrac{3}{5} \times \tfrac{3}{2} \times 500 = \frac{10 \times 3 \times 3 \times 500}{10} = 4500.$$

$$\text{Divisor} \quad = \tfrac{5}{6} \times 1 \times 270 = \frac{5 \times 1 \times 270}{6} = 5 \times 45 = 225.$$

$$\therefore \text{ Required number of cannon} = \frac{4500}{225} = 20.$$

Ex. 10. A town which is defended by 1200 men, with provisions enough to sustain them 42 days, supposing each man to receive 18 oz. a day, obtains an increase of 200 men to its garrison; what must now be the allowance to each man, in order that the provisions may serve the whole garrison for 54 days?

In this case all the three given quantities which appear to express the conditions of the question must be regarded as producing quantities; for the 1200 men, in the 42 days, receiving 18 oz. a day each, would produce or cause a consumption of the provisions. And the two given quantities in the latter part of the question are also producing quantities, since the 1400 men, in the 54 days, with the required allowance, are to produce or cause the consumption of the same provisions. At first sight, therefore, it might appear that there is no produced quantity given; but, in fact, the thing produced is the consumption, and this is measured by, or equal to, the quantity of provisions consumed, which would be

$$18\text{oz.} \times 1200 \times 42 = 907200 \text{ oz.}$$

But it is clear that as the quantity to be consumed is the same in both cases, it will answer our purpose to consider it as represented, in each case, by some smaller number, such as unity, instead of taking the larger number 907,200. This, in effect, will be only the same thing as expressing the larger number in a higher denomination, though that may be one which is not in common use.

Our arrangement will thus become:

men	days	oz.	quantity consumed
1200	42	18	1
1400	54		1.

$$\text{Dividend} = 1200 \times 42 \times 18 \times 1 = 907200$$

$$\text{Divisor} \quad = 1400 \times 54 \times 1 = 75600.$$

Effecting the division, we have

$$\begin{array}{r} 756{,}00)\ 9072{,}00\ (12 \\ \underline{756}\phantom{{,}00\ (12} \\ 1512\phantom{{,}00\ (12} \\ \underline{1512}\phantom{{,}00\ (12} \end{array}$$

So that 12 oz. will be the answer.

Or thus:

$$\text{Required allowance} = \left(\frac{1200 \times 42 \times 18 \times 1}{1400 \times 54 \times 1}\right) \text{oz.}$$

$$= \left(\frac{12 \times 42 \times 18}{14 \times 54}\right) \text{oz.}$$

$$= \left(\frac{12 \times 3}{3}\right) \text{oz.} = 12 \text{ oz.}$$

Ex. 11. If the carriage of 37 stone, 6 lbs. for 7 miles cost £2. 5*s.*, what weight should be carried 12 miles for £3. 10*s.*?

Our arrangement may be the following:

st.	lbs.	miles	£.	*s.*
37	. 6	7	2	. 5
		12	3	. 10

where the producing quantities in the first line will be the 37 st., 6 lbs., and the 7 miles, since these together, that is, the weight carried over the distance, produce the £2. 5*s.* to the carrier.

Reducing the quantities according to the Rule, the arrangement becomes

lbs.	miles	*s.*
524	7	45
	12	70

$$\text{Dividend} = 524 \times 7 \times 70 = 256760$$

$$\text{Divisor} \ \ = 12 \times 45 \qquad = 540.$$

$$\therefore \text{ req}^{\text{d}}\text{. no. of lbs.} = \frac{256760}{540}$$

$$= 475 \text{ lbs., } 7 \text{ oz., } 11\tfrac{7}{27} \text{ drs.}$$

$$= 33 \text{ st., } 13 \text{ lbs., } 7 \text{ oz., } 11\tfrac{7}{27} \text{ drs.}$$

Instead of reducing the quantities to lower denominations, as in the above operation, we might have kept them in the higher denominations, by reducing any part which was expressed in a lower to a fraction of the higher denomination. Thus, observing that 6 lbs $= \frac{6}{14}$ st. $= \frac{3}{7}$ st., and 5*s.* $=$ £$\frac{1}{4}$, and 10*s.* $=$ £$\frac{1}{2}$, we have

st.	miles	£
$37\frac{3}{7}$	7	$2\frac{1}{4}$
	12	$3\frac{1}{2}$

$$\text{Dividend} = 37\tfrac{3}{7} \times 7 \times 3\tfrac{1}{2} = \tfrac{262}{7} \times 7 \times \tfrac{7}{2} = 131 \times 7 = 917.$$

$$\text{Divisor} = 12 \times 2\tfrac{1}{4} = 12 \times \tfrac{9}{4} = 3 \times 9 = 27.$$

$$\text{Required weight} = \tfrac{917}{27} \text{ stones}$$

$$= 33 \text{ st.}, 13 \text{ lbs.}, 7 \text{ oz.}, 11\tfrac{7}{27} \text{ drs.}$$

Note. It will be observed throughout the above Examples, that in the first line of our arrangement, we have put down first the causes, and then, after them, the effect or effects; thus determining of course the places of the numbers in the second line also. This order was adopted for each line, not because it is necessary, but because it enables us more readily to distinguish between the causes and the effects, when we require them in the subsequent process.

Ex. LVII.

1. If 7 men can reap 6 acres in 12 hours, how many men will reap 15 acres in 14 hours?

2. If 3 men earn £15 in 20 days, how many men will earn 15 guineas in 9 days, at the same rate?

3. If 16 horses eat 96 bushels of corn in 42 days, in how many days will 7 horses eat 66 bushels?

4. If 800 soldiers consume 5 sacks of flour in 6 days, how many will consume 15 sacks in 2 days?

5. If 17 bushels be consumed by 6 horses in 13 days, what quantity will 8 horses eat in 11 days, at the same rate?

6. 16 horses can plough 1280 acres in 8 days, how many acres will 12 horses plough in 5 days?

7. If 11 cwt. can be carried 12 miles for £1. 5 c. how far can 36 cwt., 23 lbs. be carried for £5. 2 fl. 5 c.?

8. If the carriage of 8 cwt. of goods for 124 miles be 6 guineas, what weight ought to be carried 53 miles for half the money?

9. If 5 men on a tour of 11 months, spend £641. 13*s.* 4*d.*, how much at the same rate would it cost a party of 7 men for 4 months?

10. If with a capital of £1000 a tradesman gain £100 in 5 months, in what time will he gain £49. 5 fl. with a capital of £225?

11. If it cost £59. 2*s.* 1½*d.* to keep 3 horses for 7 months, what will it cost to keep 2 horses for 11 months?

12. The carriage of 4 cwt., 3 qrs., for 160 miles costs £3. 8 fl. 5 c. what weight ought to be carried 100 miles for £6. 0*s.* 3¾*d.*?

13. If 1 man can reap 345$\frac{5}{7}$ sq. yds. in an hour, how long will 7 such men take to reap 6 acres?

14. If 20 men in 3 weeks earn £90., in what time will 12 men earn £150?

15. If the carriage of 1 cwt., 3 qrs., 21 lbs. for 52$\frac{1}{4}$ miles come to 17*s.* 5*d.*, what will be charged for 2$\frac{1}{2}$ tons for 46$\frac{1}{2}$ miles?

16. If 10 men can reap a field of 7$\frac{1}{2}$ acres in 3 days of 12 hours each, how long will it take 8 men to reap 9 acres, working 16 hours a-day?

17. If 25 men can do a piece of work in 24 days, working 8 hours a-day, how many hours a-day would 30 men have to work in order to do the same piece of work in 16 days?

18. If the rent of a farm of 17 ac., 3 ro., 2 po. be £39. 4*s.* 7*d.*, what would be the rent of another farm, containing 26 ac., 2 ro., 33 po. if 6 acres of the former be worth 7 acres of the latter?

19. If 1500 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of 25 sheets, of the same size as the former?

20. If 5 men can reap a rectangular field whose length is 800 ft. and breadth 700 ft. in 3$\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each can 7 men reap a field whose length is 1800 ft. and breadth 960 ft.?

21. If a thousand men besieged in a town with provisions for 5 weeks, allowing each man 16 oz. a-day, be reinforced with 500 men more, and have their daily allowance reduced to 6$\frac{2}{3}$ oz; how long will the provisions last them?

22. If 20 masons build a wall 50 feet long, 2 feet thick, and 14 feet high, in 12 days of 7 hrs. each, in how many days of 10 hrs. each will 60 masons build a wall 500 feet long, 4 thick, and 16 high?

23. If 10 men can perform a piece of work in 24 days, how many men will perform another piece of work 7 times as great, in one-fifth of the time?

24. If 125 men can make an embankment 100 yards long, 20 feet wide, and 4 feet high, in 4 days, working 12 hours a-day, how many men must be employed to make an embankment 1000 yards long, 16 feet wide, and 6 feet high, in 3 days, working 10 hours a-day?

25. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft., 3 in. deep, when a block of the same stone 5 ft. long, 3 ft., 9 in. broad, and 2 ft., 6 in. deep, weighs 7500 lbs.?

26. If 100 men drink £20 worth of wine at 4*s.* 6*d.* per bottle, how many men will drink £72 worth at 5*s.* per bottle, in the same time, at the same rate of drinking?

27. If 5 horses require as much corn as 8 ponies, and 15 quarters last 12 ponies for 64 days, how long may 25 horses be kept for £41. 5*s*. when corn is 22 shillings a quarter?

28. If 42½ yds. of cloth which is 18 in. wide cost £59. 14*s*. 2*d*., what will 118¼ yds. of yard-wide cloth of the same quality cost?

29. 124 men dig a trench 110 yds. long, 3 ft. wide, and 4 ft. deep, in 5 days of 11 hours each; another trench is dug by half the number of men in 7 days of 9 hours each; how many feet of water is it capable of holding?

30. If the fourpenny loaf weigh 3·35 lbs. when wheat is at 4·75*s*. a bus., what ought to be paid for 47½ lbs. of bread when wheat is at 13·4*s*. a bus.?

31. A pit 24 ft. deep, 14 sq. ft. horizontal section cost £3. to dig out; how deep will a pit be of horizontal section 7 ft. by 9 ft., which costs £4. 10*s*.?

32. The value of the paper required for papering a room, supposing it ¾ yard wide, and 4½*d*. a yard, is £2. 3*s*. 1½*d*.; what would it come to, if it were 2 feet wide and 4*d*. a yard?

33. 7 men working 16 days can mow a field of corn 1320 yards long and 880 wide; what will be the length of the side of a field 1320 yards broad which 4 men can mow in 42 days?

34. A beam 16 feet long, 2¼ feet broad, and 8 inches thick, weighs 1280 lbs.; what must be the length of another beam of the same material, whose breadth is 3¼ feet, thickness 7½ inches, and weight 2028 lbs.?

35. If 12 oxen and 35 sheep eat 12 tons, 12 cwt. of hay in 8 days, how much will it cost per month (of 28 days) to feed 9 oxen and 12 sheep, the price of hay being 4 guineas a ton, and 3 oxen being supposed to eat as much as 7 sheep?

36. If 1 man and 2 women do a piece of work in 10 days, find in how long a time 2 men and 1 woman will do a piece of work 4 times as great, the rates of working of a man and woman being as 3 to 2.

37. A person is able to perform a journey of 142·2 miles in 4½ days when the day is 10·164 hours long; how many days will he be in travelling 505·6 miles when the days are 8·4 hours long?

38. If the sixpenny loaf weigh 4·35 lbs. when wheat is at 5·75*s*. per bushel, what weight of bread, when wheat is at 18·4*s*. per bushel, ought to be purchased for $18\cdot1\dot{3}$*s*.?

39. If a family of 9 people can live comfortably in England for 1560 guineas a year, what will it cost a family of 8 to live in Belgium in the same style for seven months, prices being supposed to be ⅗ of what they *would be in England*?

INTEREST.

162. **Def.** **Interest** is the sum of money paid for the loan or use of some other sum of money, lent for a certain time at a fixed rate; generally at so much for each £100 for one year.

The money lent is called **the Principal.**

The interest of £100 for a year is called **the Rate per Cent.**

The principal + the interest is called **the Amount.**

Interest is divided into Simple and Compound. When interest is reckoned only on the original principal, it is called **Simple Interest.**

When the interest at the end of the first period, instead of being paid by the borrower, is retained by him and added on as principal to the former principal, interest being calculated on the new principal for the next period, and this interest again, instead of being paid, is retained and added on to the last principal for a new principal, and so on; it is called **Compound Interest.**

SIMPLE INTEREST.

163. *To find the Interest of a given sum of money at a given rate per cent. for a year.*

Rule. "Multiply the principal by the rate per cent., and divide the product by 100, as in (Art. 126)."

Note 1. The interest for any given number of years will of course be found by multiplying the interest for one year by the number of years; and the interest for any parts of a year may be found from the interest for one year, by Practice, or by the Rule of Three.

Note 2. If the interest has to be calculated from one given day to another, as for instance from the 30th of January to the 7th of February, the 30th of January must be left out in the calculation, and the 7th of February must be taken into account, for the borrower will not have had the use of the money for one day till the 31st of January.

Note 3. If the amount be required, the interest has first to be found for the given time, and the principal has then to be added to it.

Ex. Find the simple interest of £250 for one year at 5 per cent. per annum.

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Proceeding according to the Rule given above,

$$\begin{array}{r}
\pounds. \\
250 \\
5 \\
\hline
\pounds 12{\cdot}50 \\
20 \\
\hline
10{\cdot}00s.
\end{array}$$

therefore the interest is £12. 10*s.*

Reason for the Process.

The sum of £100 must have the same relation in respect of magnitude to £250 as the simple interest of £100 for a year has to the simple interest of £250 for a year; and thus the £100, £250, £5, and the required interest must form a proportion. (Art. 148).

We have then

$$\pounds 100 : \pounds 250 :: \pounds 5 : \text{required interest},$$

$$\text{whence, required interest} = \pounds\,\frac{250 \times 5}{100} \text{ (Art. 155)},$$

which agrees with the Rule given above.

Examples worked out.

Ex. 1. Find the simple interest and amount of £417. 7*s.* 9*d.* for 1 year, 10 months, at $4\frac{3}{8}$ per cent.

$$\begin{array}{rrr}
\pounds. & s. & d. \\
417\ . & 7\ . & 9 \\
 & & 4\frac{3}{8} \\
\hline
1669\ . & 11\ . & 0 \\
156\ . & 10\ . & 4\frac{7}{8} \\
\hline
\pounds 18{\cdot}26\ . & 1\ . & 4\frac{7}{8} \\
20 & & \\
\hline
5{\cdot}21s. & & \\
12 & & \\
\hline
2{\cdot}56d. & &
\end{array}
\qquad\qquad
\begin{array}{r|rrr}
 & \pounds. & s. & d. \\
 & 417\ . & 7\ . & 9 \\
 & & & 3 \\
\hline
8 & 1252\ . & 3\ . & 3 \\
\hline
 & 156\ . & 10\ . & 4\frac{7}{8}
\end{array}$$

$$\begin{array}{lrrr}
 & \pounds. & s. & d. \\
\therefore \text{ Int. for 1 year } \ldots\ldots\ldots\ldots\ldots\ldots & =18\ . & 5\ . & 2\dfrac{56\frac{7}{8}}{100} \\
 & =18\ . & 5\ . & 2\frac{91}{160} \\
\text{Int. for 6 mo., or } \frac{1}{2} \text{ of 1 year} & = 9\ . & 2\ . & 7\frac{91}{320} \\
\text{Int. for 4 mo., or } \frac{1}{3} \text{ of 1 year} & = 6\ . & 1\ . & 8\frac{137}{160} \\
\hline
\therefore \text{ Int. for 1 y., 10 mo. } \ldots\ldots\ldots & =33\ . & 9\ . & 6\frac{227}{320}
\end{array}$$

$$\therefore \text{ amount} = \pounds 417.\ 7s.\ 9d. + \pounds 33\ .\ 9s\ .\ 6\tfrac{227}{320}\,d.$$

$$= \pounds 450.\ 17s.\ 3\tfrac{227}{320}d.$$

Note. In examples like the above we may reckon 12 months to the year; but if Calendar months are given, the interest will then be best found by the Rule of Three; as for instance in the following example:

Ex. 2. Find the simple interest and the amount of £106. 13*s.* 4*d.* from June 15, 1843, to Sept. 18, 1843, at $4\frac{1}{2}$ per cent.

$$\begin{array}{rrr}
\pounds. & s. & d. \\
106\ . & 13\ . & 4 \\
 & & 4\frac{1}{2} \\ \hline
426\ . & 13\ . & 4 \\
53\ . & 6\ . & 8 \\ \hline
\pounds 4{\cdot}80\ . & 0\ . & 0 \\
20 & & \\ \hline
16{\cdot}00s. & &
\end{array}$$

∴ £4. 16*s.* is the interest for one year.

Now the number of days from June 15 to Sept. 18

$$=15+31+31+18$$
$$=95.$$

Hence, 365 days : 95 days :: £4. 16*s.* : interest required,

whence, it will be found, that interest required = £1. 4*s.* $11\frac{3}{4}d.$ $\frac{25}{73}q.$;

∴ amount = £106. 13*s.* 4*d.* + £1. 4*s.* $11\frac{3}{4}d.$ $\frac{25}{73}q.$ = £107. 18*s.* $3\frac{3}{4}d.$ $\frac{25}{73}q.$

164. Defs. Commission is the sum of money which a merchant charges for buying or selling goods for another.

Brokerage is of the same nature as Commission, but has relation to money transactions, rather than dealings in goods or merchandise.

Insurance is a contract, by which one party, on being paid a certain sum or *Premium* by another party on property which is subject to risk, undertakes, in case of loss, to make good to the owner the value of that property.

Now, questions on Commission, Brokerage, and Insurance, these charges being usually made at so much per cent., amount to the same thing as finding the interest on a given amount at a given rate for one year, and may therefore be worked by the Rule given above for Simple Interest.

There is, however, one case of Insurance which it may be well to notice by an example worked out.

15—2

Ex. If goods worth £1200 be insured at £1. 10*s.* per cent., to what amount must they be insured, so that in case of loss the party insuring may recover the value of the goods and the premium?

If they be insured at their actual worth the premium paid will be lost, since the insurer will get £1200. only.

But if every (£100 − £1. 10*s.*), or £98. 10*s.*, be insured for £100, then, in case of loss, the value of the goods £98. 10*s.* + £1. 10*s.* (the premium paid) will be recovered.

Thus we have

£98½ : £1200 :: £100 : sum which is required to be insured:
whence, sum required to be insured = £1218. 5*s.* 6*d.* nearly.

Ex. LVIII.

1. Find the simple Interest
   - (1) On £85 for 1 year at 5 per cent.
   - (2) On £310 for 1 year at 4 per cent.
   - (3) On £1000 for 1 year at 4½ per cent.
   - (4) On £475 for 3 years at 5 per cent.
   - (5) On £936. 11*s.* 3*d.* for 2 years at 4 per cent.
   - (6) On £556. 13*s.* 4*d.* for 6 years at 5 per cent.
   - (7) On £945. 10*s.* for 2 years at 4 per cent.
   - (8) On £198. 6*s.* 8*d.* for 1 year at 3½ per cent.
   - (9) On £236. 6*s.* 8*d.* for 2½ years at 3 per cent.
   - (10) On £98. 15*s.* 10*d.* for ½ year at 2½ per cent.
2. Find the amount
   - (1) Of £1000 for 2 years at 4¼ per cent.
   - (2) Of £2833. 6*s.* 8*d.* for 4½ years at 3 per cent.
   - (3) Of £1050. 6 fl. 2c. 5 m. for 6 years at 4¼ per cent.
   - (4) Of £139. 12*s.* 6*d.* for 3½ years at 5½ per cent.
   - (5) Of 1895 guineas for 4¾ years at 2¾ per cent.
   - (6) Of £1534. 6*s.* 3*d.* for 1⅝ years at 3¼ per cent.
   - (7) Of £411. 10*s.* for ¼ year at 4⅞ per cent.
   - (8) Of £1595. 1 fl. 2 c. 5 m. for 5¾ years at 3¾ per cent.
3. Find the Simple Interest and Amount
   - (1) Of £375 for 3 years, 8 months, at 3½ per cent.
   - (2) Of £446. 10*s.* for 3 years, 3 months, at 5 per cent.
   - (3) Of £220 for 7 months at 3¾ per cent.
   - (4) Of £243. 10*s.* for 2 years, 5 months, at 4¾ per cent.
   - *(5) Of 10* guineas for 117 days at 3½ per cent.

(6) Of £684. 18*s.* 8*d.* for 1 year, 11 months, at 4⅛ per cent.

(7) Of 40 guineas from March 16, 1850, to Jan. 23, 1851, at 3⅛ per cent.

(8) Of £320. 15*s.* for 2 years, 35 days, at 4¾ per cent.

(9) Of £34. 10*s.* from August 10 to October 21, at 4½ per cent.

4. Find the brokerage on £715. 12*s.* 6*d.* at 4½ per cent.

5. What is the annual cost of insuring £4000 worth of property at ½ per cent.?

6. What must be the sum insured at 4½ per cent. on goods worth £1910, so that in case of loss the worth of the goods and the premium may be recovered?

7. At 7½ per cent., what will be the cost of insuring property worth 500 guineas, so that in the event of loss the worth of the goods and the premium of insurance may be recovered?

165. *In all questions of Interest, if any three of the four, principal, rate per cent., time, amount, be given, the fourth may be found; as, for instance, in the following examples.*

Ex. 1. Find the amount of £225 in 4 years at 3½ per cent. simple interest.

$$\begin{array}{r}
\text{£.}\\
225\\
\underline{3\tfrac{1}{2}}\\
675\\
\underline{112\ .\ 10}\\
\text{£}7\cdot87\ .\ 10\\
\underline{20}\\
17\cdot50s.\\
\underline{12}\\
6\cdot00d.
\end{array}$$

∴ Int. for 1 year = £7. 17*s.* 6*d.*

............4 years = £31. 10*s.*;

∴ amount is £225 + £31. 10*s.*, or £256. 10*s.*

Ex. 2. In what time will £225 amount to £256. 10*s.* at 3½ per cent. simple interest?

Now £256. 10*s.* – £225. = £31. 10*s.*, which is the interest to be obtained on £225 in order that it may amount to £256. 10*s.*

But Int. of £225 for one year = £7. 17*s.* 6*d.*; which must have the same relation in respect of magnitude to the £31. 10s. as the 1 year has *to the required time;*

∴ £7. 17*s.* 6*d.* : £31. 10*s.* :: 1 year : required number of years,
whence, required number of years = 4.

Ex. 3. At what rate per cent., simple interest, will £225 amount to £256. 10*s.* in 4 years?

In other words, at what rate per cent. will £225 give £31. 10*s.* for interest in 4 years?

Find what it will give at 1 per cent.

$$\begin{array}{r} \pounds. \\ 2{\cdot}25 \\ \underline{20} \\ 5{\cdot}00s. \end{array}$$

∴ (£2. 5*s.* × 4) or £9 is the interest in 4 years at 1 per cent.; and this £9 must be to the £31. 10*s.* as the 1 per cent. is to the required rate per cent.;

∴ £9. : £31½. :: 1 : required rate per cent.,
whence, required rate per cent. = 3½.

Ex. 4. What sum of money will amount to £256. 10*s.* in 4 years at 3½ per cent. simple interest?

£100 in 4 years at 3½ per cent. amounts to £100 + (3½ × 4)£, or £114; and this £114 must be to the £256. 10*s.* as the £100 is to the required sum of money;

∴ £114 : £256½ :: £100 : required number of pounds,
whence, required number of pounds = £225.

Ex. LIX.

1. What sum will amount to £150. 8*s.* in 4 years at 5 per cent. simple interest?

2. At what rate per cent. will £540 amount to £734. 8*s.* in 9 years, at simple interest?

3. In what time will £350 amount to £402. 5 fl. at 3 per cent. simple interest?

4. At what rate per cent. will £325. 16*s.* 8*d.* amount to £374. 6*s.* 0¼*d.* in 3½ years at simple interest?

5. In what time will £142. 10*s.* amount to £227. 5*s.* 9*d.* at 3½ per cent. simple interest?

6. At what rate will £157. 15*s.* 4*d.* amount to £295. 16*s.* 3*d.* in 25 years at simple interest?

7. What sum will produce for interest £56. 14*s.* in 2¼ years at 4½ per cent. simple interest?

8. What sum will amount to £105. 6*s.* 0½*d.* in 3½ years at 4¼ per cent., simple interest?

9. What sum will amount to £387. 7*s.* $7\frac{1}{5}$*d.* in 3 years at 4 per cent., simple interest?

10. In what time will £1275 amount to £1549. 11*s.* at $3\frac{4}{5}$ per cent. simple interest?

11. At what rate per cent., simple interest, will £936. 13*s.* 4*d.* amount to £1157. 7*s.* $4\frac{1}{2}$*d.*, in $4\frac{7}{8}$ years?

12. In what time will £125 double itself at 5 per cent. simple interest?

13. What sum will amount to £425. 19*s.* $4\frac{4}{5}$*d.* in 10 years at $3\frac{1}{2}$ per cent. simple interest, and in how many more years will it amount to £453. 11*s.* 7*d.*?

14. What sum of principal money, lent out at 5 per cent. per annum, simple interest, will produce in 4 years the same amount of interest as £250, lent out at 3 per cent. per annum, will produce in 6 years?

COMPOUND INTEREST.

166. *To find the Compound Interest of a given sum of money at a given rate per cent. for any number of years.*

RULE. "At the end of each year add the interest of that year, found by (Art. 163), to the principal at the beginning of it; this will be the principal for the next year; proceed in the same way as far as may be required by the question. Add together the interests so arising in the several years, and the result will be the compound interest for the given period."

The reason for the above Rule is clear from what has been stated in (Arts. 162 and 163).

Ex. Required the compound interest and the amount of £720. for 3 years at 5 per cent.

Proceeding as in Simple Interest for the 1st year;

$$\begin{array}{r} £720 \\ \underline{5} \\ £36{\cdot}00 \end{array}$$

$$\begin{array}{lrl} & £720 & = \text{1st principal,} \\ & \underline{36} & = \text{1st interest,} \\ \text{by addition,} & £756 & = \text{2nd principal, of which find interest at 5 per cent.} \\ & \underline{5} & \\ & £37{\cdot}80 & \\ & \underline{20} & \\ & 16{\cdot}00s. & \end{array}$$

£756 = 2nd principal,
37 . 16 = 2nd interest,
793 . 16 = 3rd principal, of which find interest as above,
5
£39·69 . 0
20
13·80s.
12
9·60d.

£793 . 16 = principal for 3rd year,
39 . 13 . 9$\frac{3}{5}$ = interest for 3rd year,
∴ £833 . 9 . 9$\frac{3}{5}$ = amount of £720. in 3 years at 5 per cent. compound interest.

The compound interest for that time
= sum of interests for each year,
= £36 + £37. 16s. + £39. 13s. 9$\frac{3}{5}$d. = £113. 9s. 9$\frac{3}{5}$d.

Note 1. It is customary, if the compound interest be required for any number of entire years and a part of a year, (for instance for 5$\frac{3}{4}$ years), to find the compound interest for the 6th year, and then take $\frac{3}{4}$ths of the last interest for the $\frac{3}{4}$ths of the 6th year.

Note 2. If the interest be payable half-yearly, or quarterly, it is clear that the compound interest of a given sum for a given time will be greater as the length of each given period is less; the simple interest will not be affected by the length of each period.

Ex. LX.

1. Find the compound interest of £2000 in 2 years at 4 per cent. per annum.
2. Find the amount of £800 in 3 years at 3$\frac{3}{4}$ per cent., allowing compound interest.
3. Find the compound interest of £270 in 2 years, at 3 per cent.
4. Find the amount of £690 for 3 years at 4$\frac{1}{2}$ per cent., compound interest.
5. Find the amount of £230. 15s. for 3 years, at 5 per cent., compound interest.
6. Find the difference in the amount of £415. 10s., put out for 4 years at 2$\frac{1}{2}$ per cent., 1st at simple, 2nd at compound interest.

7. Find the compound interest of £130 in 3 years at 4 per cent. (interest being payable half-yearly).

8. What will £1760. 10*s.* amount to in $2\frac{1}{2}$ years, allowing 4 per cent. compound interest?

9. A person lays by £230 at the end of each year, and employs the money at $3\frac{1}{2}$ per cent. compound interest; what will he be worth at the end of 3 years?

10. Find the difference between the simple and compound interest of £416. 13*s.* 4*d.*, for 2 years at $2\frac{1}{4}$ per cent.

11. What is the difference between the simple and the compound interest of £13,333. 6*s.* 8*d.* for 5 years, at 5 per cent.?

12. Find the amount of £180 in 3 years at $4\frac{1}{2}$ per cent. compound interest.

13. What sum of money put out to compound interest for 2 years at 5 per cent. will amount to £100?

14. What sum at 5 per cent. compound interest will amount in 2 years to £264. 12*s.*?

15. *A* and *B* each lend £256 for 3 years at $4\frac{1}{2}$ per cent. per annum, one at simple interest, the other at compound interest: find the difference in the amount of interest they respectively receive.

PRESENT WORTH AND DISCOUNT.

167. *A* owes *B* £500, which is to be paid at the end of 9 months from the present time: now it is clear that, if the debt be discharged at once (interest being reckoned, we will suppose, at 4 per cent. per annum), *B* ought to receive a less sum of money than £500; in fact such a sum of money as will, being now put out at 4 per cent. interest, amount to £500 at the end of 9 months. The sum which *B* ought to receive *now* is called the Present Worth of the £500 due 9 months hence, and the sum to be deducted from the £500, in consequence of immediate payment, which is in fact the interest of the Present Worth, is called the Discount of the £500 discharged 9 months before it is due.

DEF. We may therefore define PRESENT WORTH to be the actual worth at the present time of a sum of money due some time hence, at a given rate of interest; and we may define the DISCOUNT of a sum of money to be the interest of the Present Worth of that sum, calculated from the present time to the time when the sum would be properly payable.

PRESENT WORTH.

168. Rule. "Find the interest of £100 for the given time at the given rate per cent., and state thus:

£100 + its interest for the given time at the given rate per cent. : given sum :: £100. : present worth required."

Ex. 1. Find the present worth of £500, due 9 months hence, at 4 per cent. per annum.

Proceeding according to the above Rule,

Interest of £100 for 9 months at 4 per cent. is £3,

$\therefore$ £103 : £500 :: £100 : required present worth,

whence, required present worth = £485. 8*s*. $8\frac{88}{103}$*d*.

The reason for the above process is clear from the consideration, that £100 in 9 months at 4 per cent. interest would amount to £103, and therefore £100 is the present value of £103 due 9 months hence: and consequently we have

1st debt : 2nd debt :: 1st present worth : 2nd present worth.

Ex. 2. Find the present worth of £838, due 19 months hence, at 3 per cent. simple interest.

Since the interest of £100 for 19 months, at 3 per cent.

$$= £\left(\frac{19}{12} \times 3\right) = £\frac{19}{4} = £4\frac{3}{4},$$

$\therefore$ £$104\frac{3}{4}$: £838 :: £100 : required present worth,

whence, required present worth = £800.

Ex. 3. What is the value, at 16 years of age, of a legacy of £1000 payable at 21 years of age, allowing simple interest at 4 per cent.?

Since £100 at 4 per cent. simple interest will in 5 years amount to £120, therefore the present worth of £120 due 5 years hence will at that rate be £100.

Hence £120 : £1000 :: £100 : required value,

whence, required value = £833. 6*s*. 8*d*.

DISCOUNT.

169. Rule. "Find the interest of £100 for the given time at the given rate per cent., and state thus:

£100 + its interest for the given time at the given rate per cent. : given sum :: interest of £100 for the given time at the given rate per cent. : *discount required.*"

Ex. 1. Find the discount of £500, due 9 months hence, at 4 per cent. per annum.

Proceeding according to the above Rule,

The interest of £100 for 9 months at 4 per cent. = £3; therefore proceeding according to the Rule,

£103 : £500 :: £3 : required discount,

whence, required discount = £14. 11*s*. $3\frac{15}{103}$*d*.

The reason for the above process is clear from the consideration, that £3 is the interest for 9 months, at 4 per cent., of £100, the present worth of £103 due at the end of that time; and consequently we have

1st debt : 2nd debt :: discount on 1st debt : discount on 2nd debt.

Ex. 2. Find the discount on £1000, due 15 months hence, at 5 per cent. per annum.

The interest of £100 for 15 months at 5 per cent. = £6. 5*s*.;

∴ £106. 5*s*. : £1000 :: £6. 5*s*. : required discount,

whence, required discount = £58. 16*s*. $5\frac{11}{17}$*d*.

Ex. 3. Find the discount on £127. 2*s*. for half a year at 5 per cent.

£$100\frac{5}{2}$: £$127\frac{1}{10}$ :: £$\frac{5}{2}$: required discount;

whence, required discount = £3. 2*s*.

Note 1. Discount = given sum *less* Present Worth; Present Worth = given sum *less* Discount.

Note 2. In the discharge of a tradesman's bill it is usual to deduct interest instead of discount; thus, if *B* contracts with *A* a debt of £100, *A* giving 12 months' credit, it is usual in business, if the interest of money be reckoned at 5 per cent. per annum, and the bill be discharged at once, for *A* to throw off £5, or for *A* to receive £95 instead of £100; but if *A* were to put out the £95 at 5 per cent. interest it will not amount to £100 in 12 months; therefore such a procceding is to the advantage of *B*: the sum of money which in strictness ought to have been deducted, was not £5, the interest on the whole debt, but £4. 15*s*. $2\frac{6}{7}$*d*., the interest of the present worth of the debt, *i.e.* the discount.

Note 3. Bankers and Merchants in discounting bills calculate interest, instead of discount, on the sum drawn for in the bill, from the time of their discounting it to the time when it becomes due, adding THREE DAYS OF GRACE, which days are allowed in England after the time a bill is

NOMINALLY due, before it is LEGALLY due; which is of course an additional advantage. When a bill is payable on demand, the days of grace are not allowed.

Note 4. If a bill, without the days of grace, should appear to be due on the 31st of any month which contains only 30 days, the last day of that month, and not the first day of the next, is considered as the day on which the bill is due. Thus a bill drawn on the 31st of October, at 4 months, would be really due, adding in the days of grace, on the 3rd of March. Also bills which fall due on a Sunday, are paid in England on the previous Saturday.

Ex. A bill of £1000 is drawn on Feb. 16th, 1851, at 7 months' date; it is discounted on the 8th of July at 5 per cent. What does the banker gain by the transaction?

The bill is legally due on Sept. 19; and from July 8 to Sept. 19 are 73 days.

	£.	s.
The interest of £1000 for that time =	10 .	0
The true discount .... =	9 .	$18\frac{2}{101}$
∴ the banker's gain		$1\frac{99}{101}$s.

Ex. LXI.

1. Find the Present Worth of

(1) £283. 10*s*. due 1 year hence, at 5 per cent. per annum, simple [interest.
(2) £252. 19*s*. 3*d*.... .... $3\frac{1}{4}$ ....
(3) £676. 13*s*. 4*d*.... 6 months ...... 3 ....
(4) £284. 18*s*. ... 6 .... $3\frac{1}{2}$ ....
(5) £460. 10*s*. ... 7 .... 4 ....
(6) £390 ... 7 .... $3\frac{1}{4}$ ....
(7) £572 ... 8 .... $3\frac{3}{4}$ ....
(8) £1261. 1*s*. ... 1 .... 1 ....
(9) £35 ... 4 .... $4\frac{1}{2}$ ....
(10) £1250 ... 3 .... $3\frac{1}{2}$ ....
(11) £2110 ... 11 .... 6 ....
(12) £275. 6*s*. 8*d*. ... 15 .... 4 ....
(13) £918 ... 4 years ...... 5 ....
(14) £500 ... 19 months ...... $5\frac{1}{8}$ ....
(15) 800 gs. ... 20 years ...... $5\frac{1}{2}$ ....
(16) £2197 ... 3 years ...... 4 .... compound interest

2. Find the Discount on

(1)	£63. 6*s*. 8*d*.	due 4 months hence,	at 4 per cent. per annum,	[simple interest.
(2)	£1380. 7*s*. 6*d*.	... 9	3	....
(3)	£107. 5*s*.	... 6	5	....
(4)	£125. 10*s*.	... 3	3½	....
(5)	£487	... 5	3½	....
(6)	£340	... 5	4	....
(7)	£3640	... 10	4⅘	....
(8)	£813. 9*s*.	... 1⅓ year	4¾	....
(9)	£250. 15*s*.	... 17 months	5	....
(10)	£55.	... 146 days	4¾	....

(11) A bill of £649 is dated on June 23, 1853, at 6 months, and is discounted on July 8, at 3¼ per cent.; what does the banker gain thereby?

(12) Find the true discount on a bill drawn March 17, 1853, at 3 months, and discounted May 2, at 5¾ per cent.

(13) Find the simple interest on £545 in 2 years, at 3½ per cent. per annum; and the discount on £583. 3*s*. due 2 years hence, at the same rate of interest. Explain clearly why these two sums are identical.

(14) Explain the difference between Discount and Interest.
Five volumes of a work can be bought for a certain sum, payable at the end of a year; and six volumes of the same work can be bought for the same sum in ready money: what is the rate of discount?

(15) A tradesman marks his goods with two prices, one for ready money, and the other for one year's credit allowing discount at 5 per cent.; if the credit price be marked at £2. 9*s*., what ought to be the cash price?

STOCKS.

170. If the 3 per cent. Consols be quoted in the money-market at 96⅜, the meaning of this is, that for £96. 7*s*. 6*d*. of money a person can purchase £100 stock for which he will receive an acknowledgment which will entitle him to half-yearly dividends from Government, at the rate of 3 per cent. per annum on the stock held by him.

Similarly, if shares in any trading company, which were originally fixed at any given amount, say £100 each, be advertised in the share-market at 86, the meaning of this is, that for £86 of money one share can be obtained, and the holder of such share will receive dividends at the end of each half-year upon the £100 share, according to the state of the finances of the company.

Def. Stock may therefore be defined to be the capital of trading companies; or to be the money borrowed by our or any other Government, at so much per cent., to defray the expenses of the nation.

The amount of debt owing by the Government is called the National Debt, or the Funds. The Funds represent the credit of the country, which is bound to pay whatever debts are contracted by its Government. The government, however, reserves to itself the option of paying off the principal at any future time whatever; pledging itself, nevertheless, to pay the interest on it regularly at fixed periods, in the mean time.

From a variety of causes the price of stock is continually varying. A fundholder can at any time convert his stock into money, and it will depend upon the price at which he disposes of his stock, as compared with that at which he bought it, whether he will gain or lose by the transaction.

Note 1. Purchases or sales of stock are generally made through brokers, who charge £$\frac{1}{8}$, or 2*s.* 6*d.*, per cent. upon the stock bought or sold: so that in practice, when stock is bought by any party, every £100 stock costs that party £$\frac{1}{8}$ more than the market-price of the stock: and when stock is sold, the seller gets £$\frac{1}{8}$ less for every £100 stock sold than the market-price.

Thus, the actual cost of £100 stock in the 3 per cents. at $94\frac{1}{8}$, is £$(94\frac{1}{8}+\frac{1}{8})$, or £$94\frac{1}{4}$. The actual sum received for £100 stock in the 3 per cents. at $94\frac{1}{8}$, is £$(94\frac{1}{8}-\frac{1}{8})$, or £94.

Unless the brokerage is mentioned, it need not be noticed in working examples in stocks.

Note 2. When the price of £100 stock is £100 in money, the stock is said to be at *par.*

When the price of £100 stock is more than £100 in money, the stock is said to be at *a premium.*

When the price of £100 stock is less than £100 in money, the stock is said to be at *a discount.*

All examples in Stocks depend on the principles of proportion: those of most frequent occurrence will now be given.

Ex. 1. Required the sum which will purchase £1500 in the 3 per cents. at 82.

In this case £100 stock costs £82 in money;

∴ £100 stock : £1500 stock :: £82 money : required sum of money;
whence, required sum of money = £1230.

Ex. 2. What amount of stock in the 3¼ per cents. at 90 will £4050 purchase?

In this case £90 money will purchase £100 stock;

∴ £90 : £4050 :: £100 stock : required amount of stock;
whence, required amount of stock = £4500.

Ex. 3. If I buy £1520, 3 per cent. consols at 93¼, and pay £⅛ for brokerage, what does it cost me?

Every £100 stock costs me £(93¼ + ⅛), or £93⅜;

∴ £100 stock : £1520 stock :: £93⅜ : required sum of money;
whence, required sum of money = £1419. 6*s.*

Ex. 4. What sterling money shall I receive for £1920. 13*s.* 4*d.* in the 3½ per cents. at 98⅞, brokerage being £⅛ per cent.?

£100 stock realizes £(98⅞ − ⅛) = £98¾;

∴ £100 stock : £1920⅔ stock :: £98¾ : required sterling money;
whence, required sterling money = £1896. 13*s.* 2*d.*

Ex. 5. If I invest £7927. 10*s.* in the 3 per cents. at 94⅜, what annual income shall I receive from the investment?

For every £94⅜ I get £100 stock, and the interest on £100 stock is £3; therefore for every £94⅜ of money I get £3 interest;

∴ £94⅜ : £7927 10*s.* :: £3 : required annual income;
whence, required annual income = £252.

Note 3. If it be required to find the income arising from a certain quantity of stock, the question is merely one of simple interest.

Note 4. It may be noticed in the above examples, that when the question was simply to find amount of stock, or money realized by sale of stock, the 3, 4, or other rate per cent. never entered into the *statement;* and when the question was simply to find income arising from any sum invested in the funds, then the £100 never entered into the *statement.*

Ex. 6. Which is the best stock to invest £1000 in, the 3 per cents. at 89½, or the 3½ per cents. at 98½?

In the first case,

every £89½ of money gives £3 interest;

∴ every £1 of money gives £ $\frac{3}{89\frac{1}{2}}$, or £ $\frac{6}{179}$ interest.

In the second case,

every £$98\frac{1}{2}$ of money gives £$3\frac{1}{2}$ interest;

$\therefore$ every £1 of money gives £ $\frac{3\frac{1}{2}}{98\frac{1}{2}}$, or £ $\frac{7}{197}$, interest;

and comparing the fractions $\frac{6}{179}$ and $\frac{7}{197}$,

since 7×179 is $> 6 \times 197$,

the 2nd fraction is greater than the 1st, and therefore the second investment the best.

Ex. 7. How much stock can be purchased by the transfer of £2000 from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 96; and what change will be effected in income by it?

In order to find how much stock at 96 can be purchased for £2000 stock at 90, we must consider that the higher the price of the stock the less will the quantity of it produced be by the purchase, so that we must state as follows;

96 : 90 :: £2000 stock : required amount of stock,

whence, required amount of stock = £1875.

Income in first case = £60, income in second case = £65. 12*s*. 6*d*.;

$\therefore$ income is increased by £5. 12*s*. 6*d*.

Note 5. All questions of the transfer of stock from one kind to another, belong to the Rule of Three Inverse.

Note 6. The last question might have been worked thus: first sell out the stock at 90, and then invest the proceeds in $3\frac{1}{2}$ per cents. at 96.

Ex. 8. A person purchases £1000 3 per cent. consols at $97\frac{1}{8}$, and sells out again when they have sunk to $83\frac{1}{2}$; how much does he lose by the transaction?

He loses on every £100 stock £$(97\frac{1}{8} - 83\frac{1}{2})$, or £$13\frac{5}{8}$;

therefore his total loss = £$(13\frac{5}{8} \times 10)$ = £136. 5*s*.

Ex. LXII.

Find the quantity of stock purchased by investing:

(1) £2850 in the 3 per cents. at 75.
(2) £712 in the $3\frac{1}{2}$ per cents. at 89.
(3) £504 in the 4 per cents. at 96.
(4) £883. 5 fl. in the 4 per cents. at 93.

(5) £3741 in the 3¾ per cents. at 87.
(6) £500 in the 3 per cents. at 83¾.
(7) £800 in the 4 per cents. at 75½.
(8) £4311. 8*s*. 9*d*. in the 3½ per cents. at 85⅜.
(9) 2000 guineas in the 3½ per cents. at 94.
(10) £2353 in the 3 per cents. at 90⅜, brokerage ⅛ per cent.
(11) £3277 in the 4 per cents. at 105⅞, brokerage ⅛ per cent.
(12) 10000 guineas in the 3¼ per cents. at 99¼, brokerage ⅛ per cent.

2. Find the value in sterling money of

(1) £2600 in the 4 per cents. at 93.
(2) £1920 in the 3 per cents. at 77½.
(3) £3000 in the 3¾ per cents. at 92½.
(4) £2240 in the 3¼ per cents. at 81⅞.
(5) £3416 3 per cent. stock at 89 per cent.
(6) £1743 3¼ per cent. stock at 82⅞ per cent.
(7) £2675 4 per cent. stock at 91⅗ per cent.
(8) £1000 4 per cent. stock at 97⅜ per cent., brokerage ⅛ per cent.
(9) £2153. 10*s*. bank stock at 188⅞ per cent., brokerage ⅛ per cent.

3. Find the yearly income arising from the investment of

(1) £1008 in the 3 per cents. at 84.
(2) £5580 in the 4 per cents. at 93.
(3) £1138. 5 fl. in the 4½ per cents. at 92.
(4) £1638 in the 4½ per cents. at 93⅗.
(5) £2000 in the 3 per cents. at 88½.
(6) £3425. 15*s*. 2*d*. in the 3 per cents. at 91¾.
(7) £4788 in the 3½ per cents. at 105.
(8) £3500 in the 3 per cent. consols at 94⅛, brokerage ⅛ per cent.
(9) 5000 guineas in the 3½ per cents. at 102⅜, brokerage ⅛ per cent.

4. What sums of money must be invested in the undermentioned stocks in order to produce the following incomes?

(1) £60 in the 3 per cents. at 85.
(2) £288 in the 3 per cents at 67.
(3) £70 in the 3½ per cents. at 90.
(4) £83. 2 fl. 5 c. in the 4½ per cents. at 94.

(5) £87 in the 3 per cents. at 74½, brokerage ⅛ per cent.
(6) £37. 10*s*. in the 4 per cents. at 93¼, brokerage ⅛ per cent.

5. At what rate per cent. will a person receive interest who invests his capital?

(1) In the 3 per cents. at 91.
(2) In the 3½ per cents. at 94.
(3) In the 4½ per cents. at 96½, brokerage ⅛ per cent.
(4) In the 5 per cents. at 102⅞, brokerage ⅛ per cent.

6. If £7927. 10*s*. be laid out in purchasing 3 per cent. stock at 94⅜, what annual income will be derived from this investment, after deducting an income-tax of 7*d*. in the pound?

7. A person invested money in the 3 per cent. consols when they were at 90, and some more when they were at 80; find the rate of interest he obtained in each case, and the advantage per cent. of the second purchase over the first.

8. Find the income which will be derived from a capital of £2000, if ⅜ths of it be invested in the 3 per cents. at 98, and the remainder in the 3½ per cents. at par.

9. If a person receives 4½ per cent. interest on his capital by investing in the 3½ per cents., what is the price of the stock, and how much stock can be purchased for £1200?

10. How much money must a broker invest in the funds when consols are at 90, so as to procure the same income as if he had invested £1100 when consols were at 99?

11. A person buys £500 stock at 98¾, and sells out at 103; what does he gain by the transaction?

12. A person invests 9000 guineas in the 3 per cents. at 81, and sells out when they have sunk to 67½; how much does he lose by the transaction?

13. When £100 stock may be purchased in the 3 per cents. for £89½, at what rate may the same quantity of stock be purchased in the 3½ per cents. with equal advantage?

14. A person invests his share of a legacy of £1000, which is a third, in the 3 per cents. at 88⅜ per cent., find his half-yearly dividends.

15. A person transfers £1000 stock from the 4 per cents. at 90, to the 3 per cents. at 72; find the alteration in his income.

16. What incomes will £5000 of 3½ per cent. stock, and £5000 sterling invested in the 3½ per cent. stock at 102⅜, respectively produce?

17. Find the income produced by £12600 of 3 per cent. stock; and its sterling value, when the stocks are at 95.

18. A person transfers £3000 stock from the 3 per cent. consols at $89\frac{3}{8}$, to the reduced $3\frac{1}{2}$ per cents. at $98\frac{1}{4}$: find what quantity of the latter he will hold, and the alteration in his income.

19. Which is the best stock to invest £10000 in, the 3 per cents. at $90\frac{1}{2}$, or the 4 per cents. at 101?

20. A person invests £1037. 10*s.* in the 3 per cents. at 83, and when the funds have risen 1 per cent. he transfers his capital to the 4 per cents. at 96: find the alteration in his income.

21. Which is the better investment, the $3\frac{1}{2}$ per cents. at 96, or the 4 per cents. at 111, and what is the difference per cent. between them?

22. If £512 be invested in the 3 per cents. at 96, what will be the half-yearly interest, after deducting an income-tax of 7*d.* in the pound?

23. How much in the 3 per cents. at 96 must be sold out to pay a bill of £1654, 9 months before it becomes due, real discount being allowed at $4\frac{1}{2}$ per cent. per annum?

24. Which is the better investment, £1896 in the $3\frac{1}{2}$ per cents. at 87, or in railway shares at £89 per share, the dividends in the latter case being $3\frac{3}{4}$ per cent. on the sum invested?

25. A person has £2950 in the 3 per cents. at $83\frac{1}{2}$; when the funds have fallen $2\frac{1}{2}$ per cent., he transfers his capital into the 5 per cents. at 108; find the alteration in his income.

26. Which would be the best investment, 3 per cent. stock at $87\frac{3}{8}$, or shares at £233 each, on each of which a dividend of £7. 13*s.* 4*d.* is paid annually? What sum must be invested in the former to produce an annual income of £460? and what in the latter?

27. If the $3\frac{1}{2}$ per cents. be at 91, how much must a person invest in order that he may have a yearly income of £460, after paying 7*d.* in the pound for income-tax?

28. The dividends on a certain amount of 3 per cent. stock accumulated in 13 years to £3081. How much stock was there, and what will it be worth if the stock be sold at $79\frac{7}{8}$?

29. A person possesses £3200 3 per cents., which he sells at $99\frac{3}{4}$: he invests the proceeds in railway shares at £56 a share, which shares pay 5 per cent. interest on £45, the amount paid on each share. How much is his income altered by the transaction?

30. If I lay out £1911 in the purchase of 3 per cent. consols, when they are at $79\frac{1}{2}$, at what price should I sell out my stock again in order to

realize on the whole a gain of £150, after having paid ⅛th per cent. for commission on each transaction?

31. A person had £10,000 in the 3 per cent. South Sea Annuities, and the Government offered to give £110 bearing interest at the rate of 2½ per cent. for every £100 of these annuities, or to pay the £10,000 in cash on a certain day. The latter proposal was preferred, and on the money being paid it was re-invested in consols at 93. How much would he have lost in income had he accepted the first proposal, and what will he now gain by the new investment?

32. What sum would be saved annually if the interest on a public debt of £4,000,000 were reduced from 3½ per cent. to 3 per cent.? If in consequence the price of this stock fell from £101 to £95⅜, how much would the whole property of the fundholders be diminished?

PROFIT AND LOSS.

171. Def. All questions in Arithmetic which relate to gain or loss in mercantile transactions, fall under the head of Profit and Loss.

Examples in Profit and Loss are worked by the principle of Proportion: various examples will now be worked out by way of illustration.

Ex. 1. If a cask of wine containing 84 gallons cost £112. 5*s*., what is gained by selling it at 31*s*. 6*d*. per gallon?

The gain = selling price *less* first cost;
the selling price = (31½ × 84)*s*. = £132. 6*s*.;
therefore the gain = £132. 6*s*. − £112. 5*s*. = £20. 1*s*.

Ex. 2. A ream of paper cost me 21*s*. 6*d*., what must I sell it at, so as to realize 20 per cent.?

The reasoning in this case is, If £100 gain £20, or produce £120, what will 21*s*. 6*d*. produce?

∴ £100 : 21*s*. 6*d*. :: £120 : required amount in pounds,
whence, required amount = £1. 5*s*. 9⅗*d*.

Ex. 3. If I buy hay at £4. 16*s*. a ton, what must I sell it at to lose 15 per cent.?

In this case, every £100 would realize £(100 − 15), or £85;
∴ £100 : £4. 16*s*. :: £85 : required amount in pounds,
whence, required amount = £4. 1*s*. 7⅕*d*.

Ex. 4. A man buys 33 geese for £10; at how much per head must he sell them to gain 10 per cent. on his outlay?

In this case,

£100 : £10 :: £110 : selling price of the geese in pounds,
whence, selling price = £11,
∴ selling price of each goose = £$\frac{11}{33}$ = 6*s*. 8*d*.

Ex. 5. A person buys shares in a railway when they are at £19½, £15 having been paid, and sells them at £32. 9*s*. when £25 has been paid: how much per cent. does he gain?

He buys each share at £19½, and he afterwards pays upon it £(25 − 15), or £10; therefore at the time he sells, he has paid on each share £29. 10*s*.; so that, by selling at £32. 9*s*., he gains on every £29. 10*s*. paid by him, (£32. 9*s*. − £29. 10*s*.) = £2. 19*s*.;

∴ £29½ : £100 :: £2$\frac{19}{20}$: gain per cent. in pounds,
whence, gain per cent. = £10, or gain is 10 per cent.

Ex. 6. What was the prime cost of an article, which when sold for 12*s*. realized a profit of 20 per cent.?

Here what cost £100 would be sold for £120;

∴ £120 : 12*s*. :: £100 : prime cost in pounds,
whence, prime cost = £½ = 10*s*.

If the above example had been, "What was the prime cost of an article, which when sold for 12*s*., entails a loss of 20 per cent.?"

we should have £80 : 12*s*. :: £100 : prime cost in pounds,
whence, prime cost = £¾ = 15*s*.

Ex. 7. If by selling a horse for £40 I lose 20 per cent., what must I have sold him for so as to gain 10 per cent.?

Here what would cost me £100 must be sold in one case for £80, and in the other for £110; and therefore we get this statement; selling price of £100 in 1st case : selling price of horse in 1st case :: selling price of £100 in 2nd case : selling price of horse in 2nd case;

£80 : £40 :: £110 : selling price in pounds;
whence, selling price = £55.

Ex. 8. A grocer buys 3 cwt. of sugar at 6*d*. a lb., 2 cwt. of sugar at 10½*d*. a lb., and 2½ qrs. of sugar at 1*s*. a lb.; and mixes them: he sells 4 cwt. of the mixture at 9*d*. a lb. What must he sell the remainder at, in order to gain 25 per cent. on his outlay?

	£.	s.	d.
3 cwt., or 336 lbs., at 6*d.* a lb., cost………	8	8	0
2 cwt., or 224 lbs., at 10½*d.* a lb., cost……	9	16	0
2½ qrs., or 70 lbs., at 1*s.* a lb., cost………	3	10	0
∴ 630 lbs. cost …	21	14	0

In order to gain 25 per cent. on £21. 14*s.*, it must realize £27. 2*s.* 6*d.*

	£.	s.	d.
∴ he must sell 630 lbs. for…	27	2	6
but he sells 448 lbs. for…	16	16	0
∴ by Subtraction he must sell 182 lbs. for…	10	6	6

∴ he must sell 1 lb. for $\frac{£10.\ 6s.\ 6d.}{182}$, or $13\frac{8}{13}d.$

Ex. LXIII.

1. Bought 5 cwt., 3 qrs., 14 lbs. of cheese, at £1. 12*s.* per cwt., and sold it again for £2. 0*s.* 8*d.* per cwt. What was the gain upon the whole?

2. If 5 cwt., 3 qrs., 14 lbs. be bought for £9. 8*s.* and sold for £11. 18*s.* 11*d.*, what is the rate of gain per cwt.?

3. Find the total value of 43 articles at £4. 6*s.* 8*d.* each, 57 at £11. 8*s.* 6*d.* each, and 4 at £13. 15*s.* 4*d.* each. What is gained or lost by selling them at the rate of 3 for £28?

4. A person buys 400 yards of silk at £80, and sells 300 yards at 5*s.* 6*d.* a yard, and the rest, which is damaged, at 2*s.* a yard; find how much per cent. he gains or loses.

5. A grocer buys 2 cwt. of sugar at 6*d.* per pound, and 4 cwt. at 4½*d.*; he sells 3 cwt. at 5½*d.* per pound; at what rate per pound will he be able to sell the remainder so as neither to gain nor lose by the bargain?

6. If a commodity be bought for £3. 8*s.* 5*d.* a cwt. and sold for 8*d.* a lb., find the rate of profit per cent.

7. Bought goods at 6½*d.* per pound, and sold them at £4. 10*s.* per cwt.; what is the gain or loss per cent.?

8. An article which cost 3*s.* 6*d.* is sold for 3*s.* 10½*d.*; find the gain per cent.

9. Goods were sold at 12 guineas, at a profit of $22\frac{38}{41}$ per cent.; what was the prime cost?

10. If a tradesman gain 5*s.* 6*d.* on an article which he sells for 22*s.* what is his gain per cent.?

11. A man sells a horse for £24. 12*s.*, and loses £18. per cent. on what *the horse cost him*, what was the original cost?

12. By selling an article for *5s.* a person loses 5 per cent.; what was the prime cost, and what must he sell it at to gain 4½ per cent.?

13. The cost price of a book is *6s.* *8d.*; the expense of sale 5 per cent. upon the cost price; and the profit 25 per cent. upon the whole outlay: find the selling price of the book.

14. If by selling an article for £25. *10s.* 8 per cent. be lost, what per cent. is gained or lost if it be sold at £38?

15. I bought 500 sheep at £2. *2s.* a-head; their food cost me *5s.* *6d.* a-head: I then sold them at £2. *8s.* *6d.* a-head. Find my whole gain, and also my gain per cent.

16. A person having bought goods for £40 sells half of them at a gain of 5 per cent.; for how much must he sell the remainder so as to gain 20 per cent. on the whole?

17. A vintner buys a cask of wine containing 36 gallons at *10s.* per gallon; he keeps it for four years, and then finds that he has lost 6 gallons by leakage; at what price per gallon must he sell the remainder in order that he may realize 20 per cent. upon his outlay?

18. A person rents a piece of land for £120 a year. He lays out £625 in buying 50 bullocks. At the end of the year he sells them, having expended £12. *10s.* in labour. How much per head must he gain by them in order to realize his rent and expenses, and 10 per cent. upon his original outlay?

19. A grocer mixes two kinds of tea which cost him *3s.* *8d.* and *4s.* *4d.* per lb. respectively; what must be the selling price of the mixture in order that he may gain 15 per cent. on his outlay?

20. A person has goods worth £30; he sells one-third of them so as to lose 10 per cent.: what must he sell the remainder at so as to gain 20 per cent. on the whole?

21. I buy a house for 500 guineas, and sell it immediately at a profit of 30 per cent.; what do I receive, supposing the expenses of the sale to be 5 per cent.?

22. The prime cost of a 76-gallon cask is £23. *12s.* *6d.*, but 13 gallons are lost by leakage; 9 gallons of water is then mixed with the remainder, and it is sold at *7s.* *6d.* a gallon. Find the whole gain, and also the gain per cent.

23. A stationer sold quills at *11s.* a thousand, by which he cleared ⅜ of the money; he raises the price to *13s.* *6d.* What does he clear per cent. by the latter price?

24. A person sold 72 yards of cloth for £8. *14s.*; his profit being the cost of 11·52 yards: how much did he gain per cent.?

25. A smuggler buys 6 cwt. of tobacco at 1*s.* 3*d.* per lb.; he meets with a revenue-officer, who seizes ⅓rd of it: at what rate per lb. must he sell the remainder, so as, 1st, neither to gain or lose; 2nd, to gain 5 guineas; and 3rd, to gain cent. per cent.?

26. A person expends £3000 in railway shares at 15½ per cent. discount, and sells them at par; what does he gain by the transaction, and what per cent.?

27. A wine-merchant bought 14½ pipes of wine, which having received damage, he sold for £1120$\frac{1}{2}\frac{4}{5}$, thereby losing 20 per cent.; find the cost of the wine per pipe, and the selling price of it per gallon.

28. A farm is let for £96 and the value of a certain number of quarters of wheat. When wheat is 38*s.* a quarter, the whole rent is 15 per cent. lower than when it is 56*s.* a quarter. Find the number of quarters of wheat which are paid as part of the rent.

29. A man having bought a lot of goods for £150, sells ⅓rd at a loss of 4 per cent.; by what increase per cent. must he raise that selling price, in order that by selling the rest at the increased rate, he may gain 4 per cent. on the whole transaction?

30. A person bought a French watch, bearing a duty of 25 per cent., and sold it at a loss of 5 per cent.; had he sold it for £3 more, he would have cleared 1 per cent. on his bargain. What had the French maker for it?

DIVISION INTO PROPORTIONAL PARTS.

172. *To divide a given number into parts, which shall be proportional to certain other given numbers.*

This is merely an application of the Rule of Three; still it may be well to state a general rule, by which examples which come under the above head may be worked.

RULE. State thus: "As the sum of the given parts : any one of them :: the entire quantity to be divided : the corresponding part of it."

This statement must be repeated for each of the parts, or at all events for all but the last part, which of course may either be found by the Rule, or by subtracting the sum of the values of the other parts from the entire quantity to be divided.

Ex. 1. Divide 40 guineas among *A*, *B*, and *C*, so that their portions *may be as* 7, 11, and 14 respectively.

Proceeding according to the Rule given above,

$$32 : 7 :: 40 \text{ guineas} : A\text{'s share},$$

$$32 : 11 :: 40 \text{ guineas} : B\text{'s share},$$

whence, A's share = £9. 3*s*. 9*d*., and B's share = £14. 8*s*. 9*d*.

C's share may be found from the proportion

$$32 : 14 :: 40 \text{ guineas} : C\text{'s share},$$

whence, C's share = £18. 7*s*. 6*d*.;

or by subtracting £9. 3*s*. 9*d*. + £14. 8*s*. 9*d*., or £23. 12*s*. 6*d*. from £42, which leaves £18. 7*s*. 6*d*., as above.

The reason for the above process is clear from the consideration, that 40 guineas is to be divided into 32 equal parts, of which A is to have 7 parts, B 11, and C 14.

Ex. 2. Divide £11000 among 4 persons, A, B, C, D, in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

Sum of shares = $\frac{77}{60}$;

$\therefore$ $\frac{77}{60} : \frac{1}{2}$ :: £11000 : A's share in pounds,

whence A's share = £4285. 14*s*. $3\frac{3}{7}$*d*.

Similarly,

B's share = £2857. 2*s*. $10\frac{2}{7}$*d*., C's share = £2142. 17*s*. $1\frac{5}{7}$*d*.,

D's share = £1714. 5*s*. $8\frac{4}{7}$*d*.

Ex. 3. Divide £45000 among A, B, C, and D, so that A's share : B's share :: 1 : 2, B's : C's :: 3 : 4, and C's : D's :: 4 : 5.

In this case,

B's share = $2A$'s share, $3C$'s share = $4B$'s share,

$4D$'s share = $5C$'s share;

$\therefore$ we have, C's share = $\frac{4}{3}B$'s share = $\frac{8}{3}A$'s share,

and D's share = $\frac{5}{4}C$'s share = $\frac{10}{3}A$'s share;

$\therefore$ A's share + B's share + C's share + D's share

= A's share $\times \left(1 + 2 + \frac{8}{3} + \frac{10}{3}\right)$,

= $9A$'s share;

$\therefore$ A's share = £5000, B's = £100000, C's = £13333. 6*s*. 8*d*.,

D's = £16666. 13*s*. 4*d*.

FELLOWSHIP OR PARTNERSHIP.

173. Def. **Fellowship** or **Partnership** is a method by which the respective gains or losses of partners in any mercantile transactions are determined.

Fellowship is divided into **Simple** and **Compound Fellowship**: in the former, the sums of money put in by the several partners continue in the business for the same time; in the latter, for different periods of time.

SIMPLE FELLOWSHIP.

174. Examples in this Rule are merely particular applications of the Rule in Art. (172), and that Rule therefore applies.

Ex. 1. Two merchants, A and B, form a joint capital; A puts in £240, and B £360: they gain £80. How ought the gain to be divided between them?

$$£\,(240+360) : £240 :: £80 : A\text{'s share in pounds,}$$
$$\text{whence, } A\text{'s share} = £32, \text{ and } \therefore B\text{'s share} = £48.$$

Note. The estate of a Bankrupt may be divided among his creditors by the same method.

Ex. 2. A bankrupt owes three creditors, A, B, and C, £175, £210, and £265, respectively; his property is worth £422. 10*s.*: what ought they each to receive?

$$£650 : £175 :: £422\tfrac{1}{2} : A\text{'s share,}$$
$$£650 : £210 :: £422\tfrac{1}{2} : B\text{'s share,}$$
$$\text{whence, } A\text{'s share} = £113.\ 15s.,\ B\text{'s share} = £136.\ 10s.;$$
$$\therefore C\text{'s share} = £172.\ 5s.$$

COMPOUND FELLOWSHIP.

175. Rule. "Reduce all the times into the same denomination, and multiply each man's stock by the time of its continuance, and then state thus:

As the sum of all the products : each particular product :: the whole quantity to be divided : the corresponding share."

Ex. 1. *A* and *B* enter into partnership; *A* contributes £3000 for 9 months, and *B* £2400 for 6 months, they gain £1150: find each man's share of the gain.

Proceeding by the Rule given above,

£ (3000 × 9 + 2400 × 6) : £ (3000 × 9) :: £1150 : *A*'s share of gain,
or £41400 : £27000 :: £1150 : *A*'s share of gain,
and £41400 : £14400 :: £1150 : *B*'s share of gain,
whence, *A*'s share = £750, and *B*'s share = £400.

The reason for the above process is evident from the consideration, that a stock of £3000 for 9 months would be equivalent to a stock of 9 times £3000 for 1 month; and one of £2400 for 6 months, to one of 6 times £2400 for 1 month: hence, the increased stocks being considered, the question then becomes one of Simple Fellowship.

Ex. 2. There were at a feast 20 men, 30 women, and 15 servants; for every 10*s*. that a man paid, a woman paid 6*s*., and a servant 2*s*.; the bill amounted to £41: how much did each man, woman, and servant pay?

20 men at 10*s*. each = 200 at 1*s*., 30 women at 6*s*. = 180 at 1*s*., and 15 servants at 2*s*. = 30 at 1*s*.; and 200 + 180 + 30 = 410.

Hence we have

410 : 200 :: £41 : 20 men's share (in pounds),
410 : 180 :: £41 : 30 women's share (in pounds),
410 : 30 :: £41 : 15 servants' share (in pounds);
∴ 20 men's shares = £20., 30 women's shares = £18,
and 15 servants' shares = £3.;
∴ each man paid £1., each woman 12*s*., and each servant 4*s*.

EQUATION OF PAYMENTS.

176. **Def.** When a person owes another several sums of money, due at different times, the Rule by which we determine the just time when the whole debt may be discharged at one payment, is called the EQUATION OF PAYMENTS.

Note. It is assumed in this Rule that the sum of the interests of the several debts for their respective times equals the interest of the sum of the debts for the equated time.

RULE. "Multiply each debt into the time which will elapse before it becomes due, and then divide the sum of the products by the sum of the debts; the quotient will be the equated time required."

Ex. 1. *A* owes *B* £100, whereof £40 is to be paid in 3 months, and £60 in 5 months: find the equated time.

Proceeding according to the Rule given above,

$$\text{then } (40\times3+60\times5)=(40+60)\times\text{equated time in months,}$$

$$\text{whence, equated time}=4\tfrac{1}{5}\text{ months.}$$

The reason for the above process, in accordance with our assumption, is clear from the consideration, that the sum of the interests of £40 for 3 months and £60 for 5 months, is the same as the interest of £(120 + 300), or £420, for 1 month; if therefore *A* has to pay £100 in one sum, the question is, how long ought he to hold it so that the interest on it may be the same as the interest on £420 for 1 month? The statement therefore will be this:

$$\pounds100 : \pounds420 :: 1\text{ month} : \text{required number of months.}$$

$$\text{whence, required number of months}=4\tfrac{1}{5}\text{ months;}$$

which is evidently the equated time of payment, and agrees with the result obtained by the Rule given above.

Ex. 2. *A* owed *B* £1000, to be paid at the end of 9 months: he pays, however, £200 at the end of 3 months, and £300 at the end of 8 months: when was the remainder due?

In this case,

$$(200\times3+300\times8+500\times\text{number of months required})=1000\times9,$$

or $500\times$ number of months required $=6000$; ∴ number of months required $=12$.

Ex. LXIV.

1. A company of militia consisting of 72 men is to be raised from 3 towns, which contain respectively 1500, 7000, and 9500 men. How many must each town provide?

2. Divide £17. 11*s*. 9*d*. into two parts which shall be to each other as 5 : 16.

3. Divide 4472 into parts which shall be to each other in the ratio of 3, 5, 7, 11; and also £500 into parts which shall be in the ratio of $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{4}{7}$.

4. A bankrupt owes *A* £256. 6*s*. 8*d*., *B* £203. 10*s*., and *C* £141. 13*s*. 4*d*.; his estate is worth £421. 1*s*.; how much will *A*, *B*, and *C* receive respectively?

5. A mass of counterfeit metal is composed of fine gold 15 parts, silver 4 parts, and copper 3 parts: find how much of each is required in making 18 cwt. of the composition.

6. Two persons have gained in trade £720; the one put in £2200 and the other £1800; what is each person's share of the profits?

7. Sugar being composed of 49·856 per cent. of oxygen, 43·265 per cent. of carbon, and the remainder hydrogen: find how many lbs. of each of these materials there are in one ton of sugar.

8. A man leaves his property amounting to £13000 to be divided amongst his children, consisting of 4 sons and 3 daughters; the three younger sons are each to have twice the share of each of the daughters, and the eldest son as much as a younger son and a daughter together; find the share of each.

9. Two persons, *A* and *B*, are partners in a mercantile concern, and contribute £1200 and £2000 capital respectively; *A* is to have 10 per cent. of the profits for managing the business, and the remaining profits to be divided in proportion to the capital contributed by each; the entire profit at the year's end is £800; how much of it must each receive?

10. Divide £100 among *A*, *B*, *C*, and *D*, so that *B* may receive as much as *A*; *C* as much as *A* and *B* together; and *D* as much as *A*, *B*, and *C* together.

11. Divide £11875 among *A*, *B*, and *C*, so that as often as *A* gets £4, *B* shall get £3, and as often as *B* gets £6, *C* shall get £5.

12. *A* commences business with a capital of £1000, two years afterwards he takes *B* into partnership with a capital of £15000, and in 3 years more they divide a profit of £1500; required *B*'s share.

13. £700 is due in 3 months, £800 in 5 months, and £500 in 10 months: find the equated time of payment.

14. Find the equated time of payment of £750, one half of which is due in 4 months, $\frac{3}{8}$ in 5 months, and the rest in 6 months.

15. *A* owes *B* a debt payable in $7\frac{1}{30}$ months, but he pays $\frac{1}{3}$ in 4 months, $\frac{1}{4}$ in 6 months, $\frac{1}{5}$ in 8 months: when ought the remainder to be paid?

16. *A*, *B*, and *C* rent a field for £11. 6*s.*; *A* puts in 70 cattle for 6 months; *B*, 40 for 9 months; and *C*, 50 for 7 months: what ought *C* to pay?

17. *A*, *B*, and *C* invest capital to the amount of £700, £500, and £300 respectively; *A* was to have 25 per cent. of the profits, which amount to £450; what share of the profits ought *C* to have?

18. *A* and *B* enter into a speculation; *A* puts in £50 and *B* puts in £45; at the end of 4 months *A* withdraws $\frac{1}{2}$ his capital, and at the end of 6 months *B* withdraws $\frac{1}{3}$ of his; *C* then enters with a capital of £70; at the end of 12 months their profits are £254; how ought this to be divided amongst them?

EXCHANGE.

177. **Def.** **Exchange** is the Rule by which we find how much money of one country is equivalent to a given sum of another country, according to a given *course* of Exchange.

Def. By the **Course** of **Exchange** is meant the *variable* sum of the money of any place which is given in exchange for a *fixed* sum of money of another place: thus, for instance, in London one pound sterling, a fixed sum, is given for a variable number of French francs, more or less, according to circumstances. By the **Par** of Exchange is meant the intrinsic value of the coin of one country as compared with a given fixed sum of money of another.

Exchanges between merchants are effected by written instruments, called **Bills of Exchange**; and a bill on London entitles the holder to obtain gold in London for the value of the amount mentioned in the bill.

Examples in Exchange worked out.

Ex. 1. A merchant in Paris draws a bill of 1500 francs upon a merchant in London for goods supplied: what sterling money will the latter have to pay, exchange being 24·25 francs for £1 sterling?

Here 24·25 francs : 1500 francs :: £1 : required amount of money in pounds;

whence, required amount of money = £61. 17*s*. $1\frac{35}{97}$*d*.

Ex. 2. What is the course of exchange between London and Lisbon when 594 milrees, 480 rees are received for £158. 16*s*. 9*d*.? (1 milree = 1000 rees).

Here 594 mils., 480 rees : 1 mil. :: £158. 16*s*. 9*d*. : course of exchange,

or 594·48 mils. : 1 mil. :: 38121*d*. : course of exchange,

whence, course of exchange = 64·124...*d*.

that is, 64·124*d*. or rather more than 5*s*. 4*d*. English money, would be paid for 1 milree of Portuguese.

Def. **Arbitration, or Comparison of Exchanges**, is the method of fixing upon the rate of Exchange, called the **Par of Arbitration**, between the first and last of a given number of places, where the course of Exchange between the first and second, the second and third, &c. of these places is known. It is called **Simple or Compound Arbitration**, as three or more places are concerned.

Ex. 1. If the Exchange between Paris and Frankfort be at 20 francs for 9 florins, 20 kreutzers; and the Exchange between London and Frankfort 11 florins, 54 kreutzers for the £1 sterling, what is the course of exchange between London and Paris? (1 florin = 60 kreutzers).

$$9\tfrac{1}{3}\text{ florins} = 20\text{ francs, therefore }1\text{ florin} = \tfrac{60}{28}\text{ francs,}$$

$$11\text{ fl. }54\text{ kreutz., or }11\tfrac{9}{10}\text{ fl.} = £1,\ \therefore\ 1\text{ fl.} = £\tfrac{10}{119},$$

$$\therefore\ £\tfrac{10}{119} = \tfrac{60}{28}\text{ francs,}$$

$$\text{or }£1 = \left(\tfrac{60}{28} \times \tfrac{119}{10}\right)\text{ francs} = 25{\cdot}5\text{ francs.}$$

Ex. 2. £1 English being = 25·4 francs, 3·75 francs being = 105 kreutzers, 60 kreutzers being = 1 florin; find in English money the value of 1143 florins.

$$\begin{aligned} 1143\text{ florins} &= (1143 \times 60)\text{ kreutzers,} \\ &= \left(1143 \times 60 \times \frac{3{\cdot}75}{105}\right)\text{ francs} \\ &= £\left(1143 \times 60 \times \frac{3{\cdot}75}{105} \times \frac{1}{25{\cdot}4}\right) \\ &= £96.\ 8s.\ 6\tfrac{6}{7}d. \end{aligned}$$

Ex. LXV.

1. Convert £1519. 17*s.* 6*d.* into francs and centimes, at 23·45 francs per £. sterling. (1 franc = 100 centimes).

2. Convert 4750 milrees, 280 rees into English money, at 64¼*d.* a milree.

3. Convert £246. 15*s.* 6*d.* into piastres and rials, exchange being at 47½*d.* a piastre. (1 piastre = 8 rials).

4. A merchant at Lisbon draws a bill of 2000 milrees upon London. What sterling money will the latter have to pay, exchange being 1 milree = 68*d.*?

5. If London exchanges with Holland at a gain of 6½ per cent. when the course of exchange is at 35*s.* 6*d.* per £. sterling; what is the par of exchange?

6. A bill bought in London at 25·6 francs per £. sterling, is sold in Lisbon at 172 rees per franc; what is the exchange between London and Lisbon?

7. A merchant in London is indebted to one at St Petersburg 15000 rubles: the exchange between St Petersburg and England is 50*d.* per ruble, between St Petersburg and Amsterdam 91*d.* per ruble, and between Amsterdam and London 36*s.* 3*d.* per £. sterling: which will be the most advantageous way for the London merchant to be drawn upon?

8. What sum in English money must be given for 500 francs, when

25·6 francs is exchanged for £1? What is the arbitrated price between London and Paris, when 3 francs = 480 rees, 400 rees = 3½*s*. Flemish, and 35*s*. Flemish = £1?

9. A person in London owes another in Petersburg a debt of 460 rubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 23 francs for £1, and between Paris and Petersburg 2 francs for one ruble. The remittance is delayed until the rates of exchange are 24 francs for £1., and 3 francs for 2 rubles. What does the broker gain or lose by the transaction?

10. A trader in London owes a debt of 508 pistoles to one in Cadiz: is it more advantageous to him to remit directly to Cadiz, or circuitously through France? the exchanges being £1 = 25·4 francs, 19 francs = 1 Spanish pistole, 4 Spanish pistoles = £3.

SQUARE ROOT.

178. The SQUARE of a given number is the product of that number multiplied by itself. Thus 36 is the square of 6.

The square of a number is frequently denoted by placing the figure 2 above the number, a little to the right. Thus 6^2 denotes the square of 6, so that $6^2 = 36$.

179. The SQUARE ROOT of a given number is a number, which, when multiplied by itself, will produce the given number.

The square root of a number is sometimes denoted by placing the sign $\sqrt{}$ before the number, or by placing the fraction $\frac{1}{2}$ above the number a little to the right. Thus $\sqrt{36}$ or $(36)^{\frac{1}{2}}$ denotes the square root of 36; so that $\sqrt{36}$ or $(36)^{\frac{1}{2}} = 6$.

180. The number of figures in the integral part of the Square Root of any whole number may readily be known from the following considerations:

The square root of	1	is	1
	100	is	10
	10000	is	100
	1000000	is	1000
	&c.	is	&c.

Hence it follows that the square root of any number between 1 and 100 must lie between 1 and 10, that is, will have one figure for its integral part; of any number between 100 and 10000, must lie between 10 and 100, that is, will have two figures in its integral part; of any number between 10000 and 1000000, must lie between 100 and 1000, that is, must have three figures in its integral part; and so on. Wherefore, if a point be placed over the units' place of the number, and thence over every second figure to the left of that place, the points will shew the number of figures in the integral part of the root. Thus the square root of $9\dot{9}$ consists, so far as it is integral, of *one* figure; that of $\dot{1}9\dot{8}$ of *two* figures; that of $1\dot{7}6\dot{4}3\dot{2}$ of *three* figures; that of $\dot{1}7\dot{6}4\dot{3}2\dot{1}$ of *four* figures; and so on.

181. The following Rule may be laid down for extracting the square root of a whole number.

Rule. "Place a point or dot over the units' place of the given number, and thence over every second figure to the left of that place, thus dividing the whole number into several periods. The number of points will shew the number of figures in the required root (Art. 180).

Find the greatest number whose square is contained in the first period at the left; this is the first figure in the root, which place in the form of a quotient to the right of the given number. Subtract its square from the first period, and to the remainder bring down the second period. Divide the number thus formed, omitting the last figure, by twice the part of the root already obtained, and annex the result to the root and also to the divisor. Then multiply the divisor, as it now stands, by the part of the root last obtained, and subtract the product from the number formed, as above mentioned, by the first remainder and second period. If there be more periods to be brought down, the operation must be repeated."

Ex. 1. Find the square root of 1369.

$$\begin{array}{r|l} & 1\dot{3}6\dot{9}\ (37 \\ & 9 \\ \hline 67 & 469 \\ & 469 \\ \hline \end{array}$$

After pointing, according to the Rule, we take the first period, or 13, and find the greatest number whose square is contained in it. Since the square of 3 is 9, and that of 4 is 16, it is clear that 3 is the greatest number whose square is contained in 13; therefore place 3 in the form of a quotient to the right of the given number. Square this number, and put down the square under the 13; subtract it from the 13, and to the remainder 4 affix the next period 69, thus forming the number 469. Take 2×3, or 6, for a divisor; divide the 469, omitting the last figure, that is, divide the 46 by the 6, and we obtain 7. Annex the 7 to the 3 before obtained, and to the divisor 6; then multiplying the 67 by the 7 we obtain 469, which being subtracted from the 469 before formed, leaves no remainder; therefore 37 is the square root of 1369.

Reason for the above process.

Since $(37)^2 = 1369$, and therefore 37 is the square root of 1369; we have to investigate the proper Rule by which the 37, or $30+7$, may be obtained from the 1369.

$$\begin{aligned} \text{Now }\ 1369 &= 900+469=900+49+420 \\ &= (30)^2+7^2+2\times30\times7 \\ &= (30)^2+2\times30\times7+7^2 \end{aligned}$$

where we see that the 1369 is separated into parts in which the 30 and the 7, together constituting the square root, or 37, are made distinctly apparent. Treating then the number 1369 in the following form, viz.

$$(30)^2+2\times30\times7+7^2$$

we observe that the square root of the first part or of $(30)^2$, is 30; which is one part of the required root. Subtract the square of the 30 from the whole quantity $(30)^2+2\times30\times7+7^2$, and we have $2\times30\times7+7^2$ remaining. Multiply the 30 before obtained by 2, and we see that the product is contained 7 times in the first part of the remainder, or in $2\times30\times7$; and adding the 7 to the 2×30, thus making $2\times30+7$ or 67, this latter quantity is contained 7 times exactly in the remainder $2\times30\times7+7$ or 469; so that by this division we shall gain the 7, the remaining part of the root. If we had found that the $2\times30+7$ or 67, when multipled by the 7, had produced a larger number than the 469, the 7 would have been too large, and we should have had to try a smaller number, as 6, in its place.

The process will be shewn as follows;

$$\begin{array}{l|l} & (30)^2+2\times30\times7+7^2\ (30+7 \\ & (30)^2 \\ \hline 2\times30+7 & 2\times30\times7+7^2 \\ & 2\times30\times7+7^2 \\ \hline \end{array}$$

This operation is clearly equivalent to the following:

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & \multicolumn{1}{l}{900+420+49\ (30+7} \\
\multicolumn{1}{r}{} & \multicolumn{1}{l}{900} \\ \cline{2-2}
60+7 & \quad 420+49 \\
 & \quad 420+49 \\ \cline{2-2}
\end{array}$$

This again is equivalent to the following:

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & \multicolumn{1}{l}{1\dot{3}6\dot{9}\ (37} \\
\multicolumn{1}{r}{} & \multicolumn{1}{l}{\ \ 9} \\ \cline{2-2}
67 & 469 \\
 & 469 \\ \cline{2-2}
\end{array}$$

which is the mode of operation pointed out in the Rule.

Note 1. The reasoning will be better understood when the Student has made some progress in Algebra.

Note 2. The divisor obtained by doubling the part of the root already obtained, is often called a *trial divisor*, because the quotient first obtained from it by the Rule in (Art. 181), will sometimes be too large. It will be readily found, in the process, whether this is the case or not, for when, according to our Rule, we have annexed the quotient to the trial divisor, and multiplied the divisor as it then stands by that quotient, the resulting number should not be greater than the number from which it ought to be subtracted. If it be, the quotient is too large, and the number next smaller should be tried in its place.

Note 3. If at any point of the operation, the number to be divided by the trial divisor be less than it; we then affix a cypher to the root, and also to the trial divisor, bring down the next period, and proceed according to the Rule.

Ex. 2. Find the square root of 74684164.

$$\begin{array}{lr|l}
 & \multicolumn{1}{r}{} & \multicolumn{1}{l}{7\dot{4}6\dot{8}4\dot{1}6\dot{4}\ (8642} \\
 & \multicolumn{1}{r}{} & \multicolumn{1}{l}{64} \\ \cline{3-3}
\{2\times 8=16\} & 166 & 1068 \\
 & & \ \ 996 \\ \cline{3-3}
\{2\times 86=172\} & 1724 & \ \ 7241 \\
 & & \ \ 6896 \\ \cline{3-3}
\{2\times 864=1728\} & 17282 & \ \ 34564 \\
 & & \ \ 34564 \\ \cline{3-3}
\end{array}$$

Therefore 8642 is the square root of 74684164.

Ex. 3. Find the square root of 71690512350625.

$$
\begin{array}{lr|l}
 & & 7\dot{1}6\dot{9}0\dot{5}1\dot{2}3\dot{5}0\dot{6}2\dot{5}\ (8467025 \\
 & & \underline{64} \\
\{2\times8=16\} & 164 & 769 \\
 & & \underline{656} \\
\{2\times84=168\} & 1686 & 11305 \\
 & & \underline{10116} \\
\{2\times846=1692\} & 16927 & 118912 \\
 & & \underline{118489} \\
\left\{\begin{array}{l}(2\times8467=16934)\\(2\times84670=169340)\end{array}\right\} & 1693402 & 4233506 \\
 & & \underline{3386804} \\
 & 16934045 & 84670225 \\
 & & \underline{84670225} \\
\end{array}
$$

∴ 8467025 is the required square root.

182. Again, since the square root of

·01	is	·1
·0001	is	·01
·000001	is	·001
·00000001	is	·0001
&c.		&c.

it appears, that in extracting the square root of decimals, the decimal places must first of all be made even in number, by affixing a cypher to the right, if this be necessary; and then if points be placed over every second figure to the right, beginning as before from the units' place of whole numbers, the number of such points will shew the number of decimal places in the root.

183. If there be no whole number, or integral part in the given number, we must, in pointing begin with the second figure from that which would be the units' place, if there were a whole number, and mark successively over every second figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the units' place and point over every second figure to the right and left of it. The number of points over the whole numbers and decimals will shew respectively the numbers of figures in the integral and decimal parts of the root. Thus if the given number were 6115·23, place the first point over the 5, and mark from it to the right and left, thus $6\dot{1}1\dot{5}\cdot2\dot{3}$. If the given number were 58·432, first make the decimal places even in number thus, 58·4320, and then point thus $5\dot{8}\cdot4\dot{3}2\dot{0}$.

184. With the above explanation (Arts. 180 and 182) on the subject of pointing, the rule for extracting the square root of a decimal, or of a number consisting partly of a whole number and partly of a decimal, will be the same as that before given (Art. 181) for finding the square root of a whole number. As the decimal notation is only an extension or continuance of the ordinary integral notation, and quite in agreement with it, the reason before given for the process, will in fact apply also here.

185. To extract the square root of a vulgar fraction, if the numerator and denominator of the fraction be perfect squares, we may find the square root of each separately, and the answer will thus be obtained as a vulgar fraction; if not, we can first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting number. The answer will thus be obtained either as a decimal, or as a whole number and decimal, according to the case. Also a mixed number may be reduced to an improper fraction, and its root extracted in the same way.

Ex. 4. Extract the square root of ·4 to four places of decimals.

$$\begin{array}{r|l}
 & \cdot 4\dot{0}0\dot{0}0\dot{0}0\dot{0}\ (\cdot 6324 \\
 & 36 \\
\hline
123 & 400 \\
 & 369 \\
\hline
1262 & 3100 \\
 & 2524 \\
\hline
12644 & 57600 \\
 & 50576 \\
\hline
 & 7024
\end{array}$$

Ex. 5. Extract the square root of ·0006 to four places of decimals.

$$\begin{array}{r|l}
 & \cdot 0\dot{0}0\dot{6}0\dot{0}0\dot{0}\ (\cdot 0244 \\
 & 4 \\
\hline
44 & 200 \\
 & 176 \\
\hline
484 & 2400 \\
 & 1936 \\
\hline
 & 464
\end{array}$$

Ex. 6. Extract the square root of ·0365 to five places of decimals.

$$\begin{array}{r|l}
 & \cdot 0\dot{3}6\dot{5}0\dot{0}0\dot{0}0\dot{0}\ (\cdot 19104 \\
 & 1 \\ \hline
29 & 265 \\
 & 261 \\ \hline
381 & 400 \\
 & 381 \\ \hline
38204 & 190000 \\
 & 152816 \\ \hline
 & 37184
\end{array}$$

Ex. 7. Extract the square root of 53111·8116.

$$\begin{array}{r|l}
 & \dot{5}3\dot{1}1\dot{1}\cdot 8\dot{1}1\dot{6}\ (230\cdot 46 \\
 & 4 \\ \hline
43 & 131 \\
 & 129 \\ \hline
4604 & 21181 \\
 & 18416 \\ \hline
46086 & 276516 \\
 & 276516 \\ \hline
\end{array}$$

Ex. 8. Find the square root of $\frac{529}{2401}$.

$$\begin{array}{r|l}
 & \dot{5}2\dot{9}\ (23 \\
 & 4 \\ \hline
43 & 129 \\
 & 129 \\ \hline
\end{array}
\qquad\qquad
\begin{array}{r|l}
 & 2\dot{4}0\dot{1}\ (49 \\
 & 16 \\ \hline
89 & 801 \\
 & 801 \\ \hline
\end{array}$$

therefore square root required $=\frac{23}{49}$.

Ex. 9. Find the square root of $\frac{5}{7}$.

This may be done by first reducing $\frac{5}{7}$ to a decimal, and then by extracting the square root of the decimal, thus $\frac{5}{7}=\cdot 714285\ldots$

$$\begin{array}{r|l}
 & \cdot 7\dot{1}4\dot{2}8\dot{5}\ (\cdot 845\ldots \\
 & 64 \\ \hline
164 & 742 \\
 & 656 \\ \hline
1685 & 8685 \\
 & 8425 \\ \hline
 & 260
\end{array}$$

or thus, $\sqrt{\frac{5}{7}} = \sqrt{\left(\frac{5 \times 7}{7 \times 7}\right)} = \frac{\sqrt{35}}{7}$.

$$\begin{array}{r|l} & 3\dot{5}\cdot0\dot{0}0\dot{0}0\dot{0}\ (5\cdot916 \\ & 25 \\ \hline 109 & 1000 \\ & 981 \\ \hline 1181 & 1900 \\ & 1181 \\ \hline 11826 & 71900 \\ & 70956 \\ \hline & \quad 944 \end{array}$$

therefore $\sqrt{\frac{5}{7}} = \frac{5\cdot916\ldots}{7} = \cdot845\ldots$

Ex. LXVI.

1. Find the square roots of

(1) 289; 576; 1444; 4096. (2) 6561; 21025; 173056.
(3) 98596; 37249; 11664. (4) 998001; 978121; 824464.
(5) 29506624; 14356521; 5345344.
(6) 236144689; 282429536481; 282475249.
(7) 295066240000; 4160580062500.

2. Find the square roots of

(1) 167·9616; 28·8369; 57648·01. (2) ·3486784401; 39·15380329
(3) ·042849; ·00139876; ·00203401. (4) 5774409; 5·774409.
(5) 120888·68379025; 240398·012416.

3. Extract the square roots of

(1) 16; 1·6; ·16; ·016. (2) 235·6; ·1; ·01; 5; ·5.
(3) ·0004; ·00081; 379·864. (4) $20\frac{1}{4}$; $153\frac{7}{9}$; $\frac{1}{3}$; $\frac{2209}{9801}$.
(5) $\frac{3}{5}$; $\frac{1}{17}$; $2\frac{1}{6}$; $\frac{3\frac{1}{2}}{4\frac{1}{2}}$. (6) $\frac{5\cdot04}{\cdot021}$; $1\frac{56}{169}$; 23·1; 42

to four places of decimals in each case where the root does not terminate.

4. Extract the square root of ·0019140625 and reduce the result to the corresponding equivalent fraction in its lowest terms.

5. Find the side of a square field equal in area to a rectangular field 700 yards wide and 2800 yards long.

6. A square field contains 1 ac., 22 po., $7\frac{9}{16}$ yds.; find the length of its side.

7. A rectangular field measures 225 yards in length, and 120 yards in breadth; what will be the length of a diagonal path across it?

8. Find the length of the side of a square enclosure, the paving of which cost £27. 1*s*. 6*d*. at 8*d*. per sq. yard.

9. The hypothenuse of a right-angled triangle is 51 yards, and the perpendicular is 24 yards, find the base.

10. A ladder, whose length is 91 feet, reaches from the extremity of a path 35 feet wide, to a point in a building on the other side, which is within 9 inches of the top of it; find the height of the building.

11. Extract the square root of ·0050722884, and find within an inch the length of a side of a square field the area of which is 2 acres.

12. Two persons start from a certain point at the same time, the one goes due east at the rate of 12 miles an hour, and the other due north at the rate of 9 miles an hour; how far are they distant from each other at the end of six hours?

13. A ladder 36 feet long will reach to a window 28 feet from the ground, on one side of a street; and if the foot of the ladder be retained in the same position, will reach to a window 26 feet high on the other side. Find the breadth of the street.

14. A society collected among themselves for certain purposes a fund of £45. 18*s*. 9*d*.: each person paid as many pence as there were members in the whole society. Find the number of members.

15. The area of a circular lake is 295066·24 square yards, how many yards are contained in the side of a square of equal superficies?

CUBE ROOT.

186. The Cube of a given number is the product which arises from multiplying that number by itself, and then multiplying the result again by the same number. Thus $6 \times 6 \times 6$ or 216 is the cube of 6.

The cube of a number is frequently denoted by placing the figure 3 above the number, a little to the right. Thus 6^3 denotes the cube of 6, so that $6^3 = 6 \times 6 \times 6$ or 216.

187. The Cube Root of a given number is a number, which, when multiplied into itself, and the result again multiplied by it, will produce the given number. Thus 6 is the cube root of 216; for $6 \times 6 \times 6$ is $= 216$.

The cube root of a number is sometimes denoted by placing the sign $\sqrt[3]{}$ before the number, or placing the fraction $\frac{1}{3}$ above the number, a little to the right. Thus $\sqrt[3]{216}$ or $(216)^{\frac{1}{3}}$ denotes the cube root of 216; so that $\sqrt[3]{216}$ or $(216)^{\frac{1}{3}} = 6$.

188. The number of figures in the integral part of the cube root of any whole number may readily be known from the following considerations:

The cube root of 1 is 1
1000 is 10
1000000 is 100
1000000000 is 1000
&c.

Hence it follows that the cube root of any number between 1 and 1000 must lie between 1 and 10, that is, will have one figure for its integral part; of any number between 1000 and 1000000, must lie between 10 and 100, that is, will have two figures in its integral part; of any number between 1000000 and 1000000000, must lie between 100 and 1000, that is, must have three figures in its integral part; and so on. Wherefore, if a point be placed over the units' place of the number, and thence over every third figure to the left of that place, the points will shew the number of figures in the integral part of the root. Thus the cube root of $67\dot{7}$ consists, so far as it is integral, of *one* figure; that of $198\dot{9}9\dot{9}$ of *two* figures; that of $13\dot{4}19\dot{8}99\dot{9}$ of *three* figures; and so on.

189. The following Rule may be laid down for extracting the Cube Root of a whole number.

Rule. "Place a point or dot over the units' place of the given number, and thence over every third figure to the left of that place, thus dividing the whole number into several periods. The number of points will shew the number of figures in the required root. (Art. 188.)

Find the greatest number whose cube is contained in the first period at the left; this is the first figure in the root, which place in the form of a quotient to the right of the given number.

Subtract its cube from the first period, and to the remainder bring down the second period.

Divide the number thus formed, omitting the two last figures, by 3 times the square of the part of the root already obtained, and annex the result to the root.

Now calculate the value of 3 times the square of the first figure in the root (which of course has the value of so many tens) + 3 times the product of the two figures in the root + the square of the last figure in the root. Multiply the value thus found by the second figure in the root, and subtract the result from the number formed, as above mentioned, by the first remainder and the second period. If there be more periods to be brought down the operation must be repeated."

Ex. 1. Find the cube root of 15625.

$$\begin{array}{r|l}
\multicolumn{2}{r}{1\dot{5}62\dot{5}\ (25} \\
\multicolumn{2}{r}{2^3=8\qquad\quad} \\
3\times2^2=12 & 7625 \\
3\times(20)^2=3\times400=1200 & \\
3\times20\times5=\ \ 100 & \\
5^2=\ \ \ \ 25 & \\
\hline
1525 & \\
\text{Multiply by}\ \ \ \ 5 & \\
\hline
7625 & 7625 \\
\end{array}$$

After pointing according to the Rule we take the first period, or 15, and find the greatest number whose cube is contained in it. Since the cube of 2 is 8, and that of 3 is 27, it is clear that 2 is the greatest number whose cube is contained in 15; therefore place 2 in the form of a quotient to the right of the given number.

Cube 2, and put down its cube, viz. 8, under the 15; subtract it from the 15, and to the remainder 7 affix the next period 625, thus forming the number 7625. Take 3×2^2, or 12, for a divisor; divide 76 by 12, 12 is contained 6 times in 76; but when the other terms of the divisor are brought down 6 would be found too great, therefore take 5. Annex the 5 to the 2 before obtained; and calculate the value of $3\times(20)^2+3\times20\times5+5^2$, which is 1525; multiplying 1525 by 5 we obtain 7625, which being subtracted from 7625 before formed leaves no remainder, therefore 25 is the cube root required.

Reason for the above process.

Since $(25)^3=15625$, and therefore 25 is the cube root of 15625; we have to investigate the proper Rule by which the 25, or 20+5, may be obtained from 15625.

$$\begin{aligned}
\text{Now } 15625&=8000+7500+125 \\
&=8000+6000+1500+125 \\
&=(20)^3+3\times(20)^2\times5+3\times20\times5^2+5^3,
\end{aligned}$$

where we see that the 15625 is separated into parts in which the 20 and the 5, together constituting the cube root, or 25, are made distinctly apparent. Treating then the number 15625 in the following form, viz.

$$(20)^3+3\times(20)^2\times5+3\times20\times5^2+5^3,$$

we observe that the cube root of the first part or of $(20)^3$ is 20; which is one part of the required root. Subtract the cube of the 20 from the whole quantity, and we have $3\times(20)^2\times5+3\times20\times5^2+5^3$ remaining. Multiply the square of the 20 before obtained by 3, and we see that the product is contained 5 times in the first part of the remainder, or in $3\times(20)^2\times5$; and adding 3 times the product of the two terms of the root + the square of the last term of the root, thus making $3\times(20)^2+3\times20\times5+5^2$, we see that this latter quantity is contained 5 times exactly in the remainder $3\times(20)^2\times5+3\times20\times5^2+5^3$, so that by this division we shall obtain the 5, the remaining part of the root.

The process will be shewn as follows:

$$\begin{array}{l|l}
 & (20)^3+3\times(20)^2\times5+3\times(20)\times5^2+5^3\ (20+5 \\
 & \underline{(20)^3} \\
\text{divisor}=3\times(20)^2, & 3\times(20)^2\times5+3\times20\times5^2+5^3 \\
\text{and } \dfrac{3\times(20)^2\times5}{3\times(20)^2}=5; & \\
\therefore\ \{3\times(20)^2+3\times20\times5+5^2\}\times5= & \underline{3\times(20)^2\times5+3\times20\times5^2+5^3}
\end{array}$$

This operation is clearly equivalent to the following:

$$\begin{array}{r|l}
 & 8000+6000+1500+125\ (20+5 \\
 & \underline{8000} \\
3\times(20)^2=1200,\text{ and } \frac{6000}{1200}=5 & 6000+1500+125 \\
(1200+300+25)\times5= & \underline{6000+1500+125}
\end{array}$$

This again is equivalent to the following:

$$\begin{array}{l|l}
 & 1\dot{5}62\dot{5}\ (25 \\
 & \underline{8} \\
3\times2^2=3\times4=12,\text{ and } \frac{76}{12}=5 & 7625 \\
\quad 3\times(20)^2=1200 & \\
+3\times20\times5=\ \ 300 & \\
\qquad\quad +5^2=\ \ \underline{\ \ 25} & \\
\qquad\qquad\quad 1525 & \\
\qquad\qquad\qquad \underline{5} & \\
\qquad\qquad\quad 7625 & \underline{7625}
\end{array}$$

which is the mode of operation pointed out in the Rule.

Note 1. The reasoning will be better understood when the student has made some progress in Algebra.

Note 2. The divisor which is obtained according to the Rule given in (Art. 189) is sometimes called a *trial* divisor, because the number from the division may be too large, as was the case in the above Example, in which case, we must try a smaller number. We shall readily ascertain whether the number obtained from the division is too large or not, because if it be too large, the quantity which we ought to subtract from the number formed by a remainder and a period will turn out in that case to be larger than that number, which of course it ought not to be, and so we must try a smaller number.

Note 3. If at any point of the operation, the number to be divided by the trial divisor be less than it; we affix a cypher to the root, two cyphers to the trial divisor, bring down the next period, and proceed according to the Rule.

Ex. 2. Extract the cube root of 95443993.

	$9\dot{5}44\dot{3}99\dot{3}$ (457	
$4^3=64$		
trial divisor $=3\times4^2=48$	31443	
$3\times(40)^2=4800$ $3\times40\times5=\ \ 600$ $5^2=\ \ \ 25$ 5425 5 27125		$\frac{314}{48}$ goes 6 times, but 6 will be found too large; try 5.
	27125	
trial divisor $=3\times(45)^2=6075$	4318993	
Now 45 has the value of 450; $\therefore\ 3\times(450)^2=607500$ $3\times450\times7=\ \ \ 9450$ $7^2=\ \ \ 49$ 616999 7 4318993		$\frac{43189}{6075}$ goes 7 times, and we are led to conclude that 7 is the figure, because $7^3=343$, and 3 is the final figure in the remainder.
	4318993	

$\therefore$ 457 is the cube root required.

Ex. 3. Find the cube root of 223648543.

	$22\dot{3}64\dot{8}54\dot{3}$ (607	
$6^3=216$		
trial divisor $=3\times6^2\ \ =108$	7648	76 is not divisible by 108; bring down the next period and affix 0 to the root.
trial divisor $=3\times(60)^2=10800$	7648543	
$3\times(600)^2=1080000$ $3\times600\times7=\ \ 12600$ $7^2=\ \ \ 49$ 1092649 7 7648543		$\frac{76485}{10800}$ goes 7 times, and 7 seems likely to be the figure required; since $7^3=243$, and 3 is the final figure in the remainder.
	7648543	

$\therefore$ 607 is the cube root required.

190. Again, since the cube root of ·001 is ·1,
the cube root of ·000001 is ·01,
the cube root of ·000000001 is ·001,
&c.

it appears, that in extracting the cube root of decimals, the decimal places must first of all be made three, or some multiple of three in number, by affixing cyphers to the right, if this be necessary; and then if points be placed over every *third* figure to the right, beginning as before from the units' place of *whole numbers*, the number of such points will shew the number of decimal places in the cube root.

191. If there be no whole number or integral part in the given number, we must in pointing begin with the *third* figure from that which would be the units' place, if there were a whole number, and mark successively every *third* figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the units' place, and point over every third figure to the right and left of it: the number of points over the whole numbers and decimals will shew respectively the numbers of figures in the integral and decimal parts of the root. Thus if the given number were 5623·453134, place the first point over the 3, and mark from it to the right and left, thus $\dot{5}62\dot{3}\cdot45\dot{3}13\dot{4}$. If the given number were 5·23, make the number of decimal places equal to 3, by affixing a cypher thus, 5·230; place the first point over the 5, and the second over the 0: if the root to more decimals than one is required, more cyphers must be affixed.

192. With the above explanation (Arts. 188, 190) on the subject of pointing, the rule for extracting the cube root of a decimal, or of a number consisting partly of a whole number and partly of a decimal, will be the same as that before given (Art. 189) for finding the cube root of a whole number. As the decimal notation is only an extension or continuance of the ordinary integral notation, and quite in agreement with it, the reason before given for the process, will in fact apply also here.

193. To extract the cube root of a vulgar fraction, if the numerator and denominator of the fraction be perfect cubes we may find the cube root of each separately; and the answer will thus be obtained as a vulgar fraction; if not, we can first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting num-

ber. The answer will thus be obtained either as a decimal, or as a whole number and decimal, according to the case. Also a mixed number may be reduced to an improper fraction, and its root extracted in the same way.

Ex. 4. Find the cube root of 48228·544.

$$\begin{array}{rr|l}
 & & 4\dot{8}22\dot{8}\cdot54\dot{4}\ (36\cdot4 \\
 & 3^3= & 27 \\ \hline
3\times3^2=27 & & 21228 \\
3\times(30)^2=2700 & & \\
3\times30\times6=\ \ 540 & & \\
6^2=\ \ \ \ 36 & & \\ \cline{1-1}
3276 & & \\
6 & & \\ \cline{1-1}
19656 & & 19656 \\ \hline
3\times(36)^2=3888 & & 1572544 \\
3\times(360)^2=388800 & & \\
3\times360\times4=\ \ \ \ 4320 & & \\
4^2=\ \ \ \ \ \ \ 16 & & \\ \cline{1-1}
393136 & & \\
4 & & \\ \cline{1-1}
1572544 & & 1572544 \\ \hline
\end{array}$$

∴ 36·4 is the cube root required.

Ex. 5. Find the cube root of ·000007 to three places of decimals.

$$\begin{array}{rr|l}
 & & \cdot00\dot{0}00\dot{7}00\dot{0}\ (\cdot019 \\
 & & 1 \\ \hline
 & 3\times1^2=3 & 6000 \\
3\times(10)^2=\ \ 300 & & \\
3\times10\times9=\ \ 270 & & \\
9^2=\ \ \ \ 81 & & \\ \cline{1-1}
651 & & \\
9 & & \\ \cline{1-1}
5859 & & 5859 \\ \hline
 & & 141
\end{array}$$

Ex. 6. Find the cube root of $\frac{5}{9}$ to three places of decimals.

$$\frac{5}{9}=\cdot 555555555\ldots$$

$$\begin{array}{rr|l}
 & & \cdot 55\dot{5}55\dot{5}55\dot{5}\ (\cdot 822 \\
 & 8^3= & 512 \\ \cline{3-3}
 & 3\times 8^2=192 & 43555 \\
3\times(80)^2= & 19200 & \\
3\times 80\times 2= & 480 & \\
2^2= & \underline{4} & \\
 & 19684 & \\
 & \underline{2} & \\
 & 39368 & \underline{39368} \\
 & 3\times(82)^2=20172 & 4187555 \\
3\times(820)^2= & 2017200 & \\
3\times 820\times 2= & 4920 & \\
2^2= & \underline{4} & \\
 & 2022124 & \\
 & \underline{2} & \\
 & 4044248 & \underline{4044248} \\
 & & 143307
\end{array}$$

194. Higher roots than the square and cube can sometimes be extracted by means of the Rules for square and cube root; thus the 4th root is found by taking the square root of the square root; the 6th root by taking the square root of the cube root, and so on.

Ex. LXVII.

1. Find the cube roots of

(1) 1728; 3375; 29791. (2) 54872; 110592; 300763.
(3) 681472; 804357; 941192. (4) 2406104; 69426531; 8365427.
(5) 251239591; 28372625; 482285544.
(6) 17173512; 259694072; 926859375.
(7) 27054036008; 219365327791.

2. Find the cube roots of

·389017; 32·461759; 95443·993; ·000912673;
·001906624; ·000024389.

3. Find the cube roots of

(1) 3, ·3, ·03. (2) $\frac{8}{27}$; $\frac{250}{686}$; 44·6.

(3) $405\frac{28}{125}$; $7\frac{1}{5}$; 3·00415. (4) ·0001; $\frac{1257\cdot728}{16384}$,

to three places of decimals, in those cases, where the root does not terminate.

4. Find the cube root of 233·744896, and also the cube root of the last-mentioned number multiplied by ·008.

5. The cost of a cubic mass of metal is £10481. 1*s.* 4*d.* at 10*s.* 5*d.* a cubic inch. What are the dimensions of the mass ?

6. A cubical block of stone contains 50653 solid feet, what is the area of its side ?

7. A cube contains 56 solid feet, 568 solid inches ; find its edge.

8. Find the cost of carpeting a cubical room, whose content is 21717·639 solid feet, with carpet 21 inches broad, at 3*s.* 6*d.* a yard.

9. A cubical box contains 941192 solid inches : find the cost of painting its outside surface at 6*d.* a square foot.

10. If the solid content of a cube be 37 ft., 64 in., shew that its surface will be 66 ft., 96 in.

11. The edge of a cubical vessel is 2 feet long : what is the length of the edge of another cubical vessel containing 3 times as much ?

12. Find the 4th root of 43046721 ; and the 6th root of ·000000004096.

APPENDIX.

MISCELLANEOUS PAPERS.

I.

1. Shew how to divide 4 things of the same size and material among 3 children, *A*, *B*, and *C*, by merely breaking *one* of the four, and so that *A*'s share shall be $\frac{2}{5}+\frac{5}{6}+\frac{1}{2}$ of a whole one, *B*'s share $\frac{3}{8}+\frac{7}{10}+\frac{3}{20}$ of a whole one, and *C*'s share the remainder.

2. A person employs 25 men, and 20 women, who work respectively 12 and 10 hours a-day during five days of the week, and half time on the remaining day; each man receives 3*d*., and each woman 2*d*. an hour. What is the whole expense of labour during a year? (a year = 52 weeks).

3. If 144 men can dig a trench 40 yds. long, 1 ft. 6 in. broad, and 48 ft. deep, in 3 days of 10 hours each; how long must another trench 5 ft. deep and 2 ft. 3 in. broad be, in order that 51 men may dig it in 15 days of 9 hours each?

4. Explain what is meant by compound interest. What is the difference between the simple and compound interest of £345. 5*fl*., for 2 years, at 3·5 per cent.?

5. The length of a rectangular field which contains 2 acres, 3 roods, 5 poles, is 151 yards, 9 in.; find its breadth.

6. Which is the greater $\sqrt{2}$ or $\sqrt[3]{3}$? Find the cube root of $\frac{5030·912}{65536}$.

7. What is the worth of 16 lbs. of a mixture of tea which contains 5⅓ parts of black worth 4*s*. 8*d*. per lb., and 4½ parts of green worth 6*s*. per lb., and 2⅙ parts of orange pekoe worth 3*s*. 6*d*. per lb.?

8. Find the equated time of payment of £200 due 14 months hence, and of £300 due 19 months hence; and determine the present value of the whole sum (supposed to be due at the equated time) allowing 3¾ per cent. simple interest.

9. Supposing the supply from California to become so great that the market price of gold decreases in the ratio of 7 : 5, what would be the absolute loss sustained by a fundholder upon every £100 which was paid off at par, if he had bought in before the depreciation took place, when the price of stock was at 89⅓?

10. A person buys tea at 6*s*. a lb. and also some at 4*s*. a lb. In what proportions must he mix them, so that selling his tea at 5*s*. 3*d*. a lb., he may gain 20 per cent. on each lb. sold?

11. Find the cost of papering a room 19 ft. 8 in. wide, 24 ft. 4 in. long, and 13½ ft. high, with paper 2¾ ft. wide, which costs 11*s*. per piece of 12 yards; the windows and parts not requiring paper, making up a sixth of the whole surface.

12. A certain sum produces an annual income of £201 when invested in the 3 per cent. consols, and of £260 when invested in railway 4 per cent. preference shares at par. Required the sum invested, and the price of consols.

II.

1. Explain the method of pointing in extracting the square roots of whole numbers and decimals. Find √(57214096), and also, as far as three places of decimals, √(572·14096).

2. What kinds of questions can be solved by means of the Rule of Three? Distinguish between the Rule of Three Direct, Inverse, and Double.

3. A bankrupt pays 5*fl.* 7*c.* 5*m.* in the £, what ought a creditor to receive on a debt of £1920. 7*fl.* 5*c.*?

4. A person, after paying from his rental 7*d.* in the £. for income-tax, and 3½ per cent. on a mortgage of £4000, has £1568. 13*s.* 4*d.* remaining: what was his rental?

5. On the price of 25 vols., bought at 3*s.* a vol., the bidder is allowed 5 per cent.; on that of 12 others, at 5*s.* 3*d.* a vol., 7½ per cent.; 2½ per cent. of the auction-duty is also paid by the purchaser: what will the books cost?

6. A person buys 3½ cwt. of tea at 5*s.* 4½*d.* per lb. and 4½ cwt. of tea at 3*s.* 3½*d.* per lb., and mixes them; he sells 5 cwt. at 4*s.* 6*d.* per lb.: at what rate must he sell the remainder so as to gain 20 per cent. on his outlay?

7. If 2 cub. in. of iron weigh as much as 15 of water, and a cub. ft. of water weigh 1000 oz.; find the weight of a cubic yard of iron.

8. Three horses do the same work as 5 ponies, and 12 horses can just draw a certain load; how many ponies would be necessary to draw half the load?

9. *A* after doing ⅗ths of a piece of work in 30 days, calls to his assistance *B*, and together they finish it in 6 days; in what time would each do it separately?

10. What is the difference between *Interest* and *Discount*?

A person purchased land at £60 and £56 per acre respectively; the former he let at £2, and the latter at £2. 2*s.* per acre per annum: find the rate of interest he obtained in each case, and the advantage of the second purchase over the first.

11. In a certain lake, the tip of a bud of lotus was seen a span above the surface of the water. Forced by the wind it gradually advanced, and was submerged at a distance of two cubits. Compute the depth of the water.

12. *A*, *B*, and *C* are partners; *A* receives ⅕ profits, and *B* twice as much as *C*; find the capital of *C*, *A*'s income being diminished £40 by a fall of ½ per cent. in the rate of profit.

13. A man expends £1000 in the purchase of Great Nugget shares of £5 when they are at 2 premium, and £500 in the purchase of Agna Frias of £2, when they are at ¾ discount, he sells out again when the Nuggets fall to par, and the Agna Frias rise to 3 premium. What does he gain or lose after paying the broker ¼ per cent. on all the money which passes through his hands?

III.

1. Define a Vulgar Fraction. How many kinds of Vulgar Fractions are there? Shew that multiplying the numerator of a fraction by any number is the same in effect as dividing the denominator by it.

Simplify

(1) $\left(\frac{2}{3}+\frac{5}{6}+\frac{7}{8}+\frac{11}{12}\right)\div\left(\frac{3}{4}-\frac{5}{8}\right)$.

(2) $\frac{7\frac{1}{2}}{6\frac{1}{2}}+\frac{11\frac{1}{2}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}}\times 10\frac{9}{13}-7\frac{1}{8}$.

2. If 1lb. Avoirdupois be equivalent to 7000 grains Troy, and 66 shillings weigh 1lb. Troy, find the value of 20 avoirdupois ounces of silver.

3. A gentleman dying leaves property worth £23,100 among 3 sons and 4 daughters, directing that the sons shall have alike $\frac{1}{3}$ more than their eldest sister, who should have £300 more than either of her younger sisters, they sharing alike. How much did each get?

4. Find at what rate simple interest a sum of money would amount in 2 years to the same as at 4 per cent. compound interest.

5. The edge of a cubical beam is 18 inches; what is the edge of one containing 8 times as much?

Find the side of a square field containing 2ac. 121 yds.

6. If 5 men and 7 boys can reap a field of corn of 125 acres in 15 days, in how many days will 10 men and 3 boys reap a field of corn of 75 acres, each boy's work being $\frac{1}{3}$ of a man's?

7. A person invests £962. 10*s*. in the 3 per cents. at 77, and when the funds have fallen 1 per cent. he transfers his capital to the 4 per cents at 95; find the alteration in his income.

8. Which is the more profitable investment; the purchase of 3 per cent. consols at £96, or the purchase of shares in an insurance office at £227 per share, the annual dividend on a share being £7. 10*s*.?

9. If the wholesale dealer sell to a retailer at 10 per cent. profit, and the retailer sell to the consumer at 50 per cent. profit, what proportion of the price paid by the consumer is profit?

10. Find the area of a court-yard 9yds. 2ft. 6in. in length, and 7 yds. 1ft. 8in. in breadth, by duodecimals, and explain your method of operation.

11. A wine-merchant pays £70 for a pipe of wine, and bottles it off into an equal number of quart, pint, and half-pint bottles. How many dozen of each has he, and at what must he sell it per dozen to gain 15 per cent. on his outlay?

12. With a gallon of rum which cost 15*s*. a man mixes a quart of water, and then sells it for 16*s*. a gallon: with a gallon of gin at 11*s*., he mixes $2\frac{1}{2}$ pints of water and sells it at 12*s*. a gallon: and with a gallon of brandy which cost 22*s*. he mixes 3 pints of water, and then sells it for 23*s*. a gallon; how much does he gain per cent. supposing him to sell twice as much rum as gin, and twice as much gin as brandy?

IV.

1. Find the square root of ·000961, and prove the correctness of the result obtained. What is the length in inches of the side of a cubical box which contains ·000027 cubic yards?

2. State the rules for multiplication and division of decimals. Divide 2·50892806 by 92·41035 to four places of decimals, and shew by fractions that the result is correct.

3. Define interest, simple and compound, present worth, and discount.

What sum of money lent at 5 per cent. simple interest for 3 years will amount to £828?

Find the present worth on £487. 5*fl.* 2*c.* 5*m.* due 175 days hence at 3·75 per cent. per annum.

4. A tenant holds a farm of 350 acres, subject to a tax of 3*s.* 6*d.* per acre and a corn-rent of 100 qrs. of wheat, barley, oats, and beans respectively. Find the amount of his rent when the average prices of wheat, barley, oats, and beans per quarter are 38*s.* 9*d.*, 27*s.* 4*d.*, 17*s.* 4*d.*, and 33*s.* 10*d.* respectively.

5. It is observed that 20 men, all of equal strength, build a wall 15 ft. high, 30 ft. long, in 60 days, and 35 others, also of equal strength, build a wall 20 ft. high, and 40 ft. long, in 64 days; what is the ratio of the strength of the men of the two classes?

6. What ought to be the value of £135 in the 4½ per cents., when the 3 per cents. are at 97⅛?

7. Standard gold being coined at the rate of £3. 17*s.* 10½*d.* per oz., what is the least number of ounces that can be coined into an exact number of sovereigns?

8. A person transfers £5000 from the 3¼ per cents. at 98 to the 3 per cents. at 94; how much of the latter stock will he hold, and what will be the difference in his income?

9. Out of £4. 7*s.* 6*d.*, one-third is paid to *A*, and one-seventh to *B*; after this $\frac{4}{11}$ths of the remainder is paid to *A*, and the rest to *B*: find the sums respectively received by *A* and *B*.

10. A portion of a church-roof 63 ft. long, and 27 ft. broad, is to be re-leaded. The old lead runs 8 lb. to the square foot, but it is intended that the new lead should only be 7½ lbs. What quantity of lead will be saved, and how much will it be worth at 2½*d.* per pound?

11. A bankrupt owes £2085, of which £235 are due to *A*, £325 to *B*, £525 to *C*, and the rest to *D*. How much must he pay in the pound that *D*'s receipts may be what *C*'s ought to have been, and how much will his other creditors have each to receive?

12. *A* is twice, and *B* is just one and a half times as good a workman as *C*. The three work together for two days, and then *A* works on alone for half a day, and *B* for one day. How long would it have taken *A* and *C* together to complete as much *as the three* will have thus performed?

V.

1. A man whose weekly earnings are 17*s.* 6*d.* saves a fifth part of that sum every fortnight. In what time will he have saved 50 guineas?

2. Distinguish between abstract and concrete numbers. What is the difference between 65*s.* divided by 15*s.*, and 65*s.* divided by 15?

3. 15 cwt. of cheese is divided amongst 60 men, and 104 women, on the condition that each man is to have twice as much as each woman; how much will each have?

4. A rectangular pile of wood is 12 yds. high and 10 broad, find the number of oblong pieces 18 ft. long, 8 in. broad, and 4 in. deep contained in it, supposing the cost of covering the pile with matting at 4*d.* per square foot to be £87.

5. A person bought 23, 24 and 25 quarters of wheat at 38*s.*, 39*s.* and 40*s.* per quarter respectively, and mixes them; at what price per quarter must he sell the mixture to gain 20 per cent. by the purchase?

6. A person having £2200. 3 per cent. consols sells out at 97½; and invests the proceeds of the sale in a Railway Stock paying 4¾ per cent. at 95; find the alteration in his income.

7. Bought cloth at 9 months credit for 21*s.* per yd.; how much per yd. should I be allowed for present payment, interest being reckoned at 4 per cent. per annum?

8. How much cotton 4 ft. wide at 3*d.* per square ft. must be given in exchange for 34·45 metres of French silk ¾ of a yd. wide at 4 francs per square metre, £1 being worth 25·15 francs, and the metre being 39·37 inches?

9. Explain the method of pointing in the extraction of the cube root of decimals.

Find the square root of $\frac{\cdot 00125}{\cdot 18}$ and the cube root of 423564·751.

10. The produce of the Income-Tax at 7*d.* in the pound is £525,000. What would be the gain to the revenue if the tax were at the rate of 3 per cent.?

11. Two clocks point to 2 o'clock at the same instant; one loses 7″ and the other gains 8″ in 24 hours; when will one be half an hour before the other, and what time will each clock shew?

12. A person gives 100 guineas for a pipe of wine, which contains 52 dozen bottles; what does his wine cost him a dozen if he has to keep it ten years in bottle, and 4 per cent. simple interest be allowed on the outlay?

VI.

1. Divide 7244·3 by ·000917, and prove the correctness of the result.

Find the value of $\frac{1}{7} - \left(\frac{3}{35} + \frac{4}{55}\right) + \frac{6}{385}$.

Simplify the expression

$$\frac{\frac{5}{16} + \frac{7}{12} \text{ of } 3\tfrac{1}{4} - \left(\frac{7}{8} \text{ of } \frac{37}{21} - \frac{1}{3}\right)}{\frac{5}{14} - \frac{3}{7} \text{ of } \frac{1}{2}}.$$

2. Divide 5 miles, 20 poles, 3 yds., 6 in. by 3, and express the result in furlongs.

3. A person purchases £1000. 3 per cent. consols at $96\frac{7}{8}$, and sells out again when they have sunk to $82\frac{3}{4}$; how much does he lose by the transaction?

4. A legacy of £1500 is left to three individuals in the proportions of 1, 2, and 3; find the sums received by each after deducting the legacy duty of 10 per cent.

5. Find the discount on £237. 10*s.* due 4 months hence, at 4 per cent. simple interest. What would be the amount of the error in the above case if interest were taken instead of discount, and in whose favour would the error be?

6. How many yds. of English cloth, $1\frac{1}{2}$ yds. wide, at 2*s.* 6*d.* per sq. ft., must be given for 10 metres of French velvet, $\frac{1}{3}$ of a metre wide, at 10*s.* per sq. metre? (supposing 37 English feet = 36 Rhynland feet, and 3·192 R. ft. = 1 metre).

7. A person sells out of the 3 per cents. at 96, and invests his money in Railway 5 per cent. stock. By this means his income is increased 50 per cent. Find the price of the railway stock.

8. The cost of carpeting a room twice as long as it was broad at 5*s.* a sq. yd. amounted to £6. 2*s.* 6*d.*; and the painting of the walls at 9*d.* a sq. yd. amounted to £2. 12*s.* 6*d.* Find the height of the room.

9. How much ought the price of the three per cent. consols to sink below par, in order that a broker may be enabled to obtain four per cent. on money?

10. Shew that $\sqrt{2}$ lies between $\frac{17}{12}$ and $\frac{41}{29}$; extract the cube root of 669·921875 cubic feet, and reduce the result to inches.

11. A man purchases £700 stock in the 3 per cent. consols at $94\frac{1}{2}$, and also invests £585 in the purchase of Russian 5 per cent. stock at $97\frac{1}{2}$; how much stock has he standing in his name? If he sells out of the 3 per cents. at 95 and out of the 5 per cents. at $96\frac{1}{4}$, does he gain or lose by the transaction, and how much?

12. A grocer buys coffee at the rate of £8. 10*s.* per cwt. and chicory at £2. 10*s.* per cwt., and mixes them in the proportion of 5 parts chicory to 7 coffee; at what rate must he sell the mixture so as to gain £$16\frac{2}{3}$ per cent. on his outlay?

VII.

1. Which is cheapest, an article that costs 15*s.* and will last 9 months, or one which costs 12*s.* and will last 7 months? How much will be saved in 3 yrs, 33 wks, (1 year = 52 wks.), not calculating interest, by constantly using the cheaper one?

2. Find the value of $\dfrac{\frac{1}{3} \text{ of } \frac{1}{2} + \frac{3}{2} \text{ of } 5}{9\frac{1}{3} - 1\frac{2}{3}}$; also of $\dfrac{\frac{2}{3} - \frac{1}{4}}{1 \div \left(\frac{3}{10} - \frac{1}{2} \times \frac{1}{5}\right)}$ of 2 shillings.

Express $\dfrac{2^7}{7 \times 10^7} + \dfrac{2^8}{3 \times 10^8}$ in decimals.

3. Gunter's chain is four poles long, and divided into 100 links. Shew that square links may be converted into acres by moving the decimal point five places *to the left.* (*N. B. 160* square poles = 1 acre.)

4. *A* pays *B* a debt a year before it is due, mercantile discount being allowed. If *B* had waited for payment till the end of the year, he would then, money being supposed to produce 5 per cent. interest, have been £5 richer than by the actual arrangement. Determine the amount of *A*'s debt.

5. *A* and *B* can perform a piece of work in 10 days, *A* and *C* in 12 days, and *B* and *C* in 16 days. In what time would they do it separately?

6. Having given that 125 Italian lire make 23 Roman scudi, and that 2001 Roman scudi make 12500 Austrian zwanzigers; find how many Italian lire make 100 Austrian zwanzigers.

7. A person has 4 houses, the united values of which amount to £1840. The value of the first house is two-thirds of the second, that of the second is three-fourths of the third, and the value of the third is five-sixths of the fourth house; find the value of each.

8. At what rate per cent. simple interest will 3½*d.* produce 3½*s.* interest in 33⅓ years?

9. If Government allow land-tax to be redeemed for so much stock in the 3 per cent. consols as will produce a yearly income larger by one-tenth part than the yearly tax redeemed, what sum of money sterling must be invested in stock in order to redeem a land-tax of £2. 6*s.* 8*d.* per annum; consols being at 96⅜ per cent., inclusive of brokerage?

10. Extract the cube root of 7311891877729.

11. A man buys a flock of sheep consisting of 117 for £108; he loses 3, and finds 18 others in such bad condition that he is obliged to sell them for 4 shillings apiece less than they cost him; at how much per head must he sell the remainder, in order that he may, on the whole, gain £5 by the transaction?

12. A room 27·7 ft. long, 19·55 ft. wide, and 12·4 ft. high, is hung with paper 2·7 ft. wide; find the cost of the paper at 1*s.* 3*d.* a yard.

VIII.

1. Shew how to change a vulgar fraction into a decimal, and prove that the decimal will terminate or recur according to the form of the denominator of the given fraction in its lowest terms. Which sort will the fraction $\frac{102}{136}$ produce?

2. Add together $\frac{3}{4\frac{1}{2}}$ of 4*s.* 7*d.*, 2·35 of 1*s.* and ·2375 of £1; and reduce the result to the decimal of half a guinea.

3. Extract the square root of ·142857 of $3\frac{1}{9}$ square yards, and express in inches the cube root of 41·421736 solid feet.

4. How much money must a broker invest in the funds when consols are at 90, so as to procure the same income as if he had invested £1100 when consols were at 99?

5. Of every volume of air 21 per cent. is oxygen. How much oxygen is contained in 25 cubic feet 700 cubic inches of air?

6. A person receives £533. 6*s*. 8*d*. for £560 due two years hence. At what rate is discount calculated?

7. Find the cost of overlaying with gold a table $6{\cdot}\dot{4}28571\dot{4}$ yards long, and $1{\cdot}7\dot{8}$ yards wide at 25*s*. the square inch.

8. Paid £30 including a duty of £5 per cent. for a Geneva watch; what did the duty amount to?

9. The exchange at Paris upon London is at the rate of 25 francs, 70 centimes for £1 sterling, and the exchange at Milan upon Paris is at the rate of 42 Austrian lire for 20 francs: find how many Austrian lire should be paid at Milan for a £20 note?

10. A dealer buys 80 tons of coals, and after selling them again at 1*s*. 6*d*. per sack finds that he has gained £4. Had he sold them at 1*s*. 4*d*. per sack he would have lost £6. Find the cost price per ton, and the weight of a sack of coals.

11. A piece of work has to be finished in 36 days, and 15 men are set to do it, working 9 hours a day, but after 24 days it is found that only $\frac{3}{5}$ths of the work is done: if 3 additional men be then put on, how many hours a day will they all have to labour to finish the work in time?

12. A wine-merchant having bought 7 dozens of wine at 68*s*. per dozen *starts* the same into a vat containing 130·4 gallons of wine worth 38*s*. per dozen, and sells the mixture at 46*s*. What per centage does he obtain? (a dozen bottles contain 2·7 gallons).

IX.

1. From one million take ninety-nine thousand and nine; and take one millionth from one hundred and one thousandths, expressing the result by a decimal.

What is the amount of 9 lbs. 3 oz. $12\frac{1}{2}$ dwts. repeated 6240 times?

2. Reduce $\frac{4}{5}$ of $\frac{3}{7}$ of $7\frac{1}{2}$*d*. to the fraction of half-a-guinea.

Divide $\frac{1}{4}$ of $1\frac{3}{5}$ by $\frac{3}{5}$ of $\frac{4}{9}$, and 16 ft. 3 in. by $2\frac{1}{10}$.

3. Express ·0025 by a simple fraction, and $\frac{3}{125}+6\frac{3}{40}-\frac{17}{25}$ by a decimal.

Multiply 1·25 by ·072. Divide ·010101 by ·02; 120 by ·0048; and 56·25 by 1·25; proving the truth of each result.

4. If three men working 11 hours a day can reap 20 acres in 11 days; how many men working 12 hours a day will reap a field 360 yards long and 320 yards broad in 4 days?

5. Divide 448 guineas among 7 men, 9 women, and 3 boys, so that each woman may have $\frac{3}{5}$ of each man's share, and each boy $\frac{6}{7}$ of each woman's share.

6. A person invests £4800 in the 4 per cents. at 80, and at the end of each year invests the dividend which becomes due in the same stock; supposing the funds to remain at 80 for 3 years, find his dividend at the end of the third year.

7. *A* and *B* can do a piece of work in 6 days, *B* and *C* can do it in 7 days, and *A*, *B*, and *C* can do it in 4 days. How long would *A* and *C* take to do it?

8. In how many years will £250 double itself at 2½ per cent. simple interest?

9. Find the amount of £254. 4*fl.* 8*m.* for 5 years at 3½ per cent. per annum, simple interest.

10. A room whose height is 18ft. 5in., breadth 20ft. 10in. and length 22ft. 9in. has a door 7ft. 6in. by 3ft. 4in., and two equal windows each 3ft. 6in. by 5ft. 2in.; find the cost of papering it at 3*s.* 5¼*d.* per square yard.

11. A debt is to be discharged at the expiration of 4½ months, ¼ is paid immediately, and ¼ at the expiration of 3 months; when ought the remainder to be paid?

12. Find the length of the interior edge of a cubical bin which contains 40 quarters of wheat. (An imperial bushel fills 2218·192 cubic inches.)

13. A person increased his capital annually ⅓rd part, and at the end of 4 years, one year's interest thereon at 4½ per cent. amounted to £270. What capital did he start with?

X.

1. The height of a tower on a river's bank is 50 feet, the length of a line from its top to the opposite bank is 65 feet; what is the breadth of the river?

2. *A* and *B* agree to divide their travelling expenses in the proportion of the numbers 7 and 5. *A* pays in the whole £25. 16*s.*, and *B* pays in the whole £15. 17*s.*; what has the one to pay and the other to receive in order to settle the account?

3. The posts of an electric telegraph by the side of a railway are placed at intervals of 60 yards: find the rate per hour of a train which passes over eleven of these intervals in 25″. Also, find the least distance of two posts from each other which are an exact number of miles apart.

4. The weekly receipts of a Railway Company average £2683. 7*s.* 6*d.* from passenger, and £2117. 8*s.* from goods traffic. The expenses of working are £13,303. 7*s.* 2*d.* per calendar month. Their capital is 2 millions: what interest can they pay per share of £100?

5. A grocer buys 567 cwt. of sugar at £1. 19*s.* 10½*d.* per cwt., and mixes it with 1161 cwt. at £2. 2*s.* 6½*d.*; at what price per lb. must he sell the mixture, in order to realize a profit of 12 per cent.?

6. A man buys 50 shares in a railway at £20. 10*s.* per share; and 100 more at £7. 15*s.* per share. The half-yearly dividend is 3*s.* 4*d.* per share: what interest per cent. per annum does he make of his money?

7. A person buys wheat at 39*s.* per qr., and some of superior quality at 6*s* per bushel: in what proportion must he mix the two so as to gain 25 per cent. by selling the mixture at 57*s.* 6*d.* per qr.?

8. At a game of billiards, *A* can give *B* 15 points in 50, and he can give *C* 20 in 50? how many can *B* give *C* in a game of 70?

9. If 225 quarters of wheat which were sold on Sept. 28th, 1853, for 56*s.* per qr. would have fetched 94*s.* per qr. on March 25th next following, and money be valued at the rate of 3½ per cent. per annum, what would then have been gained by having kept the wheat?

10. A tradesman marks his goods with two prices, one for ready money, and the other for credit of 6 months: what fixed proportion ought the two prices to bear to each other, allowing 5 per cent. per annum, simple interest?

11. A certain sum in ten years amounted to £10,000 at 5 per cent. per annum, simple interest: find the sum. What must be the market-value of 3 per cent. stock, in order that, after deducting the income-tax of seven-pence in the pound, it may yield 3½ per cent. interest?

12. If the 3 per cents. are at 95, and Government offer to receive tenders for a loan of £5,000,000, the lender to receive five millions in the 3 per cents., together with a certain sum in the 3¼ per cents., what sum in the 3¼ per cents. ought the lender to accept?

XI.

1. Supposing the velocity of electricity to be 288,000 miles per second, and the earth's circumference to be 25,000 miles; calculate to seven places of decimals the time of transmitting an electric telegraphic message to the Antipodes.

2. There are 5 partners in a business which produces £2100 profit; the senior partner has 6 shares, the second 5 shares, and the three junior partners 1 share each. What does each partner receive?

3. Find the cost of paving a street ½ a mile long, and 47 ft. broad, at the cost of 8½*d.* per square yard.

4. A tea-dealer buys a chest of tea containing 2 qrs. 17 lbs., at 3*s.* 1½*d.* per lb., and two chests, each containing 3 qrs. 7 lbs., at 3*s.* 5½*d.* per lb.; what will he gain per cent. by selling the mixture at 4 shillings per lb.?

5. If a person receives 4½ per cent. interest on his capital by investing in the 3½ per cents., what is the price of the stock, and how much stock can be purchased for £1200?

6. From a bill of £3. 11*s.* 8*d.*, due 18 months hence, a tradesman deducts 5*s.*; what is the rate per cent. at which the discount is calculated?

7. If a lb. Troy of English standard gold $\frac{11}{12}$ fine be worth £46. 14*s.* 6*d.*, what is the value of a rupee weighing 7 dwts. 11 grs., in which $\frac{122}{1000}$ is pure gold?

8. The sum of £1250 is invested in the 3 per cent. consols, when the price of stock is 88 per cent.; what will be the income produced, and how will the income be affected if the price of stock rise to 92 per cent., and the stock be then sold out and the money put out to interest at 4 per cent.?

9. Explain the method of pointing in extracting the cube root of whole numbers, and decimals.

10. Find the length of the side of a cube which contains 8060150125 solid inches.

11. If I borrow money at 3 per cent. interest, payable yearly, and lend it immediately at 5 per cent. payable half-yearly, (receiving compound interest for the 2nd half-year), and gain thereby at the end of the year £660; what was the sum of money which I borrowed?

12. If a four-oared boat costs £50, what would a six-oared boat cost, supposing the expense of setting up a boat is proportionate to the square of the number of oars? Which would cost the least per head?

XII.

1. Explain the nature of decimal fractions; state the rules for the addition and multiplication of them, and shew that they are the same as in the case of vulgar fractions. Write down the cube of ·2.

2. The population of Great Britain in 1851 was 21,121,967; and the increase during the previous half century had been 93·47...per cent. What was the population in 1801?

3. The sidereal year being 365d., 6h., 9m., 9·6sec.; and the tropical year 365d. 5h., 48m., 49·7sec.; reduce their difference to the decimal of a tropical year.

4. 80,000 cwt. of ammunition are to be removed from a fortress in 9 days: but it is found that in 6 days 18 horses have carried away only 4500 cwt.; how many horses are requisite to carry away the rest in the time that is left?

5. What is the discount on £775. 6*fl.* 1*c.* 2·5*m.* due 5 months hence at $4\frac{1}{2}$ per cent.?

6. Would a person increase or diminish his income by selling £1157. 3 per cent. stock at $83\frac{1}{4}$ to purchase into the $3\frac{1}{4}$ per cents. at $83\frac{7}{8}$?

7. A tradesman finds that if he asks for his goods 15 per cent. above the wholesale price, he can sell his whole stock in 4 months, whereas if he asks 20 per cent. he requires 6 months to sell the same amount. Which will he find the more profitable system at the year's end?

8. Find the respective times between 7 and 8 o'clock, when the hour and minute-hands of the watch are, 1st, exactly opposite to each other; 2nd, at right angles to each other; 3rd, coincident.

9. *A* and *B* can do a piece of work together in 4 hours, *A* and *C* together in $3\frac{3}{5}$ hours, and *B* and *C* together in $5\frac{1}{7}$ hours; in what time will each do the work alone?

10. If (a) and (b) be the sides of a rectangle, in what sense may the area of the figure be said to be $a \times b$?

Find the area of a room 25ft. 6in. by 18ft. 3in.; and the cost of covering it with carpeting 2 ft. wide, at 4*s*. per yard.

11. After deducting from a clear rental of £15000 one-fifth for personal expences, and 1000 guineas for extras, I invest the remainder in an estate at 25 years' purchase; what will be the annual value of the estate?

12. A London merchant owes a sum of money in Paris: which method of payment will be most advantageous to him, a direct exchange, or a circuitous remittance from London to Venice, from Venice to Hamburg, and from Hamburg to Paris, the exchanges being as follows: £1 = 24·6 French francs, 19 francs = 10 Hamburg marks, 1 mark = 4¼ lire of Venice, 55¼ lire = £1?

XIII.

1. What sum is the same fraction of a crown that 2*s.* 9⅗*d.* is of a guinea?

2. In what length of time will £150 amount to £165. 7*fl.* 5*c.* at 3 per cent. simple interest?

3. Define Present Worth, and Discount. State the difference between mathematical and mercantile discount. A farmer buys 57 sheep for £120, payable at the end of 12 months, and sells them directly at 35*s.* a-head ready money; what does he gain by the transaction, supposing the interest of money to be 5 per cent.?

4. Eight bells begin tolling simultaneously, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively: find after what interval of time they will again be all tolling at the same instant.

5. A person's average annual expenditure from the year 1830 to the year 1850 inclusive is £391. 9*s.* 2*d.* He finds that in 1830 he spent £391. 16*s.*, and in 1851 £445. 8*s.* 9*d.* What was his average annual expenditure from 1831 to 1851 inclusive?

6. In Austria 120 gulden (paper currency) are worth 100 silver gulden. What amount of paper money should be obtained for £10 sterling, if the value of £1 be 9 gulden, 30 kreutzers in silver? (60 kreutzers = 1 gulden).

7. If a piece of work can be done in 50 days by 35 men working at it together, and if, after working together for 12 days, 16 of the men were to leave the work; find the number of days in which the remaining men could finish the work.

8. A shilling weighs 3 dwts. 15 grs., of which three parts out of forty are alloy and the rest pure silver: if the value of silver rises 8 per cent., what must be the reduction in the weight of pure silver in a shilling?

9. A cube contains 56 solid feet, 568 solid inches; find its edge.

10. By selling a horse for £116. 17*s.*, a person lost 5 per cent.; what will be his gain or loss per cent. when he sells him for £132. 4*s.* 6*d.*

11. The sum of £6000. has been subscribed to build lodging-houses for the poor, and accommodation provided for 120 families. These are divided into 6 classes which pay respectively 7, 6, 5, 4, 3½, 3 shillings per week per family. How much per cent. per annum will the subscribers realize, supposing the yearly expenses for repairs &c. to be £20?

12. A man in mowing grass walks at the rate of ·4 miles an hour; it takes him 72 minutes to mow a grass-plot of 1056 square yards, how broad does he mow?

XIV.

1. Express in figures thirty-four and two thousandths, and by it divide 2825566·2. What alteration must be made in the quotient if the decimal point in the dividend be moved eight places to the left?

2. A bankrupt owes £6000, and has good debts to the amount of £1800, and bad debts (for which he receives on the average 10*s.* in the pound) to the amount of £1200: how much can he pay in the pound?

3. Find by Practice the value of 7 cwt. 2 qrs. 15½ lbs. at £3. 0*s.* 7*d.* a cwt.

4. On June 1, 1854, the 3 per cent. stocks are quoted at 91¼; what would be the annual income of a person investing £3411. 6*s.* 8*d.* in them, after payment of the income-tax of 10*d.* in the £1, brokerage being estimated as usual at 2*s.* 6*d.* per cent.?

5. If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden; how many gulden = £1?

6. What principal in the decimal coinage put out at simple interest for 5 years at 4 per cent. will amount to £111?

7. Divide £2025 among *A*, *B*, *C*, *D*, *E*, so that *A*'s share : *B*'s share :: 1 : 2; *C*'s share : *B*'s share :: 5 : 4; *D*'s share : *C*'s share :: 6 : 5; and *E*'s share : *D*'s share :: 4 : 3.

8. *A* can do a piece of work in 27 days, and *B* in 15 days; *A* works at it alone for 12 days, *B* then works 5 days, and afterwards *C* finishes it in 4 days; in what time could *C* have done the whole work?

9. Extract the square root of 3080¼, and the cube root of 9·528128. Find the hypothenuse of a right-angled triangle of which the other sides are 4 ft., and 1 ft. 8 in.

10. Incomes below £150 a year being subject to 5*d.* in the pound income-tax, and incomes above £150 to 7*d.* in the pound; find what income above £150 a man must have, that he may be just 7¾*d.* a year poorer than a man who has £149. 10*s.* a year.

11. A person enters France with 33 sovereigns, one half-sovereign, and 7 florins in his purse. He expends in France 577 francs, 50 centimes. How many florins will he receive in exchange for the remainder; the rate of exchange being 26 francs, 25 centimes per pound sterling?

12. *A* has £90,000 stock in the 3 per cent. South Sea Annuities, and is offered by government the choice of being paid off at par at the end of the year, or of receiving £110 of a new 2½ per cent. stock for each £100. He chooses the former alternative; and, on being paid off, is able to invest his money in the 3 per cent. consols at 92. Find the amount of his stock in consols, and the excess of his income above what it would have been if he had agreed to the proposed conversion.

XV.

1. 650 horses are conveyed in transports to the seat of war at a cost for food of £1542. A storm occurs just after one-fourth of the voyage is completed, in which 10 horses are killed. If the expense of the food of each horse be 1*s*. per day, what was the length of the voyage?

2. The shares in a certain undertaking pay 13*s*. 4*d*. each, being at the rate of 5 per cent. Find the amount of each share and what interest per cent. they will pay upon the purchase-money if bought at $\frac{7}{8}$ premium.

3. A cask which weighs 1 cwt. 19 lbs. 13 oz. floating in sea-water displaces 2 cub. ft. $61\frac{5}{9}$ cub. in. What is the weight of a cub. ft. of sea-water?

4. A man invests £1000 equally, in the shares of two railway companies; the shares of the one are 3 per cent., and the other at 5 per cent. discount; the price of stock in the former suddenly rises 7 per cent. and that of the latter is depressed 5 per cent. lower than when the man purchased; if he now sell out, what will he gain or lose?

5. An oz. of gold is worth £3. 17*s*. $10\frac{1}{2}$*d*. In making sovereigns 2 parts out of 23 consist of an alloy which is worth $1\frac{1}{2}$*d*. an oz.; how much gold and how much alloy will be respectively required for 2617 sovereigns?

6. A river 30 ft. deep and 200 yds. wide is flowing at the rate of 4 miles an hour, find how many cub. ft. of water runs into the sea per minute: also the number of tons. (A cub. ft. of water = 1000 oz.)

7. A person buys sugars at 7*d*. and 10*d*. a lb., and mixes them in the proportion of 3 : 5; what will he gain per cent. by selling the mixture at 9*d*. a lb.?

8. Find the number of cubic chains and links in a rectangle parallelopiped whose edges are 94 chains 50 links, 1 chain 5 links, and $31\frac{1}{2}$ links.

9. A person buys £500, 3 per cent. consols, at $96\frac{7}{81}$, and sells out again at $82\frac{3}{4}$: how much does he lose by the transaction?

10. *A* owes £3000 bearing interest at £5 per cent. per annum; he pays at the end of each year for interest and in part payment of the principal £500: find the amount of his debt at the end of the third year.

11. A wall, 5 times as high as it is broad, and 8 times as long as it is high, contains 18225 cub. ft. Find the breadth of the wall.

12. The capital of a mining-company is £300,000; the working expenses of the mine amount to £15,691. 13*s*. 4*d*. in a given year: what must be the gross proceeds of the mine to pay the original proprietors 4 per cent.? Also supposing the original shares to be £10, what per centage will those persons receive who have purchased at 2 discount?

XVI.

1. What is the cost of papering a room 12 ft. 4 in. high, 16 ft. long, and 14 ft. 3 in. wide; the paper being 2 ft. 6 in. wide, and costing 3*s*. 6*d*. a yard; the workmen also charging $\frac{1}{2}$*d*. for every square foot?

2. If £1 = 10 rupees = 100 cents = 1000 mils, express £37. 15*s*. 7½*d*. in this coinage. What is the difference between a farthing and a mil?

3. If silver be worth 5*s*. 6*d*. an oz., and pure gold be worth £4. 5*s*. an oz., what would be the weight of a 15*s*. piece, containing 92·5 per cent. pure gold, and the rest silver?

4. The content of a cistern is the sum of two cubes whose edges are 10 and 12 inches respectively, and the area of its base is the difference of two squares whose sides are $1\frac{1}{9}$ and $1\frac{2}{9}$ ft.: find its depth.

5. In the London General Post the proportion of unpaid letters to the whole number posted was 8 per cent.; and of the paid letters 51 per cent. were stamped; the whole number posted was 450,000: how many of them were stamped?

6. *A*, who travels 3¼ miles an hour, starts 2½ hours before *B*, who travels the same road at the rate of 4½ miles an hour: when will *B* overtake *A*?

7. The cost of carpeting a room twice as long as it was broad at 5*s*. a sq. yd. cost £6. 2*s*. 6*d*.; and the painting of the walls at 9*d*. a sq. yd. cost £2. 12. 6*d*. Find the height of the room.

8. A watch set accurately at 12 o'clock indicates 10 min. to 5 at 5 o'clock P. M. What is the time when the watch indicates 5 o'clock?

9. If a cubic foot of gold may be made in gilding to cover 402,600,000 square inches, find the thickness of the coating of gold.

10. A person purchases a hundred shares in a company at £3500; and ultimately sells at a profit of £43. per cent., after having received four dividends of 15*s*. 4*d*., 20*s*. 10*d*., 30*s*. 4*d*., and 38*s*. 9*d*. a share; by how much do his receipts exceed his outlay?

11. If 236 yards of cambric are bought at 7*s*. 10½*d*. per yard, and sold, one fourth at 10*s*. 3*d*., one third at 8*s*. 6*d*., and the remainder at 7*s*. per yard, what will be the loss or profit per cent. upon the whole outlay?

12. A person buys a number of railway shares at £12. 17*s*. 6*d*. per share; the purchase-money (including £8. 12*s*. 6*d*. expenses) being £1000. How many shares does he buy?

What yearly income will he derive when the dividend is at the rate of £3. per cent. on the original shares, which are £20?

XVII.

1. If the hands of a clock be together at 12 o'clock, when will they be together again?

2. A field of 7 acres is sown with turnips, beet, and cabbages; the areas of the crops being respectively as 1⅓ : 1½ : 1¼. If the values of an acre of each be also respectively in the same ratios, and an acre of turnips be worth £7; what is the worth of the whole crop?

3. Find at what time between 3 and 4 o'clock the minute-hand of a watch is 30 minutes before the hour-hand.

4. A cistern is filled by two pipes in 20 and 24 minutes respectively, and emptied by a tap in 30 minutes; what part of it will be filled in 15 minutes when they are all left open together?

5. The driving-wheel of a locomotive engine, 5 feet in diameter, turned 2500 times in going 6 miles; supposing the circumference of a circle to be 3·1416 times the diameter, find what distance was lost owing to the slipping of the wheel on the rail.

6. The value of a fraction is not altered if both its numerator and denominator be multiplied or be divided by any the same number.

When is a fraction said to be in its lowest terms? Shew that $\frac{835}{1472}$ is such a fraction, and explain the process.

7. The length of a decimetre is 3·937 inches; find the number of solid [illegible] in a cube the length of whose side is a decimetre.

8. If 160 horses consume a stack of hay 20 feet long, 11 feet, 3 inches broad, and 31 feet, 6 inches high, in 9 days, for how many days will a stack 16 feet long, 5 feet broad, and 14 feet high, supply 80 horses?

9. Find the present worth of £130. 0*s.* 7½*d.* due 9 months hence, supposing the 4 per cents. to be at 92.

10. Suppose the population of a country would increase annually by 3 per cent., were it not for emigration, which annually carries off ·5 per cent. of the people; what will be the increase per cent. in the population after 5 years?

What is the population of a country from which at the above rate 81,000 persons emigrate in 2 years?

11. One company guarantees to pay 5 per cent. on shares of £100 each: another guarantees at the rate of 4⅝ per cent. on shares of £7. 10*s.* each: the price of the former is £124½, and of the latter £8. 10*s.*; compare the rates of interests which they return to the purchaser.

12. Multiply and divide 52 ft. 6 in. by 5 ft. 10 in., and explain the results. I have to pack 1200 books in a box 5 ft. 3 in. long, 3 ft. wide, and 2 ft. 9 in. deep; each book is 10½ in. long, 4½ in. wide, and 1⅜ in. thick; find how many must be left out.

ANSWERS TO THE EXAMPLES.

Ex. I., (p. 3.)

Notation.

1. 63; 81; 99; 40; 13.
2. 200; 303; 764; 888.
3. 4000; 1471; 6930; 9009.
4. 27504; 33000; 9016.
5. 100000; 676050; 202593.
6. 7003000; 11108106; 54054088; 613020303.
7. 2000000000000; 9000000300021; 94000090094904.

Numeration.

1. Forty-three; sixty; eighty-eight; ninety-seven; fifty-nine; twelve; twenty-one; nineteen.

2. Two hundred and fifty-six; four hundred and one; five hundred; nine hundred and ninety-nine; three hundred and sixty-five; five hundred and seventy-eight; eight hundred and thirty-seven.

3. Two thousand; one thousand, seven hundred and twenty-four; three thousand and three; seven thousand, five hundred and eighty-four; one thousand and seventy-five; four thousand, five hundred and forty-one.

4. Thirty-seven thousand and three; forty-seven thousand and forty-nine; sixty-three thousand and ninety; eighty thousand and eight; three hundred and forty-one thousand, three hundred and twenty-three.

5. Six millions, eight hundred and fifty thousand, four hundred and six; eight millions, eighty thousand, eight hundred and eight; seven millions, eight hundred and forty-nine thousand, six hundred and thirty; four hundred and eighteen thousand, two hundred and fifty-four.

6. Ten millions and one; twenty millions, two hundred and twenty thousand and twenty-two; ninety-two millions, five hundred and sixty-eight thousand, nine hundred and eighty-seven; thirty millions, one hundred and eighty thousand and seventy.

7. Two thousand five hundred and sixty millions, five hundred and thirty thousand, two hundred; eight hundred millions, three hundred and nine thousand, five hundred and sixty; nine thousand seven hundred and thirty-eight millions, four hundred and thirteen thousand, two hundred and eight.

8. Seven thousand and seventy millions, four hundred and twenty-three; nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one; five thousand seven hundred and seven millions, sixty-eight thousand and eighty.

9. One hundred billions, one hundred and ninety-eight thousand seven hundred millions, ten thousand and ninety; forty-eight thousand seven hundred and twenty-six billions, eight hundred and seventy thousand six hundred and thirty-four millions, one hundred and three thousand, two hundred and sixty-four.

Ex. II., (p. 6.)

Addition.

2. 252. 4. 2288. 5. 2073. 6. 12854. 8. 15997. 9. 19258.
10. 112040. 11. 11001543. 13. 3611911. 14. 1148390.
15. 2923038. 16. 1881390. 17. 14547; 48829; 82391.
18. 779264; 2925620. 19. 149036957938; 16696683926; 142228910945.
20. 98929. 21. 4304268. 22. 1002733636293.

Ex. III., (p. 10.)

Subtraction.

2. 445. 3. 221. 4. 86. 5. 1147. 6. 41830.
7. 17359. 8. 432099. 9. 899899. 10. 300368384.
11. 73646889. 12. 6130908; 7036970; 111232112.
13. 115849491; 2922930923; 568990634342. 14. 8087; 4936.
15. 3999996; 99700000. 16. 14515927.

Ex. IV., (p. 11.)

1. XXX; XLVIII; LIX; CCXXII; $\overline{\text{VI}}$; M.DCCC.XLIII.

2. Twenty-three, 23; sixty-nine, 69; two hundred and eighteen, 218; five thousand and one, 5001; one hundred and fifty thousand, six hundred and three, 150603; two millions, one hundred, 2000100.

Ex. V., (p. 16.)

2. 2019. 3. 14335. 4. 44952. 5. 14399. 6. 459272.
7. 24710742. 9. 395736. 10. 1098300. 12. 1646331.
13. 4015708. 14. 949723. 15. 24642451. 17. 67248560.
18. 33075. 19. 4843162. 21. 128137428. 22. 694090141.
23. 4222404, 6802762, 12432634;
61964682, 87860370, 397683780;
586289802, 2862835536, 2581382769;
182581498641, 58943103679, 70935237485, 67108855380.
24. 16322724; 213777000; 2361710300; 21810149152;
16340824080; 121932631112635269; 40155302248305278754132.
25. 44786996200592; 2605651657240; 128572831324016;
15232906283422580; 16301880531036492032 85.
26. 1955470720; 684763647963885.
27. 3876; 54095923986; 440956790820. 28. 21084100; 1408008.

Ex. VI., (p. 23.)

1. 226. 2. 45340. 3. 130535154. 4. 2126. 5. 156950.
6. 128578. 7. 93024. 8. 11848122. 9. 67525. 10. 1975308.
11. 148351. 12. 630341. 13. 89057. 14. 11204629, with rem. 4.
15. 20608. 16. 476539. 17. 874359. 18. 4359760. 19. 37072.

20. 8352303, with rem. 9. 21. 17636. 22. 3633365, with rem. 11.
23. 31122. 24. 46267254. 25. 61728. 26. 456932. 27. 543817.
28. 18574687. 29. 930622, with rem. 36. 30. 71340987.
31. 814545, with rem. 17. 32. 11805559. 33. 234915. 34. 704745.
35. 8862. 36. 40930, with rem. 270. 37. 591863.
38. 22151337, with rem. 47191. 39. 57190870. 40. 7575.
41. 65299477. 42. 243096259. 43. 3396, with rem. 5094687. 44. 14830201.
45. 9000900090009 with rem. 1; and 900009000090, with rem. 10.
46. 3854, with rem. 26167. 47. 746115, with rem. 83337. 48. 6084. 49. 874359.
50. 764095. 51. 11717201, with rem. 645. 52. 5771, with rem. 542962567.
53. 39486, with rem. 2211. 54. 35. 55. 2826863, with rem. 55.

Ex. VII., (p. 26.)

1. (1) 8. (2) 15. (3) 9. (4) 11. (5) 4. (6) 40. (7) 17.
(8) 2. (9) 20. (10) 15. (11) 25. (12) 8. (13) 8. (14) 12.
(15) 493. (16) 13. (17) 13. (18) 2. (19) 7. (20) 6. (21) 36.
(22) 84. (23) 504. (24) 83. (25) 11. (26) 123. (27) 23. (28) 36.
(29) 2223. (30) 142857. (31) 87. (32) 37.

2. (1) 2. (2) 2. (3) 13. (4) 3. (5) 23. (6) 7. (7) 4. (8) 2.

Ex. VIII., (p. 30.)

1. (1) 48. (2) 900. (3) 105. (4) 140.
(5) 11803. (6) 18648. (7) 50337. (8) 408672.
(9) 344988. (10) 2663667. (11) 10867905. (12) 11754483.

2. (1) 72. (2) 48. (3) 30. (4) 120. (5) 1080.
(6) 204. (7) 1102. (8) 192. (9) 252. (10) 3465.
(11) 600. (12) 720. (13) 2520. (14) 1260. (15) 2268.
(16) 7200. (17) 2520. (18) 1008. (19) 22680. (20) 2017790775.

Ex. IX., (p. 31.)

I.

1. 28944. 2. Nine millions, ninety thousand, nine hundred and nine; ninety thousand, nine hundred and nine; 9181818; 9000000.

3. 1053634. 4. 36 years. 5. 548501.

II.

2. 5 years. 3. 700409000000000000. 4. 533242. 5. 19052.

III.

2. 300 days, and 75 lines remaining. 3. 21840. 4. 9376.
5. *A*, *B*, and *C* score respectively 18, 57, and 33 runs.

IV.

1. 24574. 2. 1302; 448. 3. 29; 71.

4. 100100101; one thousand and ten millions, one hundred and one thousand and 10; 1840. 5. 283.

V.

1. 69788; 48, with remainder 91. 2. 815. 3. 20000 German only; 30000 Tschech only; and 70000 both German and Tschech. 4. £63412.

5. 524.

VI.

1. M.D.LXIII, $\overline{\text{IX}}$. 3. 567342. 4. Two hundred and seventy thousand, one hundred and thirty; twenty-six thousand, seven hundred and eighty-four; 10234; 6. 5. 31, 13, 16 years are the ages of the children.

Ex. X., (p. 36.)

1. $\frac{15}{12}, \frac{45}{12}, \frac{60}{12}, \frac{180}{12}$. 2. $\frac{287}{63}, \frac{615}{63}, \frac{861}{63}, \frac{1845}{63}$.

Ex. XI., (p. 36.)

1. $\frac{5}{16}, \frac{5}{24}, \frac{5}{32}, \frac{5}{40}, \frac{5}{80}$. 2. $\frac{15}{6457}, \frac{15}{11740}, \frac{15}{14675}, \frac{15}{26415}$.

Ex. XII., (p. 37.)

1. $\frac{21}{3}, \frac{49}{7}, \frac{154}{22}; \frac{27}{3}, \frac{63}{7}, \frac{198}{22}; \frac{33}{3}, \frac{77}{7}, \frac{242}{22}$.

2. $\frac{52}{2}, \frac{130}{5}, \frac{338}{13}, \frac{598}{23}, \frac{910}{35}; \frac{218}{2}, \frac{545}{5}, \frac{1417}{13}, \frac{2507}{23}, \frac{3815}{35};$
$\frac{234}{2}, \frac{585}{5}, \frac{1521}{13}, \frac{2691}{23}, \frac{4095}{35}; \frac{250}{2}, \frac{625}{5}, \frac{1625}{13}, \frac{2875}{23}, \frac{4375}{35}$.

Ex. XIII., (p. 38.)

1. $3\frac{3}{4}$. 2. $12\frac{5}{6}$. 3. $6\frac{1}{8}$. 4. $12\frac{5}{9}$. 5. 9. 6. $36\frac{1}{12}$.
7. $45\frac{2}{13}$. 8. $3\frac{18}{55}$. 9. $31\frac{7}{24}$. 10. $92\frac{5}{63}$. 11. 6. 12. $37\frac{67}{157}$.
13. $90\frac{10}{111}$. 14. $1514\frac{108}{153}$. 15. $59\frac{163}{239}$. 16. $96\frac{46}{999}$. 17. $22\frac{17}{1163}$.
18. $363\frac{642}{1741}$. 19. $26\frac{31861}{36425}$. 20. $12\frac{1637}{3724}$.

Ex. XIV., (p. 39.)

1. $\frac{7}{3}$. 2. $\frac{38}{7}$. 3. $\frac{41}{9}$. 4. $\frac{37}{5}$. 5. $\frac{311}{12}$. 6. $\frac{480}{11}$.
7. $\frac{326}{13}$. 8. $\frac{223}{15}$. 9. $\frac{14022}{7}$. 10. $\frac{11152}{13}$. 11. $\frac{2482}{43}$

12. $\frac{1175}{84}$. 13. $\frac{767}{224}$. 14. $\frac{5453}{202}$. 15. $\frac{72442}{441}$. 16. $\frac{90325}{851}$.

17. $\frac{21631}{137}$. 18. $\frac{51208}{2859}$. 19. $\frac{472694}{1107}$. 20. $\frac{72413}{720}$.

Ex. XV., (p. 40.)

1. $\frac{8}{15}$. 2. $\frac{27}{35}$. 3. $\frac{3}{10}$. 4. $\frac{55}{72}$. 5. $\frac{245}{48}$.

6. $\frac{1}{3}$. 7. $\frac{375}{44}$. 8. $\frac{175}{8}$. 9. $\frac{14}{15}$. 10. $\frac{6399}{22}$.

Ex. XVI., (p. 41.)

1. $\frac{1}{2}$. 2. $\frac{2}{3}$. 3. $\frac{2}{3}$. 4. $\frac{2}{5}$. 5. $\frac{8}{9}$.

6. $\frac{2}{3}$. 7. $\frac{7}{15}$. 8. $\frac{6}{11}$. 9. $\frac{41}{57}$. 10. $\frac{13}{17}$.

11. $\frac{16}{25}$. 12. $\frac{5}{11}$. 13. $\frac{275}{903}$. 14. $\frac{35}{52}$. 15. $\frac{9}{17}$.

16. $\frac{117}{296}$. 17. $\frac{3}{4}$. 18. $\frac{123}{127}$. 19. $\frac{523}{6452}$. 20. $\frac{103}{136}$.

21. $\frac{5}{72}$. 22. $\frac{27}{1624}$. 23. $\frac{17}{31}$. 24. $\frac{5}{7}$. 25. $\frac{253}{5641}$.

26. $\frac{547}{741}$. 27. $\frac{183}{278}$. 28. $\frac{901}{1900}$. 29. $\frac{2}{7}$. 30. $\frac{23387}{35980}$.

31. $\frac{15}{29}$. 32. $\frac{7}{13}$.

Ex. XVII., (p. 43.)

1. $\frac{15}{30}, \frac{20}{30}, \frac{24}{30}$. 2. $\frac{16}{40}, \frac{35}{40}$. 3. $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$. 4. $\frac{6}{27}, \frac{5}{27}$.

5. $\frac{12}{28}, \frac{10}{28}, \frac{11}{28}$. 6. $\frac{18}{36}, \frac{27}{36}, \frac{20}{36}$. 7. $\frac{63}{72}, \frac{66}{72}, \frac{68}{72}$.

8. $\frac{20}{48}, \frac{21}{48}, \frac{26}{48}$. 9. $\frac{25}{30}, \frac{27}{30}, \frac{28}{30}$. 10. $\frac{36}{90}, \frac{60}{90}, \frac{50}{90}, \frac{63}{90}$.

11. $\frac{16}{24}, \frac{18}{24}, \frac{20}{24}, \frac{21}{24}$. 12. $\frac{429}{3003}, \frac{1092}{3003}, \frac{1617}{3003}, \frac{2002}{3003}$.

13. $\frac{240}{400}, \frac{175}{400}, \frac{28}{400}$. 14. $\frac{147}{252}, \frac{216}{252}, \frac{80}{252}, \frac{39}{252}$.

15. $\frac{308}{396}, \frac{180}{396}, \frac{286}{396}, \frac{54}{396}, \frac{11}{396}$. 16. $\frac{224}{672}, \frac{588}{672}, \frac{560}{672}, \frac{432}{672}, \frac{72}{672}, \frac{357}{672}$.

17. $\frac{486}{729}, \frac{324}{729}, \frac{189}{729}, \frac{72}{729}, \frac{48}{729}, \frac{31}{729}$.

18. $\frac{9000}{10000}, \frac{900}{10000}, \frac{90}{10000}, \frac{9}{10000}$.

19. $\frac{3255}{6300}, \frac{1190}{6300}, \frac{3276}{6300}, \frac{60}{6300}, \frac{3500}{6300}$.

20. $\frac{434}{756}, \frac{297}{756}, \frac{636}{756}, \frac{189}{756}$.

Ex. XVIII. (p. 45.)

In order of value the fractions will stand thus:

1. (1) $\frac{8}{9}, \frac{7}{10}, \frac{3}{5}$. (2) $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}$. (3) $\frac{4}{3}$ of $\frac{6}{7}, \frac{7}{12}, \frac{1}{5}$ of $\frac{3}{8}$.

(4) $\frac{31}{60}, \frac{10}{21}, \frac{5}{12}, \frac{3}{16}$. (5) $\frac{21}{26}, \frac{8}{11}, \frac{7}{13}, \frac{3}{7}, \frac{9}{22}$.

(6) $\frac{3}{7}$ of $\frac{5}{8}$ of 4, $\frac{2}{11}$ of $\frac{3}{5}$ of 5, $\frac{14}{28}, \frac{1}{6}$ of $\frac{1}{2}$ of $4\frac{3}{4}$.

(7) $\frac{27}{32}, \frac{7}{10}, \frac{27}{40}, \frac{9}{16}, \frac{3}{8}$. (8) $\frac{15}{4}, 3\frac{1}{3}, \frac{2}{7}$ of $9\frac{2}{5}$.

(9) $1\frac{4}{9}, \frac{6}{7}, \frac{5}{8}, \frac{29}{56}, \frac{13}{28}$. (10) $\frac{8}{9}, \frac{9}{22}, \frac{7}{18}, \frac{3}{11}, \frac{5}{36}$.

(11) $1\frac{1}{38}, \frac{700}{748}, \frac{401}{448}, \frac{113}{152}, \frac{51}{76}$.

(12) $\frac{15}{4}, 3\frac{1}{3}, \frac{2}{7}$ of $9\frac{2}{5}, \frac{2}{7}$ of $\frac{5}{9}$ of $\frac{4}{5}$.

2. (1) $\frac{3}{4}$ and $\frac{1}{6}$. (2) $\frac{47}{48}$ and $\frac{7}{16}$.

Ex. XIX., (p. 47.)

~~The sums~~ will be:

1. (1) $1\frac{8}{21}$. (2) $\frac{2}{3}$. (3) $1\frac{17}{24}$. (4) $\frac{1}{3}$. (5) $\frac{17}{64}$ (6) $1\frac{11}{42}$.

(7) $\frac{29}{36}$. (8) $\frac{25}{32}$. (9) $2\frac{23}{30}$. (10) $\frac{149}{156}$. (11) $11\frac{1}{40}$. (12) $14\frac{2}{3}$.

2. (1) $2\frac{1}{12}$. (2) $1\frac{23}{105}$. (3) 1. (4) $\frac{5}{6}$. (5) $1\frac{43}{120}$

(6) $2\frac{25}{252}$. (7) $1\frac{10}{21}$. (8) $1\frac{17}{60}$. (9) $1\frac{11}{105}$. (10) $15\frac{33}{35}$.

(11) $10\frac{1}{60}$. (12) $5\frac{153}{160}$. (13) $3\frac{4}{15}$. (14) $1\frac{11}{105}$. (15) $2\frac{1}{6}$.

(16) $1\frac{257}{360}$. (17) $7\frac{4}{21}$. (18) $585\frac{5}{9}$. (19) $444\frac{5}{6}$. (20) $2548\frac{41}{60}$.

3. (1) $1\frac{2221}{10000}$. (2) $4\frac{569}{720}$. (3) $2\frac{3031}{5814}$. (4) $11\frac{4}{17}$. (5) $23\frac{127}{140}$.

(6) $13\frac{11}{48}$. (7) $18\frac{43}{210}$. (8) $8\frac{79}{120}$. (9) 5976. (10) $3\frac{33}{128}$.

Ex. XX., (p. 49.)

1. (1) $\frac{1}{4}$. (2) $\frac{3}{8}$. (3) $\frac{17}{99}$. (4) $\frac{1}{20}$. (5) $\frac{1}{36}$.
(6) $\frac{1}{12}$. (7) $1\frac{4}{15}$. (8) $4\frac{7}{120}$. (9) $1\frac{7}{8}$. (10) $3\frac{137}{156}$.
(11) $3\frac{1}{48}$. (12) $11\frac{7}{12}$. (13) $13\frac{713}{729}$. (14) $64\frac{13793}{13875}$. (15) $19\frac{119}{298}$.
(16) 121. (17) $31\frac{11}{24}$. (18) $\frac{1}{56}$. (19) $\frac{2}{15}$. (20) $\frac{1}{6}$.

2. By $\frac{2}{27}$. 3. $3\frac{23}{72}$, $2\frac{55}{72}$. 4. $10\frac{115}{198}$. 5. $2\frac{191}{672}$.

6. The sum of the fractions is 5 times as great as their difference.

Ex. XXI., (p. 52.)

1. (1) $\frac{10}{21}$. (2) $\frac{104}{135}$. (3) $\frac{2}{3}$. (4) $\frac{2}{9}$. (5) $\frac{4}{5}$.
(6) $2\frac{1}{2}$. (7) $10\frac{2}{27}$. (8) $\frac{2}{3}$. (9) 40. (10) $5\frac{3}{8}$.
(11) 1. (12) $\frac{85}{1152}$. (13) 1. (14) 2.

2. (1) $\frac{1}{6}$. (2) $\frac{3}{425}$. (3) $1\frac{36}{41}$. (4) $242\frac{17}{24}$. (5) $\frac{1}{5}$. (6) $\frac{5}{13}$.

Ex. XXII., (p. 54.)

1. (1) 4. (2) $2\frac{1}{2}$. (3) $1\frac{3}{13}$. (4) $1\frac{1}{12}$. (5) $\frac{153}{608}$.
(6) $\frac{8}{21}$. (7) $\frac{4}{7}$. (8) $\frac{21}{50}$. (9) $\frac{45}{352}$. (10) $\frac{1}{12}$.
(11) $3\frac{9}{13}$. (12) 153. (13) $347\frac{5}{8}$. (14) $1\frac{7}{15}$.

2. $\frac{4805}{496}$ and $\frac{320}{496}$.

3. (1) $\frac{2}{5}$. (2) $\frac{2}{9}$. (3) $5\frac{1}{4}$. (4) $5\frac{2}{5}$. (5) $\frac{2}{3}$. (6) 36. (7) $7\frac{93}{125}$.

Ex. XXIII., (p. 58.)

I.

2. $4\frac{95}{136}$ and $3\frac{7}{136}$: $8\frac{1}{3}$.

3. (1) $37\frac{5}{8}$. (2) $8\frac{16}{21}$. (3) $\frac{17}{56}$. (4) $4\frac{25}{48}$. (5) $3\frac{16}{113}$. 5. $5\frac{13}{21}$.

II.

2. $21\frac{3}{4}$ and $3\frac{19}{24}$.

3. $\frac{764}{113}$ and $\frac{121}{456}$. 4. (1) $\frac{37}{975}$. (2) $14\frac{73}{180}$. (3) $\frac{9}{247}$. (4) $\frac{1}{11}$.

5. 16.

III.

2. (1) 5000. (2) $\frac{5}{18}$. (3) 2. (4) $1\frac{2}{549}$.

3. $\frac{1}{3}$ of 4 is greater by $\frac{1}{12}$. 4. $\frac{1474}{1521}$. 5. $1\frac{841}{1575}$.

IV.

1. $\frac{8}{35}$. 2. (1) $\frac{21}{22}$. (2) $\frac{7}{120}$. (3) $\frac{437}{584}$. (4) $\frac{25}{81}$. (5) $12\frac{3}{11}$.

3. $2\frac{29}{30}$ and $\frac{1}{945}$. 4. $\frac{1}{4}$.

5. The quotient is 144 times as large as the product.

V.

1. $\frac{7}{9}$: $\frac{3}{11}$. 2. (1) $3\frac{3}{5}$. (2) $\frac{55}{63}$. (3) $\frac{67}{80}$. (4) $\frac{867}{1280}$. (5) $\frac{255}{364}$.
(6) $\frac{39}{40}$. (7) $\frac{41}{84}$. 3. $\frac{2}{3}$. 4. $\frac{1}{360}$. 5. $\frac{3}{16}$.

VI.

2. (1) 1. (2) 1. (3) $\frac{45}{841}$. (4) 3. 3. $\frac{12}{47}$. 5. $1\frac{1}{3}$. 6. $2\frac{5}{28}$: $\frac{11}{20}$.

VII.

2. (1) $\frac{1424}{3725}$. (2) $13\frac{17}{168}$. (3) 2. (4) $\frac{335}{468}$. 3. $18\frac{3}{4}$ and $3\frac{43}{50}$. 4. $\frac{43}{1650}$.

5. The whole score was 240 runs, and the score of each 30, 24, 24, 12, 12, 12, 30, 30, 30, 30, 6.

Ex. XXIV., (p. 64.)

1. $\frac{1}{10}$; $\frac{3}{10}$; $\frac{31}{100}$; $\frac{311}{1000}$; $\frac{31111}{100000}$; $\frac{31111111}{100000000}$.

2. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{7}{20}$; $\frac{1}{20}$; $\frac{1}{200}$; $\frac{32}{125}$; $\frac{16}{625}$; $\frac{4}{15625}$; $\frac{13}{160000}$.

3. $\frac{3}{40}$; $\frac{106}{125}$; $\frac{151}{50}$; $\frac{1717}{500}$; $\frac{1717}{5}$; $\frac{1717}{50000}$; $\frac{10001}{200000}$;
$\frac{230409}{1000}$; $\frac{230409}{100000}$; $\frac{10686}{5}$; $\frac{114125001}{1250}$; $\frac{38401}{1600}$;
$\frac{657097353}{8000}$; $\frac{20819}{25000000}$; $\frac{10000009}{10000000}$; $\frac{1}{1000000000}$.

4. ·1; ·3; ·7; ·53; ·07; ·003; ·9178; 91·78; ·09178; ·0091; ·00009; 520·3; ·9; 3·0142; 6·72819; ·000672819; 6728·19.

70; 700; 70000. ·6; 60; 60000. 4·31; 43100.
162·01; 16201; 16201000; 9001600; 900·16.

6. ·051; ·00051; ·0000051. ·00008; ·000000008. ·005016; ·00005016.
·3780186; ·0003780186.

7. ·5; ·7; ·19; ·28; ·005; 9·7; ·000001; 14·4; 280·0004; 7·007;
100·00001; 1·0010001; ·000000000005.

8. Four-tenths; twenty-five hundredths; seventy-five hundredths; seven hundred and forty-five thousandths; one-tenth; one thousandth; one hundred thousandth; twenty-three and seventy-five hundredths; two and three hundred and seventy-five thousandths; two thousand three hundred and seventy-five ten thousandths; two thousand three hundred and seventy-five hundred millionths; one and one millionth; one million and one ten millionths; one hundred millionth.

Ex. XXV., (p. 66.)

1. (1) 47·09595. (2) 290·381404. (3) 6153·70427.

2. (1) 2935·5073. (2) 418·94514. (3) 406·529522. (4) 953·77386.
(5) 370·430375. 3. (1) 62·5358119. (2) 9181·6074975.
(3) 5082·3192995. (4) 1000011022959·090989011001.

Ex. XXVI., (p. 67.)

1. 1·0918; 5·8345; 141·03; ·0001; ·304317.

2. (1) 211·6875. (2) ·0421813. (3) 602·3415997. (4) 4·4954.
(5) ·48553. (6) 9·1794.

3. ·09; 655·30283; 21·068124; 9788·852. 4. 6·3; ·699993; 99·706.

Ex. XXVII., (p. 68.)

1. (1) 159·6; ·1596; 15·96; ·0001596.
(2) 173·889; ·173889; 1·7989.
(3) ·0063612; 3·72812; ·12376.

2. (1) 3·07930896. (2) 210·6144185. (3) ·00329875.

3. (1) ·03611. (2) ·0000274104. (3) ·0006594. (4) ·00007614.
(5) ·055757592. (6) ·27492. 4. ·001; ·20736.

5. (1) 32·86164. (2) 1549795·52.

Ex. XXVIII., (p. 72.)

1. (1) 2·1; 91·78. (2) ·025; 24·3. (3) ·00003; ·374.
(4) 10, 100, 10000. (5) 250; 16·25. (6) 51472; ·0000051472.
(7) ·057; 813·4. (8) ·0072; 59640. (9) 10500; 137·56.
(10) 5020; 543. (11) 326000; 32·6; ·0097. (12) 1·3; 13; ·13; 130.
(13) ·002; ·000002; 2. (14) 2·01; 20100; ·001875.
(15) 948·7096; 9487096. (16) 26153·4; 21·4.
(17) 2040000; ·00082175. (18) 7934·7; 79347; 79347000.
(19) ·00002; ·000002; 20. (20) ·57; 57000.

2. (1) 3·7356; ·0117; 76·9230. (2) 1·4895; 50830·1313.
(3) 320911·4782; 1·9005; 1·3157. (4) 14036019·0930; ·0011.
(5) 12·8413; ·0026.

3. (1) ·0000186; ·00186. (2) ·00256256; ·256256; 256·256.
(3) 4360; 103·36; ·04545.

Ex. XXIX., (p. 75)

1. (1) ·25; ·75; ·625; ·36; ·3125; ·95.
(2) ·515625; ·432; ·285; 1·36; ·00625.
(3) 6·171875; ·2375; ·05078125; ·005859375; 15·0075264.

2. (1) ·007080078125. (2) ·84375. (3) ·0001. (4) ·661. (5) ·575.
(6) ·79375. (7) ·5. (8) 9·6953125. (9) 86·497. (10) 562·926.

Ex. XXX., (p. 80.)

1. (1) $\cdot\dot{5}$; $\cdot\dot{1}\dot{8}$; $\cdot\dot{0}2\dot{7}$; $\cdot\dot{4}2857\dot{1}$.
(2) $\cdot5\dot{6}$; $\cdot7\dot{4}\dot{3}$; $\cdot\dot{1}9753086\dot{4}$; $15\cdot\dot{1}5\dot{6}$.
(3) $\cdot917897\dot{7}\dot{2}$; $7\cdot\dot{2}8571\dot{4}$; $\cdot000\dot{1}\dot{7}$.
(4) $24\cdot008\dot{4}9713\dot{3}$; $17\cdot01\dot{8}5714\dot{2}$; $2\cdot16\dot{7}843\dot{2}$.

2. (1) $\frac{7}{9}$; $\frac{7}{90}$; $\frac{5}{22}$. (2) $\frac{289}{495}$; $\frac{5}{37}$; $\frac{79}{300}$. (3) $\frac{37}{19980}$; $\frac{1007}{333}$; $\frac{153}{12375}$.
(4) $\frac{481}{3367}$; $\frac{191}{480}$; $\frac{51467}{134680}$. (5) $\frac{1036}{3367}$; $\frac{10619}{16835}$; $\frac{39}{14}$.
(6) $\frac{114137}{333000}$; $\frac{1043}{33300}$; $\frac{385}{48}$. (7) $\frac{1284121}{15000}$; $\frac{51}{14}$; $\frac{4023367}{31680}$.

Ex. XXXI., (p. 81.)

1. (1) 31·371538. (2) 700·612301. (3) 6·116666; 1·681818; 308·05272.
2. $2\cdot2\dot{3}8461\dot{5}$; $13\cdot72\dot{6}1904\dot{7}$. 3. (1) $13\cdot\dot{2}$; $\cdot2\dot{7}$. (2) $25\cdot21\dot{3}$; 300.
(3) $363\cdot57\dot{4}\dot{0}$; $245\cdot\dot{3}$. (4) 1·35169....; $17\cdot\dot{4}\dot{5}$.
4. (1) $48\cdot7\dot{5}$; $6\cdot7\dot{6}$. (2) 303·75; $2\cdot\dot{3}$. (3) 7; 48·734; ·0134.

Ex XXXII. (p. 81).

I.

1. $\frac{1}{16}$; $\frac{314159}{100000}$; $3\cdot27\dot{3}8095\dot{2}$. 2. 13·0125.
3. 573·005754; 573·004246; ·43204577; 759953·5 ...; 1·02515; 1·00485; ·01030225; 100.
4. Yes. 5. (1) 394. (2) ·009072. (3) 1. (4) 11·8125.
6. 1·05 nearly.

II.

1. ·000000700409; $\frac{24269}{2000}$; ·0032546.

2. Three hundred and ninety seven thousand and eight, and four hundred and five thousand and nine millionths; 397008405·009; 397·008405009. Three hundred and ninety-seven millions, eight thousand four hundred and five, and nine thousandths. Three hundred and ninety-seven, and eight millions four hundred and five thousand and nine thousand-millionths.

3. ·03493. 4. 11·025; $\frac{441}{40}$; ·00053874; ·0002; ·0642.

5. (1) ·000091304...... (2) $2{\cdot}\dot{5}1\dot{8}$. (3) ·625. (4) 10·0045.

6. 2·4976096088.

III.

1. 57 and 57000; 12644·042...... 2. (1) $\frac{9}{10}$ and ·9.

(2) $\frac{1384}{1975}$ and ·7007...... (3) $\frac{968}{625}$ and 1·5488. (4) $\frac{7}{24}$ and ${\cdot}291\dot{6}$.

3. $2{\cdot}\dot{6}$; 8585; yes. 5. 15·35 miles. 6. $\frac{37}{240}$.

IV.

1. 124·36653. $31\frac{457}{999}$; $1\frac{2377}{4995}$.

2. 3006005; three hundred thousand, six hundred and five-tenths.

3. In order of magnitude they stand thus 1·5 × ·75; 2·625 ÷ 5; 5 × ·05.

4. ${\cdot}0\dot{0}4\dot{9}$; $1269\dot{3}$ *is misprinted for* ${\cdot}1269\dot{3}$. *Ans.* ·006545; 542000; ·0046; 20020; ·02002.

5. 3·14159. 6. $\frac{3}{5}$.

V.

1. $3{\cdot}08\dot{3}$; 2·87296; $8{\cdot}0267\dot{8}5714\dot{2}$.

2. (1) 851. (2) 1·60546875. (3) 35·4875. (4) $1{\cdot}\dot{6}8350\dot{1}$.

3. $\frac{333}{106}$ is the nearer. 4. 2·7182818; ·00097061; $\frac{97061}{100000000}$.

5. 7925·6 miles nearly. 6. 13·74696.

VI.

1. 1·33454412; ·0089147......; ·0730091; 7·30091. 2. $\frac{451}{369}$.

3. $\frac{49}{410}$; $6\frac{139}{446}$. 4. $\frac{6401}{49500}$. 5. ${\cdot}7\dot{2}$. 6. ·4375.

Ex. XXXIII., (p. 102)

1. (1) 13680*d.*; 617904*q.* (2) 3744*d.*; 1260000*d.*
(3) 201 halfpence; 975*q.* (4) 80425*q.*; 188663 halfpence.
(5) 88560*q*; 8550 fourpenny-pieces. (6) 1347192*q*; 945 sixpences.

2. £5909. 18*s.* 9½*d.* 3. 200 half-crowns, 1000 sixpences, 1500 fourpences.

4. 840 half-crowns; 200 half-guineas. 5. 234 half-guineas; 91 moidores.

6. (1) 343555 grs.; 6493 lbs., 19 dwts., 21 grs. (2) 195597 lbs., 2 oz., 17 dwts., 12 grs.; 51456 drs.; 154368 sc. (3) 278912 oz.; 13 tons, 3 cwt., 3 qrs., 3 lbs., 14 oz., 9 drs. (4) 162 tons, 17 cwt., 3 qrs., 25 lbs., 9 oz.; 19439375 drs. (5) 98920 grs.; 72 oz., 4 dwts., 22 grs. (6) 3005 st., 1 lb.; 3583 tons, 17 cwt., 2 qrs., 1 st. (7) 6864 yds.; 36305280 in. (8) 7 lea., 4 fur., 10 po., 5 yds., 2 ft., 4 in.; 4712544 in. (9) 2661126 barleycorns; 88016½ yds. (10) 79 chains, 2 yds.; 9 ft., 9 in. (11) 1348 nails; 1124 nails. (12) 2004 nails; 880 nails. (13) 5680 po.; 273460 sq. yds. (14) 6188724 sq. in.; 138847½ sq. ft. (15) 1575000 sq. links; 312 ac., 2 ro. (16) 783 cub. ft.; 3 cub. yds., 10 cub. ft., 1031 cub. in. (17) 794153 cub. in.; 1245888 cub. in. (18) 4504 pts.; 11432 gals., 2 qts., 3 gills. (19) 5824 pts.; 362 tier., 34 gals., 4 pts., 2 gills. (20) 2880 pts.; 4578 hhds., 38 gals., 5 pts. (21) 24344 qts.; 166 gals. (22) 1429 lds., 2 qrs., 7 bus.; 1160 pks. (23) 9000 bus.; 1291 chald., 34 bus., 3 pks. (24) 27336 sheets; 108 reams, 9 quires, 17 sheets. (25) 22266000 sec.; 2674859 sec.

(7) 668190½ pts.; 334095¼ qts.; 83523$\frac{13}{16}$ gals.; 2320$\frac{61}{64}$ bar.
(8) 873223200 sec. 9. 5025 hrs.; 18090000 sec. 10. 56209280 sq. ac.

Ex. XXXIV. (p. 105.)

1. £23. 9*s.* 8*d.* 2. £253. 10*s.* 8*d.* 3. £252. 3*s.* 3¾*d.*
4. £153. 16*s.* 3¾*d.* 5. £271. 10*s.* 6. £3329. 8*s.* 1½*d.*
7. 143 tons, 15 cwt., 3 qrs., 18 lbs. 8. 14 lbs., 9 oz., 2 drs., 19 grs.
9. 223 ac., 3 ro., 15 po.

10. £241. 18*s.* 7¼*d.*; £2778. 6*s.* 11*d.*; £1967. 12*s.* 7¼*d.*; £3722. 11*s.* 5¾*d.*; £66851. 0*s.* 4½*d.*; £79251. 16*s.* 0½*d.*; £769861. 15*s.* 2½*d.*

11. 49lbs., 10oz., 13grs.; 193lbs., 9oz., 19dwts.; 1757lbs., 1oz., 18dwts., 14grs.

12. 3oz., 2drs., 9grs.; 232lbs., 4oz., 4drs., 1sc.; 246lbs., 4oz., 2drs., 17grs.

13. 98 lbs., 10 oz., 13 drs.; 2 tons, 13 cwt., 3 qrs., 24 lbs., 8 oz.; 2214 tons, 9 cwt., 2 qrs., 26 lbs.; 153 tons, 9 cwt., 2 qrs., 2 lbs., 2 oz.

14. 176 yds., 2 ft., 5 in.; 199 m., 2 fur.; 166 m., 7 fur., 13 po., 1 yd., 2 ft., 1 in.; 125 lea., 2 m., 4 fur., 198 yds. 15. 185 yds., 1qr., 3 na.; 182 Eng. ells.

16. 181 ac., 16 po.; 87 ac., 2 ro., 26 po., 14½ sq. yd., 2 sq. ft., 93 sq. in.

17. 85 c. yds., 9 c. ft., 575 c. in.; 74 po., ¼ sq. yd., 8 sq. ft., 53 sq. in.

18. 304 gal., 1 qt.; 47 pipes, 55 gals., 1 qt.; 403 hhds., 36 gals., 7 pts.

19. 260 qrs., 3 pks., 11 pts.; 1702 lds., 6 bus., 1 pk.

20. 28 mo., 1 wk., 19 hrs., 40 m.; 216 yrs., 39 wks., 6 d., 17 hrs., 51 m., 51 sec.

21. 35 yrs., 7 mo., 2 wks., 3 d., 13 hrs.

Ex. XXXV. (p. 107).

1. £45. 14*s*. $6\frac{17}{24}$*d*. 2. £93. 7*s*. $8\frac{4}{63}$*d*. 3. £17. 19*s*. $8\frac{2}{75}$*d*.

4. £339. 12*s*. $2\frac{3}{16}$*d*. 5. £1463. 7*s*. $2\frac{11}{40}$*d*.

Ex. XXXVI. (p. 109).

1. £20. 8*s*. 10*d*. 2. £62. 11*s*. 4*d*. 3. 189. 14*s*. $0\frac{1}{2}$*d*.

4. £888. 0*s*. $2\frac{1}{2}$*d*. 5. £331. 19*s*. $11\frac{1}{2}$*d*. 6. £313. 6*s*. $1\frac{3}{4}$*d*.

7. 4 cwt., 3 qrs., 5 lbs., 7 oz. 8. 12 fur., 36 po., 1 yd.

9. 5 ac., 2 ro., 31 po. 10. 55 qrs., 5 bus., 1 pk., 1 gal.

11. £77. 17*s*. $10\frac{3}{4}$*d*.

12. (1) 38 lbs., 4 oz., 9 dwts, 10 grs. (2) 859 lbs., 4 oz., 6 dwts., 5 grs.

(3) 6 tons, 16 cwt., 3 qrs., 5 lbs. (4) 17 tons, 13 cwt., 3 qrs., 19 lbs.

(5) 15 lbs., 8 oz., 4 drs., 1 sc. (6) 30 yds., 1 qr., 2 na.

(7) 1 yd., 1 ft., 10 in. (8) 11 yds., 1 ft., 1 in.

(9) 1 m., 5 fur., 36 po., 5 yds. (10) 6 lea., 2 m., 6 fur., 23 po., 4 yds., 2 ft.

(11) 7 ac., 2 ro., 36 po. (12) 1 ro., 28 po., 28 sq. yds., 8 sq. ft.

(13) 1 cub. yd., 20 cub. ft., 1305 cub. in. (14) 15 tuns, 2 hhds., 53 gals., 1 qt., 1 pt.

(15) 5 bar., 3 fir., 3 qts. (16) 4 lds., 4 qrs., 7 bus., 3 pks.

(17) 2 mo., 1 wk., 3d. (18) 9°, 36′, 55″.

(19) £52. 0*s*. $4\frac{1}{4}$*d*. (20) £1770. 7*s*. $1\frac{3}{4}$*d*.

Ex. XXXVII. (p. 111).

1. 18*s*. $4\frac{7}{12}$*d*. 2. £4. 4*s*. $11\frac{1}{7}$*d*. 3. £3. 13*s*. $11\frac{3}{8}$*d*.

4. £110. 18*s*. $7\frac{7}{8}$*d*. 5. £1798. 19*s*. $1\frac{29}{48}$*d*.

Ex. XXXVIII. (p. 112).

1. £23. 7*s*.; £58. 7*s*. 6*d*. 2. £11. 14*s*. 6*d*.; £17. 11*s*. 9*d*.

3. £12. 15*s*. $7\frac{3}{4}$*d*.; £16. 8*s*. $8\frac{1}{4}$*d*. 4. £13. 16*s*. $1\frac{3}{4}$*d*.; £22. 1*s*. 10*d*.

5. £41. 16*s*. $8\frac{1}{4}$*d*.; £45. 12*s*. 9*d*. 6. £79. 16*s*. 3*d*.; £95. 15*s*. 6*d*.

7. £1043. 7*s*. $5\frac{1}{4}$*d*.; £3825. 13*s*. $11\frac{1}{4}$*d*. 8. £7579. 10*s*. 10*d*.; £9328. 13*s*. 4*d*.

9. £39389. 8*s*. $3\frac{3}{4}$*d*.; £120043. 18*s*. 8*d*. 10. £58402. 15*s*. $5\frac{1}{4}$*d*.; £69218. 2*s*.

11. £568135. 3*s*. 4*d*.; £649297. 6*s*. 8*d*.

12. £68236. 9*s*. $9\frac{1}{2}$*d*.; £90248. 5*s*. $2\frac{1}{2}$*d*.; £102354. 14*s*. $8\frac{1}{4}$*d*.

13. £379113. 9*s*. $2\frac{1}{4}$*d*.; £399745. 9*s*. $8\frac{1}{4}$*d*.; £1222447. 9*s*. $7\frac{1}{2}$*d*.; £6011656. 5*s*. $8\frac{1}{4}$*d*.

14. 693 lbs., 2 oz., 11 dwts., 16 grs.; 3119 lbs., 5 oz., 12 dwts., 12 grs.

15. 33 tons, 2 cwt., 2 qrs., 25 lbs., 15 oz.; 228 tons, 18 cwt., 3 qrs., 13 lbs., 12 oz.

16. 547 lbs., 5 oz., 4 drs.; 3102 lbs., 3 oz., 1 dr., 1 sc.

17. 606 yds., 1 qr., 2 na.; 3570 yds., 3 qrs., 2 na.

18. 2 fur., 10 po., 1 yd., 1 ft., 10 in.; 1 m., 1 fur., 14 po., 1 ft., 2 in.

19. 186 ac., 3 ro., 27 po., $26\frac{3}{4}$ yds., 4 ft.; 1903 ac., 36 po., 23 yds., 8 ft.

20. 4571 ac., 1 ro., 24 po.; 40380 ac., 2 ro., 32 po.

21. 577 gal., 2 qts.; 14841 gal., 3 qts.
22. 199 lds., 1 qr., 7 bus., 2 pks.; 3681 lds.
23. 1 yr., 13 wks., 4d., 8h., 34 m.; 38 yrs., 46 wks., 2 d., 13 h., 6 m.
24. 571 tuns, 1 pipe, 35 gals., 3 pts.; 7706 tuns, 1 bar., 10 gals., 2 qts.
25. 1691 bar., 17 gal., 2 qts., 1 pt.; 33135 bar., 30 gal., 2 qts.
26. £1053. 4*s.* 10$\frac{3}{4}$*d.* 27. £16464.

Ex. XXXIX. (p. 114).

1. £19. 18*s.* 2$\frac{1}{2}$*d.*; £73. 0*s.* 1$\frac{1}{6}$*d.*; £378. 5*s.* 11$\frac{1}{2}$*d.*
2. £302. 15*s.* 0$\frac{4}{5}$*d.*; £1135. 6*s.* 6*d.*; £6660. 11*s.* 5$\frac{3}{8}$*d.*
3. £4259. 7*s.* 11$\frac{1}{2}$*d.*; £18457. 7*s.* 9$\frac{5}{6}$*d.*; £85187. 19*s.* 2*d.*
4. £22079. 10*s.* 2$\frac{1}{4}$*d*; £23130. 18*s.* 3$\frac{1}{2}$*d.*; £85689. 10*s.* 5$\frac{7}{8}$*d.*
5. £6047. 18*s.* 9*d.*; £18143. 16*s.* 3*d.*; £72701. 4*s.* 11$\frac{11}{16}$*d.*

Ex. XL. (p. 116).

1. £3. 18*s.* 1*d.* 2. £2. 2*s.* 11*d.* 3. £37. 14*s.* 3$\frac{1}{4}$*d.*
4. £6. 11*s.* 7$\frac{1}{4}$*d.* 5. £61. 8*s.* 8*d.* 6. £17. 7*s.* 3$\frac{1}{2}$*d.*
7. £13. 15*s.* 7$\frac{1}{4}$*d.* 8. £62. 5*s.* 3$\frac{3}{4}$*d.* 9. £51. 3*s.* 3$\frac{1}{4}$*d.*
10. £67. 4*s.* 5$\frac{1}{2}$*d.* 11. £35. 3*s.* 3$\frac{3}{4}$*d.* 12. £39. 9*s.* 6$\frac{1}{2}$*d.*
13. £1. 9*s.* 6*d.* 14. £1. 5*s.* 2$\frac{1}{2}$*d.* 15. 6*s.* 7$\frac{1}{2}$*d.*
16. £1. 3*s.* 10$\frac{1}{4}$*d.* 17. 6*s.* 3$\frac{1}{4}$*d.* 18. £3. 13*s.* 4*d.*
19. £1. 14*s.* 2*d.* 20. 10*s.* 0$\frac{1}{2}$*d.* 21. £77. 11*s.* 4$\frac{1}{4}$*d.*
22. £1. 0*s.* 9$\frac{1}{2}$*d.* 23. £13. 16*s.* 8$\frac{1}{4}$*d.* 24. 12*s.* 9$\frac{279}{2737}$*d.*
25. £7. 8*s.* 4$\frac{3}{4}$*d.* $\frac{23}{119}$*q.* 26. £2. 13*s.* 4*d.* $\frac{11}{433}$*q.* 27. 17 cwt., 12 lbs.
28. 3 cwt., 1 qr., 14 lbs. 29. 7 mo., 26 d. 30. 14 d., 13 h., 27 m.
31. 3 lbs., 11 oz., 14$\frac{40}{57}$ drs. 32. 8 lbs., 13 oz., 6$\frac{766}{963}$ drs.
33. 2 ac., 3 ro., 27 po., 19 sq. yds., 7 sq. ft., 1$\frac{5}{13}$ sq. in.
34. 14 po., 12 sq. yds., 6 sq. ft., 119$\frac{1}{49}$ sq. in. 35. 1$\frac{562}{903}$ nails.
36. 7 bus., 1$\frac{16}{107}$ pk. 37. £19200. 38. £19200.

Ex. XLI. (p. 117).

1. £29. 7*s.* 9$\frac{1}{4}$*d.* $\frac{11}{15}$*q.* 2. £21. 4*s.* 4$\frac{1}{4}$*d.* $\frac{23}{32}$*q.* 3. £1. 4*s.* 11$\frac{3}{4}$*d.* $\frac{2}{33}$*q.*
4. £2. 9*s.* 4$\frac{1}{8}$*d.* 5. £20. 12*s.* 4$\frac{1}{2}$*d.* $\frac{1}{18}$*q.* 6. £7. 10*s.* 3$\frac{1}{4}$*d.* $\frac{11}{42}$*q.*
7. £7. 7*s.* 4$\frac{1}{2}$*d.* $\frac{19}{22}$*q.* 8. £4. 6*s.* 11*d* $\frac{7}{18}$*q.* 9. £18. 13*s.* 4*d.* $\frac{35}{36}$*q.*
10. £6. 7*s.* 4$\frac{1}{2}$*d.* $\frac{335}{528}$*q.*

Ex. XLII. (p. 119).

1. £39. 12*s.* 11*d.* 2. £17. 17*s.* 6*d.* 3. £2. 0*s.* 6*d.*
4. £1. 8*s.* 11$\frac{1}{2}$*d.* 5. 2*s.* 2$\frac{1}{4}$*d.* 6. £1. 1*s.* 1$\frac{1}{4}$*d.*
7. 1 ro., 29·94 per.; £19. 0*s.* 4$\frac{3}{4}$*d.* ·3008*q.* 8. 2779 ac.; 277 ac., 3 ro., 24 po.

Ex. XLIII. (p. 120).

1. 38. 2. 23. 3. 105. 4. 156. 5. 27. 6. 36.
7. 66. 8. 288. 9. $186\frac{34782}{58589}$. 10. $256\frac{192}{385}$.

Ex. XLIV. (p. 121).

1. £37. 18*s.* $1\frac{3}{4}$*d.* $\frac{2}{3}$*q.* 2. £26. 12*s.* $3\frac{1}{25}$*d.*
3. £2314. 17*s.* $10\frac{47}{160}$*d.* 4. 79 ac., 1 ro., 26 po., $17\frac{7}{8}$ yds.
5. 25m., 5fur., 15po., $4\frac{5}{14}$yds. 6. 1994 tons, $20\frac{8}{9}$ lbs.
7. £404. 4*s.* 8*d.* 8. 44 cwt., 3 qrs., $7\frac{3}{5}$ lbs.
9. £2. 10*s.* $6\frac{3}{4}$*d.* 10. 17 yrs., 3 mo., 1 wk., 2 d., $6\frac{2442}{4133}$ hrs.
11. £1. 12*s.* $7\frac{626}{11935}$*d.* 12. 24 yds., $1\frac{105}{166}$ na.

Ex. XLV. (p. 127).

1. 156 fl., 1560 c., 15600 m.; 63·2 c., 632 m.
2. 309·5 fl., 3095 c., 30950 m.; 961·29 fl., 9612·9 c., 96129 m.
3. 180·65 fl., 1806·5 c., 18065 m.; 9·25 fl., 92·5 c., 925 m.
4. 100·01 fl., 1000·1 c., 10001 m; 460·25 fl., 4602·5 c., 46025 m.

Ex. XLVI. (p. 129).

1. (1) £264. 1 fl. 3 c. 5 m. (2) £552. 7 fl. 7 c. 7 m.
2. (1) £3. 1 fl. 1 c. (2) 1 m. (3) £1. 5 fl. 5 m.
3. (1) £384. 1 c. 5 m; £4838. 5 fl. 8 c. 9 m. (2) £16. 6 fl. 5 c.; £932. 4 fl.
(3) £300760. 2 c. 5 m.; £2786492. 8 fl. 8 c.
4. (1) £38. 9 fl. 1 c. 5 m. (2) £978. 5 fl. 6 c. 4 m. (3) £46. 3 fl. 6 c.

Ex. XLVII. (p. 129).

I.

1. $806974\frac{1}{2}$; $169464641\frac{1}{2}$. 3. £26497. 7*s.* 11*d.* 4. $80\frac{1}{10}$.
5. £143. 4*s.* $10\frac{1}{2}$*d.* 6. £1712. 7. 135. 8. 54 gal.

II.

1. £37. 12*s.* $8\frac{1}{2}$*d.* 2. £5986. 0*s.* 8*d.* 3. 90090; 17920000.
4. £2. 4*s.* 3*d.*; £1. 9*s.* 6*d.* 5. £554. 3*s.* 6*d.* 6. £8764.
7. 56 dozens. 8. £1. 8*s.*

III.

1. 68 lbs., 13 dwts., 8 grs. 2. 116 yds. 3. 3130. 4. 17*s.* 8*d.*
6. £887. 4*s.* 7. 6 guineas, 12 guineas, 36 guineas, 144 guineas.
8. £3. 6 fl. 9 c. 1 m.

IV.

2. 37 years, and 14 years. 3. 1771. 4. 19*s*. 5. £617. 3 fl. 2 c. 6 m; £25. 3 fl. 4 c. 6 m. 6. 19 gal. 7. 1000 perches. 8. 16225866.

V.

2. 4752000000. 3. $40\frac{10}{43}$ dozens. 4. £3105. 7*s*. 6*d*.; £443. 12*s*. 6*d*.
5. $195\frac{65}{84}$ guineas. 6. 216000. 7. 6*s*. $3\frac{1}{4}$*d*. 8. £384. 5 fl. 5 c. 8 m.

VI.

1. £46. 14*s*. 6*d*. 2. 4 yds., 1 ft. 3. £372. 10*s*. $7\frac{1}{2}$*d*. 4. £5.
5. 15*s*. 6*d*. 6. £1. 13*s*.; 16*s*. 6*d*.; 5*s*. 6*d*. 7. 100000. 8. 425 quarters.

VII.

1. 5260320 min. 2. 15 tons, 15 cwt., 1 qr., 16 lbs.; 48 m., 5 fur., 1 po.; 154 moidores. 3. $26\frac{2}{3}$ yds. 4. $1205\frac{205}{324}$ days. 5. £3. 14*s*. 2*d*.
6. £19175; £45950. 7. 112 gallons. 8. 2*s*. 6*d*.

VIII.

2. 8 lbs., 4 oz., $4\frac{4}{15}$ drs. 3. 2*s*. 11*d*. 4. £211050. 5. £784. 5*s*. 9*d*.
6. £1311. 1 fl. 8 c. 1 m.; £505. 9 m.; £5727. 8 fl. 8 m. 7. £1094. 19*s*. 3*d*.
8. £108.

IX.

1. May 1, 1769. 2. 19800. 3. 7 lbs., 8 oz. 4. *A* has to give *B* £42. 15*s*. 5. $20\frac{460}{571}$ sec. 6. 144540. 7. 5 hrs., 7 m., 12 sec.
8. £5791. 10*s*.

X.

1. £809424. 2. He will have saved £8112. 3. £1. 5*s*.
4. £56260. 11*s*. 6*d*. 5. 9 fl. 2 c. 5 m. 6. 20th Oct. 1855.
7. £238. 5*s*. $5\frac{3}{4}$*d*. 8. £115. 12*s*. 3*d*.; £6011. 17*s*.

XI.

1. £1605. 7*s*. 11*d*.; £2334. 6*s*. 1*d*.; £2903. 0*s*. 5*d*. 2. £23. 3*s*. $3\frac{3}{13}$*d*.
3. $25\frac{16}{67719}$. 4. 2*s*. 5. £8364. 11*s*. 8*d*. 6. £1. 3*s*. $2\frac{1}{4}$*d*. $\frac{147}{181}$*q*.
7. 80 lbs., $13\frac{213}{215}$ oz. 8. £186. 4*s*. 2*d*.; £93. 2*s*. 1*d*.

XII.

1. 36. 2. 11*s*. $10\frac{1}{2}$*d*. $\frac{870}{913}$*q*. 3. $235\frac{29}{118}$ mo. 4. £114. 1*s*. 3*d*.
5. 1·2113. 6. 6*s*. 8*d*. 7. 6 men, 12 women, 18 boys.
8. £990. 15*s*. $0\frac{3}{4}$*d*.

Ex. XLVIII. (p. 140.)

1. (1) 7*s.* 6*d.*; 13*s.* 4*d.*; 12*s.* 6*d.*; 15*s.* 9*d.*; 17*s.* 6*d.*

(2) £1. 2*s.* 6*d.*; £1. 6*s.* 8*d.*; $11\frac{1}{4}$*d.*; 8*s.* (3) 1*s.* 1*d.*; $3\frac{3}{4}$*d.*; 4*s.*; 4*d.*; 1*s.* 6*d.*

(4) 18*s.* $2\frac{1}{2}$*d.* $\frac{2}{7}$*q.*; £1. 11*s.* 3*d.*; 10*s.* $3\frac{1}{4}$*d.* $\frac{5}{7}$*q.*; £90. 11*s.* 3*d.*

(5) 1*s.* 10*d.*; £1. 16*s.* $4\frac{3}{4}$*d.* $\frac{1}{5}$*q.*; 4*s.* 7*d.*; 1*s.* $11\frac{1}{2}$*d.* $\frac{4}{5}$*q.*

(6) 7*s.* $8\frac{1}{2}$*d.*; £3. 17*s.* 3*d.*; 1*s.* 4*d.*; £2. 2*s.*

(7) £2*s.* 6*s.* 8*d.*; £3. 14*s.* $8\frac{1}{2}$*d.* $\frac{8}{9}$*q.*; 8*s.* $9\frac{3}{4}$*d.* $\frac{24}{25}$*q.*

(8) 1 qr., 7 lbs.; 12 oz.; 6 fur., 88 yds.; 2 ro., 20 po.

(9) 3 fur., 25 po., 2 yds., 1 ft., 6 in.; 7 hrs., 12 mi.; 2 ft.; 3 qrs., 24 lbs.

(10) 7 lbs., 9 oz., $9\frac{3}{5}$ drs.; 1lb., 9oz.; 2gals., 1qt., $1\frac{1}{5}$ pt.; 4ac., 1ro., 2po., 3yds.; 1 ft. $94\frac{14}{19}$ in.

(11) 3hhds., 22gals., 2qts.; 2tuns, 1hhd., 31gals., 2qts.; 6bus., 3pks., $1\frac{5}{29}$gals.

(12) 2llds., 1comb, 2bus., 2pks., 1gal., $1\frac{1}{3}$qt.; 3 cub. yds., 5 cub. ft., $281\frac{89}{95}$ cub. in.; £9. 17*s.* 9*d.* (13) 5 hrs., 36 mi.,; £2.; 16*s.* $10\frac{1}{2}$*d.*

(14) £2. 16*s.* 3*d.*; 1*s.* $4\frac{1}{2}$*d.* $\frac{114}{121}$*q.* (15) £4.; 6*s.* 8*d.*

(16) £99. 17*s.* $0\frac{1}{2}$*d.*; £2. 7*s.* $6\frac{7}{44}$*d.*

2. (1) £1. 1*s.* 2*d.* (2) 8*s.* $9\frac{3}{4}$*d.* (3) 3*s.* $1\frac{1}{4}$*d.* $\frac{5}{42}$*q.* (4) 1*s.* $11\frac{3}{4}$*d.*

(5) 8*s.* $1\frac{1}{2}$*d.* (6) £4. 0*s.* $4\frac{3}{4}$*d.* (7) 11*s.* $5\frac{1}{2}$*d.* $\frac{2}{3}$*q.*

(8) £1. 6*s.* $11\frac{3}{4}$*d.* $\frac{5}{11}$*q.* (9) £1. 12*s.* $2\frac{1}{2}$*d.* $\frac{2}{11}$*q.*

(10) 10*s.* $11\frac{1}{4}$*d.* $\frac{17}{18}$*q.* (11) 11*s.* 9*d.* (12) £6. 15*s.* $10\frac{1}{4}$*d.* $\frac{1}{7}$*q.*

(13) £12. 2*s.* $11\frac{191}{893}$*d.* (14) 12 cwt., 2 qrs., 14 lbs., 10 oz., $10\frac{2}{3}$ drs.

(15) 1 lb., 1 oz., 12 dwts., $5\frac{1}{3}$ grs. (16) 4 fur., 39 po., 2 yds.

(17) 5 cub. ft., $110\frac{34}{37}$ cub. in. (19) 4 m., 4 fur., 1 po., $1\frac{1}{2}$ yds., $5\frac{2}{3}$ in.

(20) 7 yrs., 24 wks., 1 d., 34 m. (21) 4 ac., 1 ro., 23 po., $3\frac{3}{4}$ yds.

Ex. XLIX. (p. 142).

1. (1) $\frac{1}{3}$; $\frac{25}{168}$. (2) $\frac{5}{12}$; $\frac{27}{160}$. (3) $\frac{5}{216}$; $\frac{9}{16}$. (4) $\frac{1}{8}$; $\frac{97}{504}$.

(5) $\frac{39}{40}$; $\frac{1}{250}$. (6) $\frac{147}{16}$; $\frac{559}{651}$. (7) $\frac{7}{162}$; $\frac{20}{63}$. (8) $\frac{103}{2240}$; $\frac{495}{2}$.

(9) $\frac{11}{28}$; $\frac{2560}{1}$. (10) $\frac{59}{64}$; $\frac{3}{1100}$. (11) $\frac{761}{10560}$; $\frac{2617}{11664}$.

(12) $\frac{8}{15}$; $\frac{161}{480}$. (13) $\frac{11}{1120}$; $\frac{11}{1920}$. (14) $\frac{9}{38}$; $\frac{1681}{3840}$.

(15) $\frac{1159}{10368}$; $\frac{11}{576}$. (16) $\frac{9269}{480}$; $\frac{3}{8}$. (17) $\frac{293}{2880}$; $\frac{79}{480}$.

(18) $\frac{21}{20}$; $\frac{70}{1123}$. (19) $\frac{5}{6}$; $\frac{12663}{2}$. (20) $\frac{3025}{6596}$; $\frac{36}{1}$.

2. (1) $\frac{3}{16}$; $\frac{1}{60}$. (2) $\frac{5}{147}$; $\frac{7}{40}$. (3) $\frac{63}{100}$; $\frac{42}{5}$. (4) $\frac{21}{50}$; $\frac{5}{6}$.

(5) $\frac{1}{3}$; $\frac{2}{49}$. (6) $\frac{1}{400}$; $\frac{1}{3360}$. (7) $\frac{3}{896}$; $\frac{1}{2112}$. (8) $\frac{4}{5}$; $\frac{6}{11}$.

(9) $\frac{24}{25}$; $\frac{192}{175}$. (10) $\frac{1}{2240}$; $\frac{31}{184}$. (11) $\frac{27}{80}$; $\frac{12}{25}$. (12) $\frac{77}{108}$; $\frac{420}{79}$.

(13) $\frac{1225}{2304}$; $\frac{5}{81}$. (14) $\frac{5}{6}$; $\frac{3}{4}$. (15) $\frac{1}{1863}$; $\frac{4}{25}$. (16) $\frac{511}{1000}$; $\frac{288}{15625}$.

3. (1) $\frac{6}{49}$. (2) $\frac{1}{160}$. (3) $\frac{108}{125}$. (4) $\frac{28}{81}$. (5) $\frac{20}{9}$. (6) $\frac{7}{5400000}$.

(7) $\frac{213}{440}$. (8) $\frac{2822400}{61}$. (9) $\frac{39}{64}$. (10) $\frac{11}{24}$.

Ex. L. (p. 146.)

1. $\frac{8}{35}$ of a crown is the greatest, £$\frac{1}{19}$ the next, and $\frac{1}{20}$ of a guinea is the least. 2. The first two are equal, the third less.

3. $\frac{1}{25}$ of a day by $\frac{18}{175}$ hr. 4. $\frac{1}{15}$. 5. $\frac{92}{135}$. 6. $\frac{107}{840}$.

7. $\frac{817}{1600}$. 8. $\frac{45}{32}$. 9. $\frac{63}{400}$. 10. $\frac{233}{20}$.

Ex. LI. (p. 150.)

1. (1) 9*s.*; 3*s.* 4½*d.*; 17*s.* 6¼*d.* ·9968*q.*
(2) 5*s.* 7½*d.*; 15*s.* 11·088*d.*; 14*s.* 4½*d.*
(3) £5. 0*s.* 1½*d.*; 3*s.* 10½*d.* ·15*q.*; 11*s.*
(4) 13*s.* 1½*d.*; 7*s.* 7½*d.* ·48*q.*; 2*s.* 7½*d.*
(5) 1*s.* 2¼*d.* ·166714.; £2. 15*s.* 3½*d.* ·8128*q.*; 17*s.* 5¼*d.*
(6) £3. 0*s.* 11¾*d.* ·04*q.*; £4. 5*s.*; 5*s.* 9·12*d.*
(7) 19*s.* 8¼*d.*; 7*s.* 0½*d.*; £2. 17*s.* 3¾*d.*
(8) £1. 18*s.* 9¾*d.* ·192*q.*; £3. 4*s.* 2*d.*; £1. 7*s.* 7*d.*
(9) 2 m., 1100 yds.; 2 d., 12 hrs., 55′. 21″; 7 oz., 4 dwts.
(10) £24. 12*s.* 6·24999936*d.*; 4½ lbs.
(11) 3 qrs., 11 lbs., 4·56192 oz.; 8 lbs., 3·264 oz.; 14 po., 2 yds., 7·2 in.
(12) 4 tons, 3 cwt., 1 qr., 6 lbs., 2·56 oz.; 3 cwt., 2 qrs., 14 lbs.; 8 sq. po.
(13) 3 lbs., 10 oz., 5·568 grs.; 2 qrs., 3 bus., 3 pks.; 14 cwt., 12 lbs., 12·96 oz.
(14) 3 ro., 14 po.; 63 gals. (15) 37 po.; 9 d., 15 hrs.
(16) £506. 6*s.* 6*d.*; 19 qrs. (17) 7 ac., 3 ro., 20 p.; £3. 1*s.* 7½*d.*
(18) £5. 0*s.* 3¾*d.*; 2 m., 115 yds., 2·052 ft.
(19) 13 sq. yds., 1 sq. ft., 111·6 sq. in.; £126. 1*s.* 2·2465*d.*
(20) 4 m., 6 po., 1 yd., 2 ft., 11·97696 in.; £15. 16*s.* 6¾*d.* ·6*q.*

2. (1) 7*s.* 8*d.*; 9*s.* 5*d.*; 5 oz., 12 dwts., 16 grs.
   (2) 15*s.* 6*d.*; 1*s.* 5½*d.*; 13*s.* 4*d.* (3) 6*s.*; £2. 16*s.* 2*d.*
   (4) £6. 13*s.*; £9. 10*s.* 3¾*d.*
   (5) 6 sq. yds., 108 sq. in.; 3 fur. 10 po. 3 yds. 2 ft.; 20 d., 6 hrs.
   (6) $8\frac{19}{180}$ ac.; 20 hrs., 30 mi.

3. 7*s.*; 10*s.* 6*d.* 4. £1. 10*s.* 5. (1) £2. 16*s.* 5¼*d.*
   (2) £83. 7*s.* 2¾*d.* (3) £1. 2*s.* 9¾*d.* (4) 5*s.* 7·012*d.*
   (5) 152 wks., 5 d., 10 hrs., $54\frac{6}{11}$ sec. (6) 1 ro., 39 po., 28¼ sq. yds., $\frac{68}{550}$ sq. in.

6. ·0231 of a guinea.

Ex. LII. (p. 154.)

1. (1) $\cdot 31\dot{6}$; $\cdot 43541\dot{6}$. (2) ·23125; ·796875. (3) ·675; ·003125.
   (4) ·503125; $\cdot 057291\dot{6}$. (5) $\cdot 0847\dot{2}$; $\cdot 577\dot{3}8095\dot{2}$. (6) ·375; $\cdot 69\dot{4}$.
   (7) 1·35; 1·46875. (8) 3·590625; 1·43625.
   (9) $\cdot 2232\dot{1}4285\dot{7}$; $\cdot\dot{8}5714\dot{2}$. (10) 1·365; ·0000625.
   (11) $350\cdot\dot{9}\dot{0}$; 1·32531…… (12) $\cdot 2208\dot{3}$; $48\cdot 08\dot{3}$.
   (13) ·034375; ·30016741…… (14) $\cdot 273295\dot{4}\dot{5}$; $\cdot 07291\dot{6}$.
   (15) $\cdot 2785\dot{4}9382716\dot{0}$; ·875. (16) $\cdot 402\dot{7}$; ·61875.
   (17) $\cdot 67\dot{8}5714\dot{2}$; $\cdot 0002546\dot{2}9\dot{6}$. (18) $\cdot 9\dot{3}$; $\cdot 82\dot{2}8571\dot{4}$.
   (19) ·000015; $5\cdot\dot{8}5714\dot{2}$. (20) ·0334821……; 82·5.
   (21) $1\cdot 91\dot{6}$; 14·24. (22) $114\cdot\dot{5}\dot{4}$; ·000643……
   (23) 75·789……; $5212\cdot\dot{3}0769\dot{2}$. (24) ·01875; ·805; ·7317.

2. (1) ·45; $2\cdot\dot{1}4285\dot{7}$. (2) $\cdot 148\dot{8}0952\dot{3}$; $\cdot\dot{3}$. (3) ·28125; ·013671875.
   (4) ·225; ·511. (5) ·00243…; ·000080… (6) ·000304…; ·065625.
   (7) ·288; ·546875.

3. ·3821…… 4. ·0475. 5. $\cdot 09\dot{4}$. 6. $\cdot 102\dot{7}$. 7. 1·694……
   8. 2*s.* 3*d.*; ·45. 9. ·3140625.

10. (1) 2 c. 5 m. (2) 4 c. 1⅔ m. (3) 1 c. 8¾ m. (4) 2 fl. 5 c.
    (5) 5 fl. 2 c. 5 m. (6) 8 fl. (7) £5. 6 fl. 2 c. 5 m.
    (8) £54. 3⅔ fl. (9) £20. 9 fl. 8 c. 1¼ m. (10) 7 fl. 6 c. $9\cdot 791\dot{6}$ m.
    (11) 7 fl. 3 c. 4 m. (12) £2. 7 fl. 9 c. 6⅕ m. (13) £3. 4 c. 9 m.

Ex. LIII. (p. 160).

1. £80. 12*s.* 6*d.*; £36. 4*s.* 6*d.* 2. £49. 3*s.* 8*d.*; £5. 14*s.*
3. £9. 15*s.*; £35. 7*s.* 0¾*d.* 4. £271. 5*s.* 4*d.*; £172. 15*s.* 10*d.*
5. £17. 8*s.* 4*d.*; £10. 1*s.* 11½*d.* 6. £234. 12*s.*; £927. 13*s.* 6*d.*
7. £106. 17*s.* 6*d.*; £2600. 19*s.* 10*d.* 8. £651. 4*s.* 6*d.*; 11*s.*

9. £254. 7*s*. 6*d*.; £6899. 14*s*. 1*d*. 10. £170. 13*s*. 2¾*d*.; £790. 17*s*. 9*d*.
11. £2005. 16*s*. 8*d*.; £669. 3*s*. 9*d*. 12. £837. 3*s*. 11½*d*.; £398. 13*s*. 5¼*d*.
13. £14842. 16*s*.; £1737. 18*s*. 9¾*d*. 14. £720. 3*s*. 4½*d*.; £2358. 16*s*. 9½*d*.
15. £27820. 18*s*. 3*d*.; £82654. 9*s*. 1*d*. 16. £39061. 8*s*. 5¼*d*.; £247968. 12*s*. 5¼*d*.
17. £250014. 13*s*. 8¾*d*.; £44359. 1*s*. 10¾*d*.
18. £16982. 14*s*. 7½*d*.; £4981. 1*s*. 1¾*d*. 19. £1075. 8*s*. 8*d*.; £2173. 1*s*. 9*d*.
20. £552. 14*s*. 5⅛*d*.; £981. 19*s*. 7*d*. 21. £1927. 16*s*. 10½*d*.; £1903. 18*s*. 8¼*d*. $\frac{7}{9}$*q*.
22. £11342. 13*s*. $5\frac{5}{40}$*d*.; £5498. 9*s*. 10½*d*. $\frac{9}{11}$*q*.
23. £69241. 9*s*. $1\frac{1}{32}$*d*.; £24617. 6*s*. 1½*d*. $\frac{12}{13}$*q*.
24. £19. 7*s* 5¼*d*. $\frac{3}{20}$*q*.; £144. 14*s*. 0¾*d*. ⅛*q*.
25. £51. 12*s*. 9¼*d*. ⅓*q*.; £986. 17*s*. 10*d*.
26. £1775. 9*s*. 8½*d*. ⅔*q*.; £831. 3*s*. 2¾*d*. $\frac{1}{7}$*q*.
27. £10689. 17*s*. 8¾*d*. ⅕*q*.; £12126. 7*s*. 11½*d*. ⅔*q*.
28. £12. 15*s*. 11¼*d*. 29. £215. 16*s*. 8¾*d*. 30. £89. 6*s*. 1⅛*d*.
31. £467. 1*s*. 6¾*d*. $\frac{3}{7}$*q*. 32. £12. 5*s*. 10½*d*. $\frac{6}{7}$*q*. 33. £2. 15*s*. 11½*d*.
34. £147. 16*s*. 11¼*d*. $\frac{11}{56}$*q*. 35. £230. 16*s*. 8½*d*. ½*q*. 36. £4. 9*s*. 5¼*d*. $\frac{11}{12}$*q*.
37. £62. 8*s*. $6\frac{3}{16}$*d*. 38. £14. 19*s*. $10\frac{33}{224}$*d*. 39. £595. 6*s*. 11¾*d*. $\frac{3}{20}$*q*.
40. £51. 8*s*. $0\frac{3}{16}$*d*. 41. £308. 6*s*. 8¾*d*. $\frac{53}{70}$*q*. 42. £33. 8*s*. 3¾*d*. $\frac{181}{216}$*q*.
43. £8. 3. 4¾*d*. ½*q*.

Ex. LIV. (p. 170).

1. 3 sq. yds., 1 sq. ft., 60 sq. in. 2. 238 sq. ft., 90 sq. in. 3. 31 sq. ft., 87 sq. in.
4. $52\frac{26}{31}$ ft. 5. 32 ft. 6. 17 ft., 8 in. 7. $90\frac{6}{17}$ planks.
8. 683 sq. yds., 2 sq. ft., 125 sq. in. 9. £62. 5*s*. 5*d*. 10. £3. 1*s*. 5*d*.
11. £3. 15*s*. 6½*d*. 12. (1) £11. 15*s*. (2) £11. 6*s*. 5*d*. (3) £5. 0*s*. 7½*d*.
13. (1) 273 sq. ft., 63 sq. in. (2) 396 sq. ft., 60 sq. in. (3) 144 sq. ft., 8′. 5″. 3‴.
(4) 377 sq. ft., 10′. 5″. 7‴. 6⁗. (5) 42 sq. ft., 6′. 6″. 4‴. 11⁗.
(6) 2893 sq. ft., 8′. 4″. 10‴. (7) 274 sq. ft., 4′. 10″. 9‴.
14. $221\frac{11}{17}$ yds. 15. 26 yds., 0 ft., 4 in. 16. £10.
17. (1) £5. 2*s*. 9⅓*d*. (2) £13. 1*s*. 6¾*d*. ⅕*q*. (3) £12. 12*s*. 6¼*d*.
(4) £11. 15*s*. 6½*d*. ⅔*q*. (5) £13. 16*s*. $7\frac{11}{45}$*d*.
18. (1) 658 cub. ft., 936 cub. in.; £24. 13*s*. 10¾*d*. ½*q*.
(2) 4 cub. ft., 1088 cub. in.; £200. (3) 64 cub. yds.; £2. 8*s*.
(4) 1 cub. yd., 10½ cub. ft. (5) 826 cub. yds., 10½ cub. ft.
(6) $2533\frac{665875}{2985984}$ cub. ft.
19. 436 sq. ft., 136 sq. in.; 4915 cub. ft., 1080 cub. in.
20. 136 yds., 2 ft., 8 in. 21. £183. 12*s*. 4*d*.
22. (1) £6. $10\frac{1}{54}$*s*. (2) £4. 14*s*. 5¼*d*. ⅓*q*. (3) £1. 15*s*. 6¾*d*. $\frac{35}{54}$*q*.
(4) £3. 11*s*. 9*d*.; £30. 5*s*. 1⅓*d*.; £5. 8*s*. 4*d*.

23. 166 yds., 2 ft.; £6. 5*s*. 24. 9 ft. 25. £25. 7*s*. 2*d*.
26. $1\frac{217}{512}$ yds. 27. 96 sq. ft., 93 sq. in.; £4. 4*s*. $1\frac{47}{72}$*d*.
28. 156 sq. yds., 2 sq. ft., 36 sq. in. 29. 334 yds., $2\frac{5}{33}$ ft.
30. 391 sq. yds., 7 sq. ft., 12 sq. in. 31. $314\frac{1}{2}$ cub. ft.
32. 18 ft., 9 in.; $14\frac{1}{3}$ ft. 33. 1169 sq. ft.; £48. 14*s*. 2*d*.
34. 11520 bricks. 35. 64 tons, 9 cwt., 7 lbs. 36. $7\frac{3}{11}$ rods.
37. 4*s*. $10\frac{1}{4}$*d*. $\frac{11}{13}$*q*. 38. £9. 19*s*. $8\frac{1}{2}$*d*. $\frac{2}{3}$*q*.
39. 20 yds long, and 4 yds. broad. 40. $12\frac{1}{2}$ ft.
41. 9 yds., $1\frac{10}{27}$ ft.; £1. 15*s*. $5\frac{1}{2}$*d*. $\frac{2}{9}$*q*. 42. $24\frac{1}{216}$ cub. ft.
43. $4\frac{13}{14}$ yds. 44. 146625 stamps.
45. $256\frac{3}{4}$ sq. yds.; £15. 13*s*. $9\frac{1}{2}$*d*. $\frac{2}{3}$*q*. 46. 120000 bricks.
47. $22\frac{1}{2}$ ft. 48. 5044 bricks. 49. £69. 10*s*. 8*d*.
50. 182250. 51. £51. 9*s*; £126. 18*s*. 3*d*. 52. 12 ft., $6\frac{20}{21}$ in.
53. £37. 0*s*. $0\frac{3}{4}$*d*. 54. $469\frac{55247}{138637}$ gals.

Ex. LV. (p. 183.)

I.

1. $10{\cdot}51\dot{3}8461\dot{5}$. 2. £64. 4*s*. 3. ·625; ·3125.
4. £246. 6*s*. $7\frac{5}{8}$*d*.; £578. 19*s*. $1\frac{1}{4}$*d*. 5. £562. 3*s*. 2*d*.
6. £613. 2*s*. 8*d*. 7. £236. 8*s*. 6*d*. 8. 15*s*.
9. 2 francs, 8 centimes. 10. 10 wks.

II.

1. $\frac{10}{13}$. 2. $1668\frac{20}{27}$ metres. 3. *A*'s share = £11. 8*s*. $6\frac{3}{4}$*d*. $\frac{3}{7}$*q*., *B*'s share = £12. 17*s*. $1\frac{1}{2}$*d*. $\frac{6}{7}$*q*., *C*'s share = £15. 14*s*. $3\frac{1}{4}$*d*. $\frac{5}{7}$*q*. 4. ·0025.
5. £5118. 2*s*. 5*d*. 6. 10*d*. 7. 3361·3...rev. 8. £4. 17*s*. $\frac{17}{36}$*d*.
9. $5\frac{5}{11}$ hrs. 10. $74\frac{2}{27}$ hrs.

III.

1. 15 cwt., 2 qrs., 21 lbs.; £118. 3*s*. 7*d*.
2. 8·495; 8 chains, 4 chainlets, 9 links, 5 linklets.
4. 8*d*.; £3. 17*s*. 1*d*. 5. 5 mo. 6. £66. 19*s*. $9\frac{1}{2}$*d*.
7. 19*s*. $5\frac{1}{4}$*d*. $\frac{1}{3}$*q*. 8. 365·2425 days. 9. £50. 10*s*.

IV.

1. 26973. 2. $\cdot 0219\dot{2}3809\dot{5}$. 3. £138. 17*s*. $9\frac{1}{3}$*d*.; £236. 2*s*. $2\frac{2}{3}$*d*.
4. £36. 10*s*. $8\frac{1}{2}$*d*. $\frac{2}{7}$*q*. 5. 9 days. 6. £4. 4*s*. $2\frac{3}{4}$*d*.; 6*s*. $0\frac{11}{56}$*d*.
7. £395. 19*s*. 2*d*.; £116145. 16*s*. 8*d*. 8. 195 sq. yds.
9. *A*'s share = £600, *B*'s share = £480, *C*'s share = £320. 10. 8*s*.

V.

1. 374 quotient, and 446 remainder. 2. $\frac{42}{315}$, $\frac{225}{315}$, $\frac{81}{315}$; $1\frac{11}{105}$.
3. $3\frac{3373}{6144}$ sov. 4. £15468750. 5. $3\frac{3}{4}$ hrs.
6. 35 cub. ft.; 4 ft., $7\frac{10}{17}$ in. 7. 10 hrs., 12 m. 8. $7\frac{7}{10}$ days.
9. £4. 14*s.* 2*d.* 10. 15*s.* in the £.; £386. 14*s.* $4\frac{1}{2}$*d.*; £305. 5*s.*; £220.

VI.

1. £95335. 17*s.* 9*d.* 2. £986. 3. 10 yds., 11 in.
4. 1512 francs. 5. 1*s.* 6. 10*d.* 7. 9*d.*
8. £2. 8*s.* 8*d.* 9. $635\frac{19}{31}$ hrs. 10. £1000.

VII.

1. £2. 5*s.* 6*d.*; $\frac{91}{9500}$; ·0095......
2. 16; £4. 19*s.*; £9. 18*s.*; £19. 16*s.* 3. $129\frac{257}{987}$ yrs.
4. $5\frac{139}{540}$ days. 5. 1200 men. 6. £172. 0*s.* $9\frac{1}{2}$*d.* 7. £300.
9. 98. 10. 10234 fr., $66\frac{2}{3}$ cent.

VIII.

1. £536. 14*s.* 9*d.* 2. $15\frac{73}{96}$. 3. $\frac{1}{12}$. 4. £203. 16*s.* 2*d.*
5. $\frac{3}{1280}$; ·00234375. 6. 14 acres. 7. 8 cwt., $37\frac{1}{3}$ lbs. 8. $57\frac{3}{5}$ hrs.
9. 6*s.* 8*d.* 10. £79955. 3*s.* 4*d.*

IX.

1. 2143. 2. ·4; ·04. 3. Yes; 14*s.* 4. 80 days.
5. £19. 8 fl. 7 c. 7 m.; $\frac{1}{12}$ m. 6. 78 yds., 1 ft. 7. 4*s.*
8. The English hen. 9. 45 men. 10. 160 boys.

X.

1. ·661; ·017; 11·2. 2. $\frac{5}{2464}$. 4. £19. 1*s.* $10\frac{1}{2}$*d.*
5. $664\frac{8}{13}$ yds. 6. 6*d.* 7. £26. 14*s.* 9*d.*; 6*s.* 8. 26 yds., 4 in.
9. £66. 3*s.* $11\frac{1235}{1331}$*d.* 10. 24 days.

XI.

1. $\cdot\dot{0}8270676691729323\dot{3}$; £24. 16*s.* $2\frac{3}{4}$*d.* $\frac{73}{133}$*q.* 2. 10*d.*
3. £104. 16*s.* 4*d.* 4. 1*s.* $4\frac{1}{2}$*d.* 5. $2\frac{1}{4}$ in.
6. 25; 12·5; 5; 2·5; $1\cdot\dot{6}$; 1·25; $2\cdot08\dot{3}$. 7. $25\frac{9}{43}$ fr.; 213; $39\frac{181}{271}$.
8. $5\frac{1}{4}$ days. 9. 800,000 sheets. 10. 240 sov.; 720*s.*; 960*d.*

Ex. LVI. (p. 206.)

1. £14. 8*s*. 2. 72 yds. 3. 15*s*. 9*d*. 4. 182 ac. 5. £41. 17*s*.

6. £8. 5*s*. 9*d*. 7. 40 wks. 8. 12. 9. 12 cwt., 3 qrs.

10. £4. 10*s*. 8*d*. 11. £18. 12. £7. 10*s*. $2\frac{1}{2}$*d*. 13. £30. 12*s*. 6*d*.

14. 9*s*. 6*d*. 15. 11 qrs., 5 bus., 2 pks. 16. £75. 16*s*. $6\frac{3}{4}$*d*.

17. £690. 18. £25. 1*s*. 8*d*. 19. 17*s*. 6*d*. 20. 4 fl. 3 c. $7\frac{1}{2}$ m.

21. £1500. 22. £33. 7 fl. 23. 8*d*. 24. £2. 1 fl. 25. 50.

26. 8 days. 27. 3·8709...days. 28. $4677\frac{1}{2}$ yds. 29. £5012. 2 fl. 3 c. $7\frac{1}{2}$ m.

30. £104. 16*s*. 4*d*. 31. 18 days. 32. £47. 18*s*. 6*d*. 33. 8 men.

34. £5. 6*s*. 8*d*. 35. 8*d*; £3. 17*s*. 1*d*. 36. £6. 4*s*. $3\frac{3}{4}$*d*. 37. 24 yds.

38. 3 mo. 39. £172. 40. £4. 7 fl. 2 c. 5 m. 41. £92. 8*s*. $2\frac{4}{7}$*d*.

42. 11 yds. 43. £425. 5*s*. 44. $1\frac{11}{25}$ days. 45. 6 c. $8\frac{3}{4}$ m.

46. £8. 14*s*. $11\frac{1}{2}$*d*. $\frac{46}{625}$*q*. 47. £175. 7*s*. $7\frac{1}{4}$*d*. 48. £14. 14*s*. 49. £100.

50. 9*s*. $0\frac{3}{4}$*d*. 51. £1682. 52. £2710. 6*s*. 3*d*. 53. 10 h. 40′. $36\frac{7}{48}$″.

54. 70 ft. 8·232 in. 55. 9*s*. $8\frac{1}{2}$*d*.; £58. 13*s*. 3*d*. 56. $4\frac{68}{103}$ cwt.

57. £747. 10*s*. 58. £4. 10*s*. 59. 5*s*. 6*d*. 60. 15 hrs.

61. £40. 8*s*. 3*d*. 62. £157. 10*s*. 63. 208 days. 64. £1. 9 fl. 5 c.

65. $10\frac{1}{2}$ days. 66. $65\frac{5}{27}$ yds. 67. He loses $3\frac{1}{3}$*d*.

68. $5\frac{715}{1201}$′ before 4 o'clock. 69. £760. 70. 70 days.

71. $4166\frac{2}{3}$ yds. 72. 240000 lbs. 73. $2240\frac{2520}{3927}$. 74. £94. 10*s*.

75. 168 lbs. 76. Monday fortnight, at 6h. 36m., P.M. 77. 72 yds.

78. 13*s*. 4*d*. 79. 466650 lbs. 80. 100 days. 81. £26000.

82. £18. 10*s*. 3*d*., nearly. 83. 80 days. 84. £211. 19*s*. 3*d*.

85. £4. 4*s*. 86. $9\frac{1}{2}$ yds. 87. £39. 9*s*. $2\frac{1}{2}$*d*. $\frac{4}{5}$*q*. 88. 207; 82.

89. 3 days.

Ex. LVII. (p. 222.)

1. 15 men. 2. 7 men. 3. 66 days. 4. 7200 soldiers.

5. $19\frac{7}{89}$ bus. 6. 600 ac. 7. $18\frac{186}{811}$ mi. 8. $9\frac{19}{53}$ cwt.

9. £326. 13*s*. 4*d*. 10. 11 mo. 11. £61. 18*s*. 5*d*.

12. 11 cwt., 3 qrs., 14 lbs. 13. 12 hrs. 14. $8\frac{1}{3}$ wks. 15. £20.

16. 3 d., 6 hrs. 17. 10 hrs. 18. £50. 8*s*. 9*d*. 19. 500 reams.

20. 9 days. 21. 8 wks. 22. 64 days. 23. 350 men.

24. 2400 men. 25. 47 tons, 17 cwt., 66 lbs. 26. 324 men.

27. 48 days. 28. £332. 5*s*. $2\frac{4}{17}$*d*. 29. 2268 cub. ft. 30. 13*s*. 4*d*.

31. 8 ft. 32. £2. 3*s*. $1\frac{1}{2}$*d*. 33. 1320 yds. 34. $18\frac{18}{25}$ ft.

35. £97. 0*s*. $4\frac{4}{5}$*d*. 36. 35 days. 37. 19·36 days. 38. 49·3 lbs.

39. £509. 12*s*.

Ex. LVIII. (p. 228.)

1. (1) £4. 5*s*. (2) £12. 8*s*. (3) £45. (4) £71. 5*s*.
(5) £74. 18*s*. 6*d*. (6) £167. (7) £75. 12*s*. 9$\frac{1}{2}$*d*. $\frac{2}{5}$*q*. (8) £6. 18*s*. 10*d*.
(9) £17. 14*s*. 6*d*. (10) £1. 4*s*. 8$\frac{1}{4}$*d*. $\frac{1}{2}$*q*.

2. (1) £1085. (2) £3215. 16*s*. 8*d*. (3) £1318. 5 fl. 3 c. 4$\frac{3}{8}$ m.
(4) £166. 10*s*. 0$\frac{1}{2}$*d*. $\frac{7}{10}$*q*. (5) £2249. 13*s*. 2$\frac{1}{2}$*d*. $\frac{13}{20}$*q*. (6) £1615. 6*s*. 10$\frac{1}{4}$*d*. $\frac{103}{160}$*q*.
(7) £416. 10*s*. 3$\frac{1}{2}$*d*. $\frac{11}{20}$*q*. (8) £1939. 7 c. 3·828125 m.

3. (1) £48. 2*s*. 6*d*.; £423. 2*s*. 6*d*. (2) £72. 11*s*. 1$\frac{1}{2}$*d*.; £519. 1*s*. 1$\frac{1}{2}$*d*.
(3) £4. 16*s*. 3*d*.; £224. 16*s*. 3*d*. (4) £27. 19*s*. 0$\frac{17}{40}$*d*.; £271. 9*s*. 0$\frac{17}{40}$*d*.
(5) 2*s*. 4$\frac{497}{1825}$*d*.; £10. 12*s*. 4$\frac{497}{1825}$*d*. (6) £54. 3*s*. 0$\frac{61}{100}$*d*.; £739. 1*s*. 8$\frac{61}{100}$*d*.
(7) £1. 2*s*. 6$\frac{9}{73}$*d*.; £43. 2*s*. 6$\frac{9}{73}$*d*. (8) £31. 18*s*. 8$\frac{1}{4}$*d*. $\frac{156}{365}$*q*.;
£352. 13*s*. 8$\frac{1}{4}$*d*. $\frac{156}{365}$*q*. (9) 6*s*. 1$\frac{1}{2}$*d*. nearly; £34. 16*s*. 1$\frac{1}{2}$*d*. nearly.

4. £32. 4*s*. 0$\frac{3}{4}$*d*. 5. £20. 6. £2000. 7. £567. 11*s*. 4$\frac{8}{37}$*d*.

Ex. LIX. (p. 230.)

1. £125. 6*s*. 8*d*. 2. 4 per cent. 3. 5 years.
4. 4$\frac{1}{4}$ per cent. 5. 17 yrs. 6. 3$\frac{1}{2}$ per cent.
[illegible] £560. 8. £91. 13*s*. 4*d*. 9. £345. 17*s*. 6*d*.
10. 5$\frac{2}{3}$ years. 11. 4$\frac{5}{6}$ per cent. 12. 20 years.
13. £315. 10*s*. 8*d*.; 2$\frac{1}{2}$ years. 14. £225.

Ex. LX. (p. 232.)

1. £163. 4*s*. 2. £893. 8*s*. 4$\frac{1}{8}$*d*. 3. £16. 8*s*. 10$\frac{1}{4}$*d*. $\frac{7}{25}$*q*.
4. £787. 8*s*. 1$\frac{1103}{10000}$*d*. 5. 267. 2*s*. 5$\frac{1}{4}$*d*. $\frac{9}{100}$*q*. 6. £1. 5*s*. 7$\frac{1}{2}$ nearly.
7. £16. 8*s*. 0$\frac{1}{4}$*d*. nearly. 8. £1942. 4*s*. 9$\frac{1}{2}$*d*. nearly. 9. £714. 8*s*. 7$\frac{1}{2}$*d*. $\frac{12}{25}$*q*.
10. 4*s*. 2$\frac{1}{2}$*d*. $\frac{1}{2}$*q*. 11. £350. 8*s*. 5*d*. 12. £205. 8*s*. 2$\frac{1883}{5000}$*d*.
13. £90. 14$\frac{26}{441}$*s*. 14. £240. 15. £1·578528.

Ex. LXI. (p. 236.)

1. (1) £270. (2) £245. (3) £666. 13*s*. 4*d*. (4) £280.
(5) £450. (6) £382. 14*s*. 10$\frac{2522}{4891}$*d*. (7) £558. 0*s*. 11$\frac{29}{41}$*d*. (8) £1260.
(9) £34. 9*s*. 7$\frac{175}{203}$*d*. (10) £1239. 3*s*. 1$\frac{621}{807}$*d*. (11) £2000.
(12) £262. 4*s*. 5$\frac{1}{4}$*d*. $\frac{1}{2}$*q*. (13) £765. (14) £462. 9*s*. 5$\frac{3653}{10379}$*d*.
(15) £400. (16) £1953. 2*s*. 6*d*.

2. (1) 16*s*. 8*d*. (2) £30. 7*s*. 6*d*. (3) £2. 12*s*. $3\frac{32}{41}$*d*.
(4) £1. 1*s*. $9\frac{213}{307}$*d*. (5) £7. (6) £5. 11*s*. $5\frac{43}{61}$*d*. (7) £140.
(8) £48. 9*s*. (9) £16. 11*s*. $8\frac{200}{257}$*d*. (10) £1. 0*s*. $6\frac{126}{1019}$*d*.
(11) 2*s*. 11*d*. nearly. (12) 15*s*. $3\frac{123759}{147127}$*d*. per cent. (14) 20 per cent.
(15) £2. 6*s*. 8*d*.

Ex. LXII. (p. 240).

1. (1) £3800. (2) £800. (3) £525. (4) £950. (5) £4300.
(6) £597. 0*s*. $3\frac{39}{67}$*d*. (7) £1059. $12\frac{8}{151}$*s*. (8) £5050. (9) £$2234\frac{2}{47}$.
2600. (11) £3091. 10*s*. $2\frac{14}{53}$*d*. (12) £10566. 0*s*. $9\frac{3}{53}$*d*.
1) £2418. (2) £1488. (3) £2775. (4) £1834.
10. 4*s*. $9\frac{3}{5}$*d*. (6) £1444. 10*s*. $2\frac{1}{2}$*d*. $\frac{4}{5}$*q*. (7) £2450. 6*s*. (8) £972. 10*s*.
1*s*. $7\frac{1}{2}$*d*.
£36. (2) £240. (3) £55. 6 fl. 8 c. $7\frac{1}{2}$ m. (4) £78. 15*s*.
15*s*. $11\frac{11}{59}$*d*. (6) £112. 0*s*. $3\frac{123}{367}$*d*. (7) £159. 12*s*.
8*s*. $1\frac{151}{377}$*d*. (9) £178. 15*s*. $2\frac{766}{2467}$*d*.
£1700. (2) £6432. (3) £1800. (4) £1739.
. 2*s*. 6*d*. (6) £875. 7*s*. $9\frac{3}{4}$*d*.
£3. 5*s*. $11\frac{19}{91}$*d*. (2) £3. 14*s*. $5\frac{29}{47}$*d*. (3). £4. 13*s*. $1\frac{559}{773}$*d*.
. $1\frac{5}{103}$*d*.
13*s*. 7. £3. 6*s*. 8*d*.; £3. 15*s*.; 8*s*. 4*d*.
s. $9\frac{27}{49}$*d*. 9. $77\frac{7}{9}$; £1542. 17*s*. $1\frac{5}{7}$*d*. 10. £1000.
s. 12. £1575. 13. £104. 8*s*. 4*d*.
. $1\frac{601}{707}$*d*. 15. £2. 10*s*. 16. £175; £170. 9*s*. $1\frac{1}{11}$*d*.
£11970. 18. £$2729\frac{1}{131}$; £5. 10*s*. $3\frac{87}{131}$*d*.
er cents. 20. £6. 5*s*.
per cents.; $10\frac{5}{37}$*d*. 22. £7. 15*s*. 4*d*.
13*s*. 4*d*. 24. The Railway Shares. 25. £22. 2*s*. 6*d*.
er cents.; £13397. 10*s*.; £13980. 27. £12319. 6*s*. $3\frac{45}{233}$*d*.
£6310. 2*s*. 6*d*. 29. £32. 5*s*. 30. $85\frac{3}{4}$.
. $7\frac{11}{31}$*d*. 32. £20,000; £225000.

Ex. LXIII. (p. 246).

s. 11*d*. 2. 8*s*. 8*d*. 3. £78. 0*s*. 10*d*.
ins £15. 12*s*. 6*d*. per cent. 5. $4\frac{1}{2}$*d*. 6. £9. 2*s*. $8\frac{368}{821}$*d*.
l*s*. $0\frac{1}{4}$*d*. $\frac{53}{91}$*q*. 8. £10. 14*s*. $3\frac{1}{4}$*d*. $\frac{5}{7}$*q*. 9. £10. 5*s*.
5*s*. 8*d*. 11. £30. 12. 5*s*. $3\frac{3}{19}$*d*.; 5*s*. 6*d*. 13. 8*s*. 9*d*.

Every human born into the world
inherits the same fate as all men
[illegible]

14. £37. 1*s*. 11$\frac{1}{2}$*d*. $\frac{2}{17}$*q*. 15. £25; £2. 2*s*. 1$\frac{1}{4}$*d*. $\frac{1}{19}$*q*. 16. £27.
17. 14*s*. 4$\frac{4}{5}$*d*. 18. £3. 18*s*. 19. 4*s*. 7$\frac{1}{5}$*d*. 20. £27.
21. £648. 7*s*. 6*d*. 22. £3. 7*s*. 6*d*.; £14. 5*s*. 8$\frac{1}{2}$*d*. $\frac{2}{7}$*q*.
23. £96. 7*s*. 3$\frac{3}{11}$*d*. 24. £16. 25. 1*s*. 10$\frac{1}{2}$*d*.; 2*s*. 1$\frac{1}{4}$*d*. $\frac{1}{4}$*q*.; 3*s*. 9*d*.
26. £550. 5*s*. 11$\frac{1}{169}$*d*.; £18. 6*s*. 10$\frac{62}{169}$*d*. 27. £96. 12*s*.; 12*s*, 3$\frac{1}{5}$*d*.
28. 30 quarters. 29. 12$\frac{1}{2}$ per cent. 30. £40.

Ex. LXIV. (p. 252).

1. 6, 28, and 38. 2. £4. 3*s*. 9*d*.; £13. 8*s*. 3. 516, 860, 1204, 1892; £149. 11*s*. 5$\frac{17}{39}$*d*.; £179. 9*s*. 8$\frac{12}{13}$*d*.; £170. 18*s*. 9$\frac{25}{39}$*d*.
4. £179. 8*s*. 8*d*.; £142. 9*s*.; £99. 3*s*. 4*d*.
5. 12 cwt., 30$\frac{6}{11}$ lbs.; 3 cwt., 30$\frac{6}{11}$ lbs.; 2 cwt., 50$\frac{10}{11}$ lbs.
6. £396; £324.
7. 1116·7744 lbs. of oxygen, 969·136 lbs. of carbon, 154·0896 lbs. of hydrogen.
8. £3250; £2166. 13*s*. 4*d*.; £1083. 6*s*. 8*d*. 9. £350; £450.
10. £12. 10*s*., £12. 10*s*., £25, £50.
11. *A*'s share = £5000, *B*'s share = £3750, *C*'s share = £3125.
12. £1350. 13. 5$\frac{11}{20}$ months. 14. 4$\frac{5}{8}$ months. 15. 12 months.
16. £3. 10*s*. 17. £126. 11*s*. 3*d*.
18. *A* ought to have £80, *B* £90, and *C* £84.

Ex. LXV. (p. 255.)

1. 35641 francs, 6$\frac{7}{8}$ centimes. 2. £1271. 13*s*. 9$\frac{49}{100}$*d*.
3. 1246 pias, 6$\frac{86}{95}$ rials. 4. £566. 13*s*. 4*d*.
5. 33*s*. 4*d*. 6. 1 milree = 54$\frac{87}{172}$*d*.
7. The direct way. 8. £19. 10*s*. 7$\frac{1}{2}$*d*.; 25 francs.
9. He gains £11. 5*s*. 10. Through France.

Ex. LXVI. (p. 263.)

1. (1) 17; 24; 38; 64. (2) 81; 145; 416. (3) 314; 193; 108. (4) 999; 989; 908. (5) 5432; 3789; 2312. (6) 15367; 531441; 16807. (7) 543200; 2039750.

2. (1) 12·96; 5·37; 240·1. (2) ·59049; 6·2573. (3) ·207; ·0374; ·0451. (4) 2403; 2·403. (5) 347·6905; 490·304.

3. (1) 4; 1·2649...; ·4; ·1264...... (2) 15·3492...; ·3162...; ·1; 2·2360...; ·7071...... (3) ·02; ·0284...; 19·4901...... (4) $4\frac{1}{2}$; 12·4007...; ·5773...; $\frac{47}{99}$. (5) ·7745...; ·2425...; 1·4719...; ·8819...... (6) 15·4919...; $1\frac{2}{13}$; 4·8062......; 6·4807......

4. ·04375; $\frac{7}{160}$. 5. 1400 yds. 6. $74\frac{1}{4}$ yds. 7. 255 yds.

8. $28\frac{1}{2}$ yds. 9. 45 yds. 10. $84\frac{3}{4}$ ft. 11. ·07122; 98 yds., 1 ft., 1 in.

12. 90 miles. 13. 47·5...ft. 14. 105. 15. 543·2 yds.

Ex. LXVII. (p. 271.)

1. (1) 12; 15; 31. (2) 38; 48; 67. (3) 88; 93; 98. (4) 134; 411; 203. (5) 631; 305; 364. (6) 258; 638; 975. (7) 3002; 6031.

2. ·73; 3·19; 45·7; ·097; ·124; ·029.

3. (1) 1·442...; ·669...; ·316...... (2) $\frac{2}{3}$; $\frac{5}{7}$; 3·546......
(3) $7\frac{2}{5}$; 1·930...; 1·442...... (4) ·046...; ·425.

4. 6·16; 1·232. 5. Each edge = 27·2 in. 6. 1369 sq. ft. 7. 3 ft., 10 in.

8. £25. 18*s*. $11\frac{7}{25}$*d*. 9. £10. 0*s*. 1*d*. 11. 2·8 ft. nearly.

12. 81; ·04......

THE END.

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